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One-Group Perturbation Theory Applied to Measurements with Void

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ONE-GROUP PERTURBATION THEORY APPLIED TO SUBSTITUTION MEASUREMENTS WITH VOID

Rolf Persson

SUMMARY

Formulas suitable for evaluating progressive as well as single-rod substitution measurements are derived by means of one-group perturbation theory. The diffusion coefficient may depend on direction and position. By using the buckling concept one can derive expressions which are quite simple and the perturbed flux can be taken into account in a comparatively simple way. By using an unconventional definition of cells a transition region is introduced quite logically.

Experiments with voids around metal rods, diam. 3.05 cm, have been analysed. The agreement between extrapolated and directly measured buckling values is excellent, the buckling difference between lattices with water-filled and voided shrouds being 0.263 ± 0.015 m^{-2} and 0.267 ± 0.005 m^{-2} resp. From single-rod experiments differences between diffusion coefficients are determined to $\delta D_r/D = 0.083 \pm 0.004$ and $\delta D_z/D = 0.120 \pm 0.018$. With air-filled shrouds there is consequently anisotropy in the neutron diffusion and we have $(D_z/D_r)_{air} = 1.034 \pm 0.020$.

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1. INTRODUCTION

Substitution measurements are performed in order to get the material buckling of a whole lattice of some special fuel by replacing only a small part (5 to 15 %) of a reference lattice by the fuel elements to be investigated. By increasing the size of the test region in successive steps it is possible to make an appropriate extrapolation. The object of these notes is to derive formulas to be used when one-group perturbation theory is applied. The effect of different diffusion coefficients in different directions and in different parts of the core is taken into account. Experiments with voids around metal rods are also presented.

2. THEORY

2.1 Buckling perturbation. General case

The one-group diffusion equation*)

$$\nabla(D\nabla\phi) + (k-1)\Sigma_a \phi = 0 \qquad (1)$$

may in the case of r-z geometry be written as

$$D_r \nabla_r^2 \phi + D_z \nabla_z^2 \phi + (k-1)\Sigma_a \phi = 0 \qquad (2)$$

where ∇_r^2 and ∇_z^2 are the radical and axial parts of the Laplacian operator respectively. If we put $(k-1)\Sigma_a = D_r B_r^2$, we get

$$D_r \nabla_r^2 \phi + D_z \nabla_z^2 \phi + D_r B_r^2 \phi = 0 \qquad (3)$$

The solution of eq. (3) taking only the fundamental mode into account is

$$\phi = A \cdot J_o(\beta\, r) \cdot \sin \alpha z \qquad (4)$$

The relation between β and α is given by

$$\beta^2 + \frac{D_z}{D_r}\alpha^2 = B_r^2 \qquad (5)$$

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*) $D\nabla\phi$ means the vector $(D_x\, \partial\phi/\partial x;\ D_y\, \partial\phi/\partial y;\ D_z\, \partial\phi/\partial z)$.

It means obviously that there is a linear relationship between the radial buckling (β^2) and the axial buckling (α^2). A comparison between measurements in critical and exponential assemblies will then give information about the degree of anisotropy (D_z/D_r). However, similar information can also be found in a relatively simple way from perturbation measurements in a critical facility.

If a reactor is perturbed, but the perturbation is counterbalanced in one or another way in order to have the reactor critical all the time, this condition can be expressed [1] as

$$\int \phi \underline{P} \phi' dV = 0 \qquad (6)$$

where $\underline{P}$ is an operator representing the changes made in the core, ϕ is the unperturbed flux and ϕ' is the perturbed flux.

When analysing substitution measurements we make the following assumptions:

a) The core, which in reality consists of different regions, is at first assumed to be a uniform unit with certain fictitious properties (B_r^2, D_r, D_z).

b) The core is then divided into its regions (extra index $\underline{i}$) with their different properties (B_{ri}^2, D_{ri}, D_{zi}). Every region is considered as a perturbation to the core as a whole and has then an operator $\underline{P}_i$, which can be found from eq. (1),

$$\underline{P}_i = \nabla(D_i \nabla) - \nabla(D\nabla) + D_{ri}B_{ri}^2 - D_r B_r^2 \qquad (7)$$

where $D_r B_r^2 = (k-1)\Sigma_a$.

Eq. (6) may now be written as

$$\sum_i \int \phi \underline{P}_i \phi' dV_i =$$

$$= \sum_i \int [\phi \nabla \{(D_i - D)\nabla\phi'\} + (D_{ri}B_{ri}^2 - D_r B_r^2)\phi\phi']dV_i = 0 \qquad (8)$$

Since the values of D_r and D_z are quite arbitrary, we may also let them disappear (though D_z/D_r remains finite in order to satisfy eq. (5)) and eq. (8) should still be valid. We then get*)

$$\sum_i \int [\phi \nabla(D_i \nabla \phi') + D_{ri} B^2_{ri} \phi \phi'] dV_i = 0 \tag{9}$$

The first part of eq. (9) is transformed as follows

$$\sum_i \int \phi \nabla (D_i \nabla \phi') \, dV_i = \sum_i [\int \nabla(\phi D_i \nabla \phi') \, dV_i - D_i \int \nabla \phi \nabla \phi' dV_i] =$$

$$= - \sum_i D_i \int \nabla \phi \nabla \phi' dV_i \tag{10}$$

The sum

$$\sum_i \int \nabla(\phi D_i \nabla \phi') \, dV_i = \sum_i \int \phi D_i \nabla \phi' dS_i = 0$$

because

$$D_i \nabla \phi_i' = D_{i+1} \nabla \phi_{i+1}' \tag{11}$$

on the common boundary between the regions i and (i + 1) and $\phi = 0$ on the outer boundary.

For brevity the following symbols are introduced assuming that the radial and axial parts of the flux are separable.

$$\int_{V_i} \phi \phi' dV / \int_{\Sigma V_i} \phi \phi' dV = w_{ri} w_{zi} \tag{12}$$

$$\int_{V_i} (\nabla_r \phi \nabla_r \phi') dV / \int_{\Sigma V_i} \phi \phi' dV = \beta^2 \cdot u_{ri} w_{zi} \tag{13}$$

$$\int_{V_i} \nabla_z \phi \nabla_z \phi' dV / \int_{\Sigma V_i} \phi \phi' dV = \alpha^2 \cdot w_{ri} u_{zi} \tag{14}$$

where $\beta = 2.405/R$ and $\alpha = \pi/H$ (R and H are extrapolated quantities).

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*) The result given in eq. (9) can also be obtained by multiplying the equation $\nabla(D_i \nabla \phi_i') + D_{ri} B^2_{ri} \phi_i' = 0$ with ϕ, integrating over the volume of region i and taking the sum over all regions.

If ϕ' is replaced by the approximately equal ϕ, as given by eq. (4), we have

$$\int_{V_i} \phi\phi' dV \Big/ \int_{\Sigma V_i} \phi\phi' dV = C \cdot \int J_o^2(\beta r)\, dS_r \int \sin^2 \alpha z\, dz \tag{12a}$$

$$\int_{V_i} \nabla_r \phi \nabla_r \phi' dV \Big/ \int_{\Sigma V_i} \phi\phi' dV = C \cdot \beta^2 \int J_1^2(\beta_r)\, dS_r \int \sin^2 \alpha z\, dz \tag{13a}$$

$$\int_{V_i} \nabla_z \phi \nabla_z \phi' dV \Big/ \int_{\Sigma V_i} \phi\phi' dV = C \cdot \alpha^2 \int J_o^2(\beta r)\, dS_r \int \cos^2 \alpha z\, dz \tag{14a}$$

where $C^{-1} = \pi R^2 \cdot J_1^2(\beta R) \cdot H/2$. Thus the functions $\underline{w}$ and $\underline{u}$ are found to be

$$w_r = \int J_o^2(\beta r)\, dS_r / \pi R^2 J_1^2(\beta R).$$

$$u_r = \int J_1^2(\beta r)\, dS_r / \pi R^2 J_1^2(\beta R)$$

$$w_z = \int \sin^2 \alpha z\, dz/(H/2)$$

$$u_z = \int \cos^2 \alpha z\, dz/(H/2).$$

Bearing in mind eqs. (5) and (10) and using the symbols $\underline{w}$ and $\underline{u}$ as defined in eqs. (12) - (14) we find that eq. (9) takes the following form, if we define α_i^2 as being the axial buckling of region $\underline{i}$, when its radial buckling is β^2.

$$\sum_i [D_{ri}\beta^2(w_{ri} - u_{ri})\, w_{zi} + D_{zi}(\alpha_i^2 w_{zi} - \alpha^2 u_{zi})\, w_{ri}] = 0 \tag{15}$$

If perturbed fluxes are used in the definition of $\underline{w}$ and $\underline{u}$ there is no requirement that the perturbations have to be small. However, perturbed fluxes can only be used here when the problem is separable in $\underline{r}$ and $\underline{z}$.

2.2 Separable fluxes. Progressive substitutions

A purely *axial* perturbation gives

$$\sum_i D_{zi}(\alpha_i^2 w_{zi} - \alpha^2 u_{zi}) = 0 \tag{16}$$

since

$$w_{ri} = u_{ri} = 1$$

This case is of no interest in connection with substitution measurements and will not be discussed further. More interesting is a purely radial perturbation, which gives

$$\Sigma\,[D_{ri}\,\beta^2(w_{ri} - u_{ri}) + D_{zi}(\alpha_i^2 - \alpha^2)\,w_{ri}] = 0 \tag{17}$$

since

$$w_{zi} = u_{zi} = 1$$

If all D_i:s are equal, eq. (17) changes to

$$\sum_i (\alpha^2 - \alpha_i^2)\,w_{ri} = 0 \tag{18}$$

since

$$\sum_i u_{ri} = \sum_i w_{ri} = 1$$

Eq. (18) may also be written as

$$\alpha^2 - \alpha_1^2 = \sum_i (\alpha_i^2 - \alpha_1^2)\,w_{ri} \tag{19}$$

Because $\sum_i (w_{ri} - u_{ri}) = 0$ the first part of eq. (17) may be rewritten as follows

$$\sum_i D_{ri}\,\beta^2(w_{ri} - u_{ri}) = \sum_i (D_{ri} - D_{r1})\,\beta^2(w_{ri} - u_{ri}) \tag{20}$$

If we now let index i=1 represent the reference region and put $\delta D_{ri} = D_{ri} - D_{r1}$ eq. (17) may be given in the form

$$\sum_i (\alpha^2 - \alpha_i^2)\,w_{ri}\,D_{zi} = \beta^2 \sum_i (w_{ri} - u_{ri})\,\delta D_{ri} \tag{21}$$

In substitution measurements we get differences relative to the reference lattice (i=1) and eq. (21) may therefore be transformed to

$$(\alpha^2 - \alpha_1^2)\,(1 + \sum_i w_{ri} \cdot \delta D_{zi}/D_{z1}) =$$

$$= \sum_i (\alpha_i^2 - \alpha_1^2)\, w_{ri}(1 + \delta D_{zi}/D_{z1}) + \beta^2 \sum_i (w_{ri} - u_{ri})\, \delta D_{ri}/D_{z1} \qquad (22)$$

Here the values of $\alpha_i^2 - \alpha_1^2$ are sought and $\alpha^2 - \alpha_1^2$ is the change of the axial buckling found in the experiment.

2.3 The effect caused by $\delta D/D \neq 0$

In order to get some idea of the magnitude of the effect caused by the factors $\delta D_r/D$ and $\delta D_z/D$ we assume that i=1 and 2 only. Eq. (22) is transformed to

$$\alpha_2^2 - \alpha_1^2 = \frac{\alpha^2 - \alpha_1^2}{w_{r2}} \cdot \frac{1 + w_{r2} \cdot \delta D_{z2}/D_{z1}}{1 + \delta D_{z2}/D_{z1}} - \beta^2 \, \frac{\delta D_{r2}/D_{z1}}{1 + \delta D_{z2}/D_{z1}} \cdot \left(1 - \frac{u_{r2}}{w_{r2}}\right) \approx$$

$$\approx \frac{\alpha^2 - \alpha_1^2}{w_{r2}} - \left[\frac{\alpha^2 - \alpha_1^2}{w_{r2}} (1 - w_{r2}) \frac{\delta D_{z2}}{D_{z2}} + \beta^2 \cdot \frac{\delta D_{r2}}{D_{z2}} \right] \qquad (23)$$

If $w_{r2} \ll 1$ and $\delta D_{z2} = \delta D_{r2} = \delta D$ we have

$$\alpha_2^2 - \alpha_1^2 \approx \frac{\alpha^2 - \alpha_1^2}{w_{r2}} - [(\alpha_2^2 - \alpha_1^2) + \beta^2]\, \frac{\delta D}{D} \qquad (24)$$

Since $\beta^2 \approx 4.4\ \text{m}^{-2}$ in R0, we find that, if $\delta D/D = 0.01$ but $\alpha_2^2 - \alpha_1^2 = 0$, the last term in eq. (24) is $\beta^2 \cdot \delta D/D = 0.044\ \text{m}^{-2}$. Such an effect is therefore quite important when high accuracy is wanted. In order to get a buckling value with an accuracy of $\pm 0.01\ \text{m}^{-2}$ we have to know $\delta D/D$ within ± 0.002.

2.4 Single-rod perturbations. Evaluation of $\delta D/D$

The change δD of the one-group diffusion coefficient D_r or D_z may be estimated by calculations, but could preferably be measured in separate experiments with a single fuel element. The perturbation is either moved in the radial direction or gradually inserted in the axial direction. But in these cases the solution of ϕ' is not separable in $\underline{r}$ and $\underline{z}$ or there are azimuthal terms. Thus we use the unperturbed flux and the relations according to eqs. (12a), (13a) and (14a).

When i=1 and 2 only and i=1 represents the reference region, eq. (15) may be transformed as follows

$$\frac{\alpha^2-\alpha_1^2}{w_{r2}w_{z2}} = (\alpha_2^2-\alpha_1^2)\left(1+\frac{\delta D_{z2}}{D_{z1}}\right) - \beta^2\frac{\delta D_{r2}}{D_{z1}}\left(\frac{u_{r2}}{w_{r2}}-1\right) - \alpha_1^2\frac{\delta D_{z2}}{D_{z1}}\left(\frac{\alpha^2}{\alpha_1^2}\frac{u_{z2}}{w_{z2}}-1\right) \qquad (25)$$

In order to evaluate $\delta D_{r2}/D_{z1}$ and $\delta D_{z2}/D_{z1}$ from experiments one may use graphical methods as shown below. These provide a simple and clear analysis of the experimental data.

a) Determination of $\delta D_{r2}/D_{z1}$

Assume a perturbation (fuel element) which penetrates the whole core in the axial direction ($u_{z2}=w_{z2}=1$) and which is placed in various radial positions. Eq. (25) now takes the form

$$(\alpha^2-\alpha_1^2)\left(\frac{1}{w_{r2}}+\frac{\delta D_{z2}}{D_{z1}}\right) = (\alpha_2^2-\alpha_1^2)(D_{z2}/D_{z1}) - \beta^2\frac{\delta D_{r2}}{D_{z1}}\left(\frac{u_{r2}}{w_{r2}}-1\right) \qquad (26)$$

If we plot the left-hand side of eq. (26) versus $(u_{r2}/w_{r2})-1$, we have a linear relationship. The slope gives $\beta^2\ \delta D_{r2}/D_{z1}$ and the intercept $(\alpha_2^2-\alpha_1^2)(D_{z2}/D_{z1})$.

b) Determination of $\delta D_{z2}/D_{z1}$

Assume that a perturbation (fuel element or void) parallel to the axis is gradually inserted into the core. In this case $(\alpha^2-\alpha_1^2)/w_{r2}w_{z2}$ may be plotted against $(\alpha^2/\alpha_1^2)(u_{z2}/w_{z2})-1$. The slope of this linear function gives $\alpha_1^2\ \delta D_{z2}/D_{z1}$ according to eq. (25).

3. EXPERIMENTS

3.1 General

The formulas derived in the preceding section have been used in order to evaluate experiments with voids in shrouds (aluminium, ID 6.3 cm, OD 6.5 cm) around the reference fuel in R0 (metal rods, diam. 3.05 cm, canned in aluminium, ID 3.15 cm, OD 3.45 cm). There were 112 elements placed in a square lattice with pitch 19.0 cm. All the rods were supplied with shrouds, which individually could be either open at the bottom or closed by means of a pneumatic valve [2].

In order to analyse substitution measurements by means of one-group theory it is usually necessary to introduce a transition region [3]. By using an unconventional definition of cells as illustrated in fig. 1 it is possible to find a transition region in quite a logical way.

3.2 Single-rod experiments

The axial effect of a void in one shroud was measured as a function of the void depth. The experimental values were analysed according to eq. (25), see fig. 2. We got the result $\delta D_{z2}/D_{z1} = 0.060 \pm 0.009$ (valid for a transition cell). A test cell (index 3) as shown in fig. 1 should give twice the effect, i.e. $\delta D_{z3}/D_{z1} = 0.120 \pm 0.018$.

A single element with void was placed in different radial positions. The variation of the critical height was analysed according to eq. (26) and is shown in fig. 3. In this case the accuracy was higher than in the axial measurements, because the perturbation due to the void surrounded the total length of an element during the whole experiment, i.e. the perturbation was larger. The result was $\delta D_{r2}/D_{z1} = 0.0416 \pm 0.0020$ and consequently $\delta D_{r3}/D_{z1} = 0.0832 \pm 0.0040$.

With air-filled shrouds we have $D_z/D_r = 1.034 \pm 0.020$, i.e. there is anisotropy in the neutron migration due to the empty channels.

From the intercept in fig. 3 we get $(D_{z2}/D_{z1})\,(\alpha_2^2 - \alpha_1^2) = -0.133 \pm \pm 0.010\ \mathrm{m}^{-2}$ and since $D_{z2}/D_{z1} = 1.060 \pm 0.009$, we have $\alpha_2^2 - \alpha_1^2 = -0.125 \pm 0.010\ \mathrm{m}^{-2}$. Assuming $\alpha_3^2 - \alpha_1^2 = 2\,(\alpha_2^2 - \alpha_1^2)$ we obtain $\alpha_3^2 - \alpha_1^2 = -0.250 \pm 0.020\ \mathrm{m}^{-2}$. This value has to be compared with the result obtained from the analysis of the substitution measurements (see 3.3 below).

We exchange the indices 1 and 3 (or 3 and 1) for e and f representing shrouds with void (empty) and without void (filled) respectively. The differences between diffusion coefficients to be used in the analysis of the substitution measurements are collected in tables 1 and 2.

Table 1. Differences between diffusion coefficients having elements without void as reference (experimental values)

$\frac{D_{z2} - D_{zf}}{D_{zf}}$ δ	$\frac{D_{ze} - D_{zf}}{D_{zf}}$	$\frac{D_{r2} - D_{rf}}{D_{zf}}$	$\frac{D_{re} - D_{rf}}{D_{zf}}$
0.060 ±0.009	0.120 ±0.018	0.0416 ±0.0020	0.0832 ±0.0040

Table 2. Differences between diffusion coefficients having elements with void as reference (values calculated from table 1)

$\frac{D_{z2} - D_{ze}}{D_{ze}}$	$\frac{D_{zf} - D_{ze}}{D_{ze}}$	$\frac{D_{r2} - D_{re}}{D_{ze}}$	$\frac{D_{rf} - D_{re}}{D_{ze}}$
-0.054 ±0.008	-0.107 ±0.016	-0.0372 ±0.0024	-0.0743 ±0.0048

3.3 Progressive substitutions

Since all the rods were furnished with shrouds it was possible to make complete substitutions in both directions, i.e. with air-filled shrouds either around test elements or around reference elements. The theory could then be tested with high experimental accuracy.

The analysis has been performed by means of eq. (22), which has been rearranged as follows

$$\frac{(a^2 - a_1^2)\,(1+\xi)}{\bar{w}_r} - \beta^2 \frac{\sum_i (w_{ri} - u_{ri})\,\delta D_{ri}/D_{z1}}{\bar{w}_r} =$$

$$= (a_3^2 - a_1^2) + \delta a_2^2 \cdot \frac{\bar{w}_{r2}}{\bar{w}_r} \tag{27}$$

where

$$\xi \cdot D_{z1} = \sum_i w_{ri}\ \delta D_{zi}$$

$$\bar{w}_r\ D_{z1} = w_{r3}\,D_{z3} + \frac{1}{2}\ w_{r2}\,D_{z2}$$

$$\bar{w}_{r2}\,D_{z1} = w_{r2}\,D_{z2}$$

$$\delta a_2^2 = a_2^2 - (a_3^2 + a_1^2)/2.$$

When the diffusion coefficients are the same in the whole core we have

$$\delta D_r = \delta D_z = 0$$

$$\xi = 0$$

$$\bar{w}_r = w_{r3} + w_{r2}/2$$

$$\bar{w}_{r2} = w_{r2}.$$

The dependence of the critical level on the number of test elements with void is shown in fig. 4. The level is rather constant in the region investigated.

By plotting the left-hand side of eq. (27) versus $\bar{w}_{r2}/\bar{w}_r$ we should get a linear relationship and $a_3^2 - a_1^2$ as the intercept. The first term, $(a^2 - a_1^2)\,(1+\xi)/\bar{w}_r$, was small compared to the second term with β^2, which dominated as may be seen in the tables 3 and 4. Experimental points are given in fig. 5.

Table 3. Values used in eq. (27) and derived from substitution measurements having elements without void as reference. Diffusion coefficients were taken from table 1

Number of test elements (with void)	$\frac{(\alpha^2 - \alpha_1^2)(1 + \xi)}{\bar{w}_r}$ (m^{-2})	ξ	$\beta^2 \frac{\sum_i (w_{ri} - u_{ri})\, \delta D_{ri}/D_{z1}}{\bar{w}_r}$ (m^{-2})
1	0.088 ± 0.006	0.004	0.333
4	0.075 ± 0.002	0.014	0.324
6	0.061 ± 0.001	0.020	0.314
9	0.048	0.029	0.303
12	0.040	0.037	0.297
16	0.034	0.048	0.291
24	0.013	0.063	0.273
32	-0.006	0.075	0.252

Table 4. Values used in eq. (27) and derived from substitution measurements having elements with void as reference. Diffusion coefficients were taken from table 2

Number of test elements (without void)	$\frac{(\alpha^2 - \alpha_1^2)(1 + \xi)}{\bar{w}_r}$ (m^{-2})	ξ	$\beta^2 \frac{\sum_i (w_{ri} - u_{ri})\, \delta D_{ri}/D_{z1}}{\bar{w}_r}$ (m^{-2})
1	-0.024 ± 0.005	-0.003	-0.332
4	-0.060 ± 0.002	-0.012	-0.342
6	-0.055 ± 0.001	-0.018	-0.335
9	-0.048	-0.025	-0.325
12	-0.045	-0.032	-0.321
16	-0.045	-0.041	-0.316
24	-0.023	-0.056	-0.294
32	-0.004	-0.067	-0.273

The intercepts of the lines in fig. 5 give $\alpha_e^2 - \alpha_f^2 = -0.261\ \mathrm{m}^{-2}$ and $\alpha_f^2 - \alpha_e^2 = 0.263\ \mathrm{m}^{-2}$. These values have to be compared with the correct result $\alpha_f^2 - \alpha_e^2 = 0.267\ \mathrm{m}^{-2}$. The value measured directly is $0.257\ \mathrm{m}^{-2}$, but corrections due to the change in extrapolation lengths have to be applied in order to make the difference comparable with the results obtained by the substitution procedure. The radial correction is $0.010\ \mathrm{m}^{-2}$. Axially we get nothing, because the difference in axial extrapolation lengths is not taken into account in the other part of the analysis. For the calculation of the correction we assume that $D_{rf} = D_{zf}$ and use the number 0.0832 given in table 1 to find the change of the effective extrapolation length which also takes the epithermal neutrons into account.

The errors indicated in fig. 5 correspond only to the experimental uncertainty of the water level and do not include any systematic error, such as that of $\delta D/D$. We find that the error in $(D_{re} - D_{rf})/D_{zf}$ gives rise to a buckling uncertainty of about $0.015\ \mathrm{m}^{-2}$. Thus the buckling difference found directly ($0.267\ \mathrm{m}^{-2}$) and those obtained from the analysis of the progressive substitutions ($0.261\ \mathrm{m}^{-2}$ and $0.263\ \mathrm{m}^{-2}$ resp.) obviously agree well within the experimental errors.

4. CONCLUSIONS

The proposed method of analysing substitution measurements with voids has proved to be good within the experimental errors. The largest contribution to the uncertainty comes from the measurements with single fuel elements, i.e. the determination of $\delta D/D$. The critical water height can be determined within ±0.01 cm. In order to increase the accuracy of the experiments the following possibilities are feasible:

a) Instead of using a single fuel element we may place several test elements at the same distance from the axis i.e. equivalent positions. However, in order to interpret the results there have to be small local perturbations, and consequently the elements have to be separated from each other at least two lattice pitches.

b) By using a pile-oscillator technique the sensitivity could perhaps be increased a factor of 10.

The results reported here were taken from a single series of measurements. The accuracy could of course also be increased by repeating the measurements several times.

In the experiments described in this report the same kind of fuel was used in the whole core. The only difference between the test and reference regions was the void. Thus the experimental conditions were ideal. However, if the test fuel and the reference fuel are different, the experimental program has to be enlarged. Then the axial void effect of a single fuel assembly has to be studied in a test region as well, e.g. the central assembly in a region of 3 x 3 fuel assemblies may be used for a differential investigation of the axial void. The results of the radial void experiments valid for a transition cell can then also be transferred to a test cell.

5. ACKNOWLEDGEMENT

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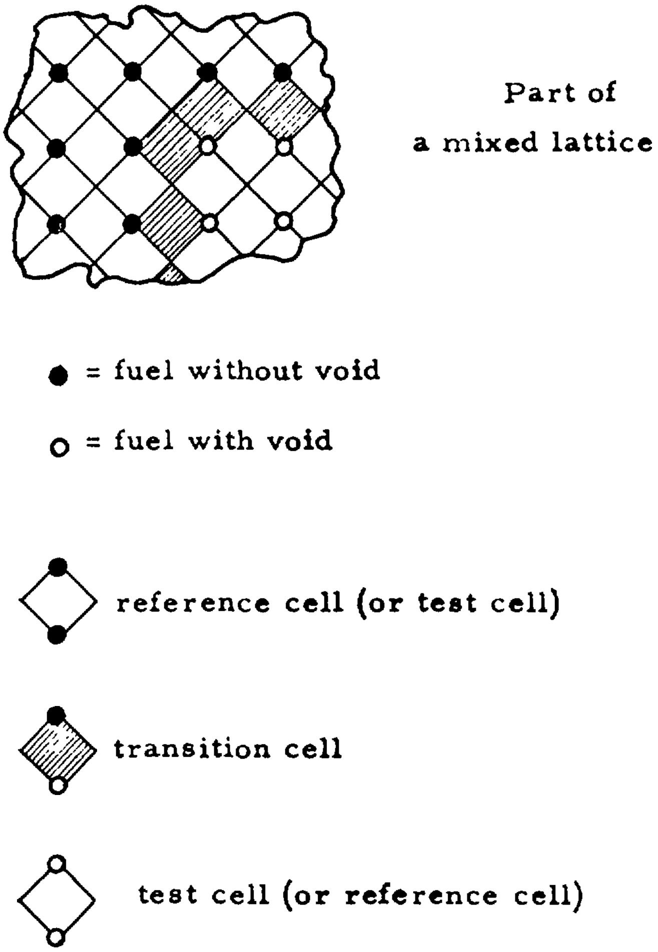


Fig. 1. Definition of cells

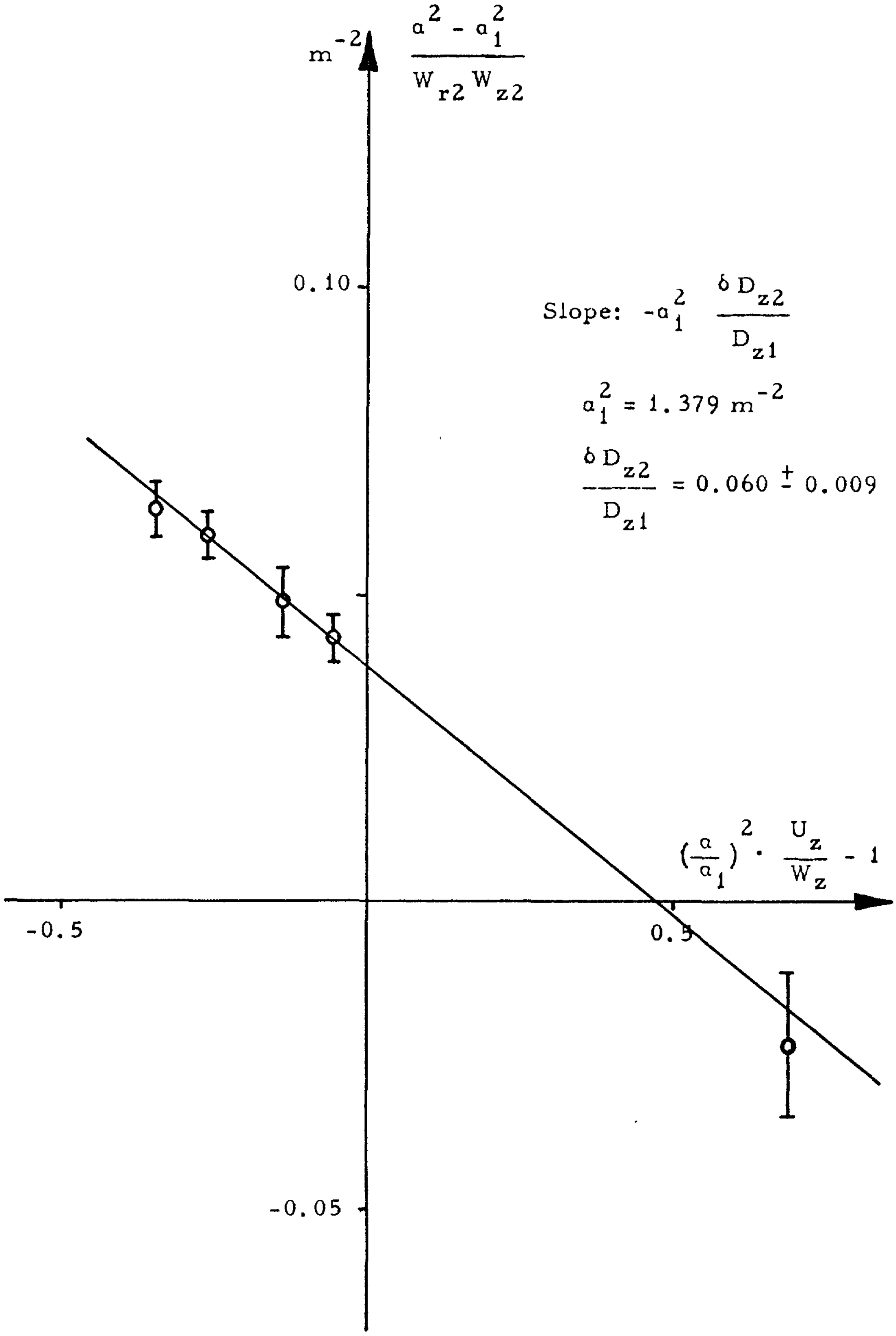
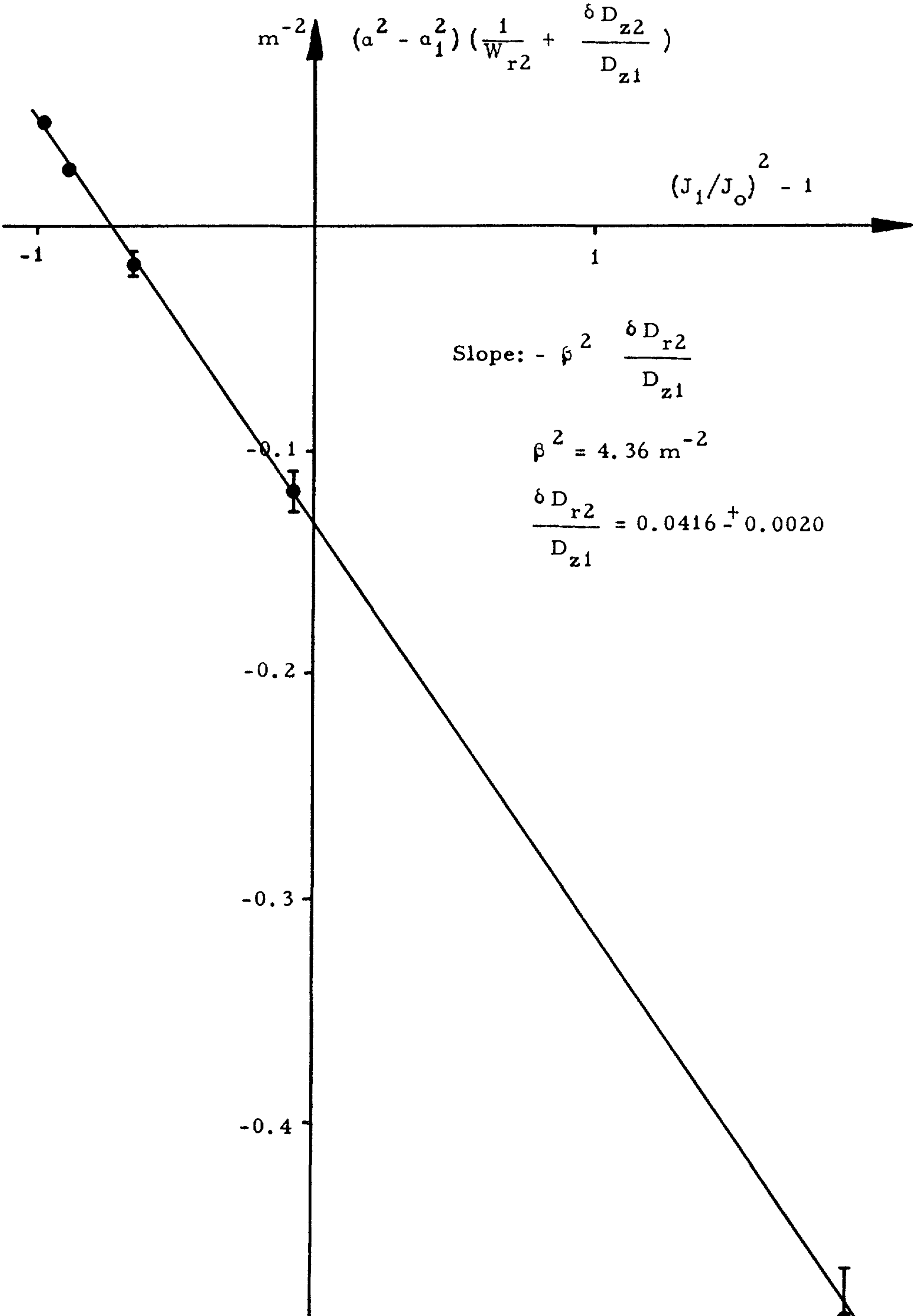


Fig. 2. Determination of $\delta D_z/D$ from axial void experiments with single element

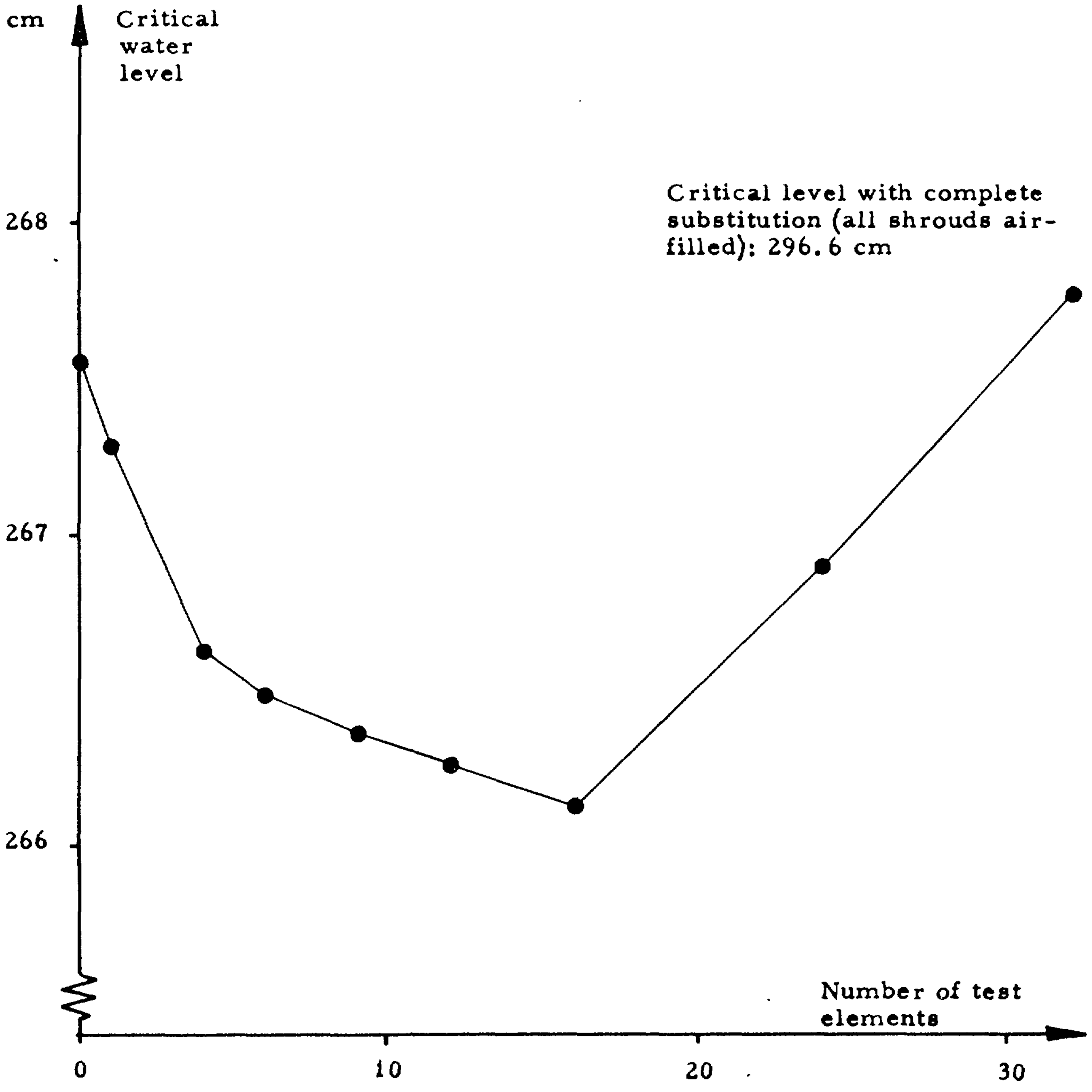


Slope: $-\beta^2 \frac{\delta D_{r2}}{D_{z1}}$

$\beta^2 = 4.36\ m^{-2}$

$\frac{\delta D_{r2}}{D_{z1}} = 0.0416 \pm 0.0020$

Fig. 3. Determination of $\delta D_r/D$ from radial void experiments with single element



Reference shrouds with D_2O.
Test shrouds with air.

Fig. 4. Critical water level
versus
number of test elements

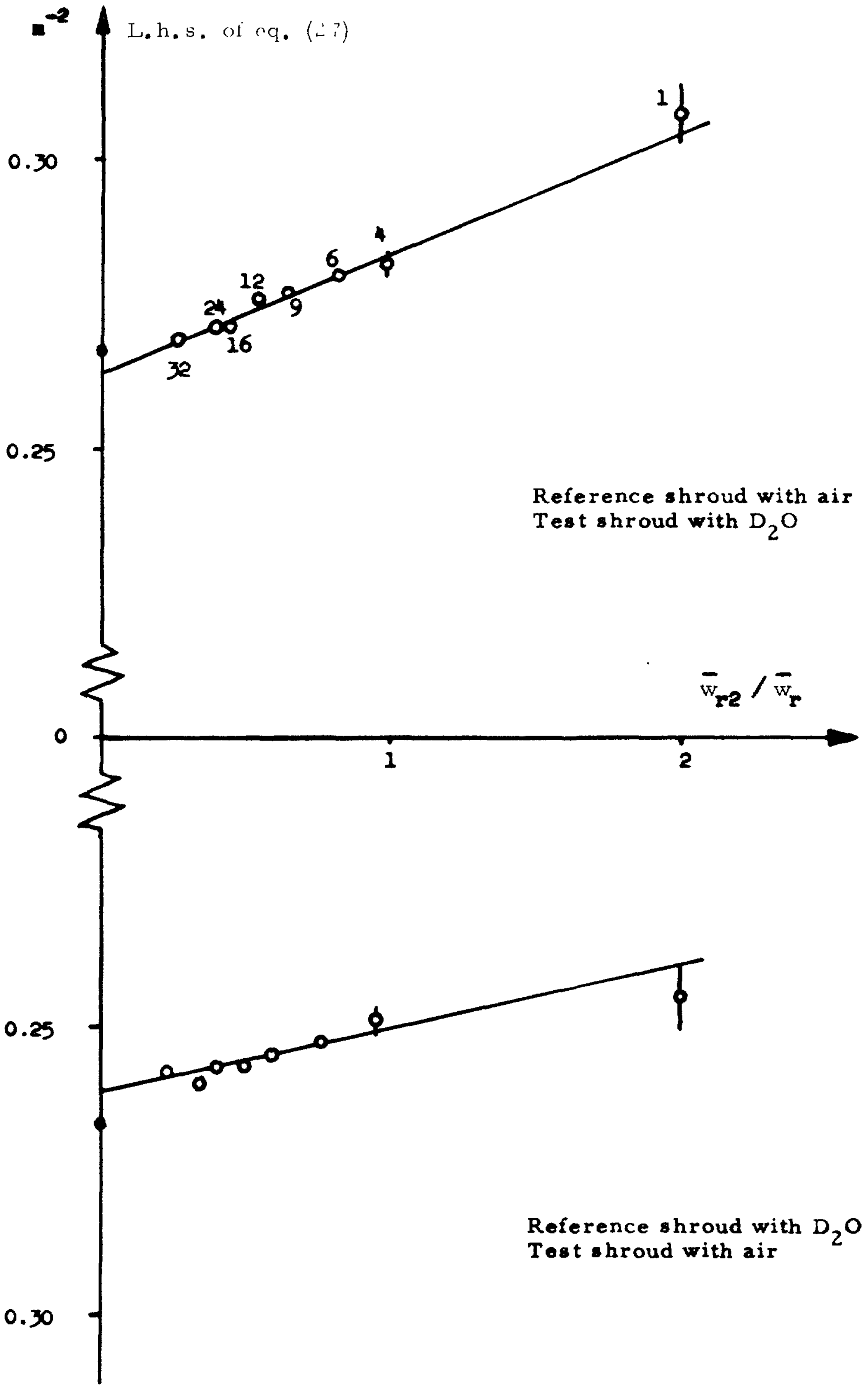


Fig. 5. Analysis of substitution measurements with void

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