Hall Effect Influence on a Highly Conducting Fluid

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### ABSTRACT

The properties of an incompressible perfect fluid exhibiting Hall effect is investigated in the limit of infinite electrical conductivity and mobility. The magnetic field strength and the fluid velocity are found to obey the equations  $\underline{B} = \frac{\mu\rho}{\sigma} \operatorname{curl} \underline{V}$  and  $\underline{V} = -\frac{\mu}{\sigma\mu_0} \operatorname{curl} \underline{B}$  (MKS units) where  $\rho$ ,  $\sigma$  and  $\mu$  denote mass density, conductivity and charge carrier mobility. Some physical interpretations and applications are given.

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### 1. INTRODUCTION

The concept of infinite electrical conductivity is very often used in magnetohydrodynamic theory. The main reason is of course a reduction of the complexity in the mathematics but in many cases it actually is a very good approximation. It may be noted here that electrical conductivity is that physical property of matter which has the largest known range of values.

There are two ways of interpreting infinite conductivity. Usually one assumes such a multitude of charge carriers that they form a continuum and consequently the specific motion of any of them is then disregarded. On the other hand, infinite conductivity can also be achieved by a finite number of charge carriers which respond infinitely quickly to an electric field. \* It will be shown that these two interpretations give entirely different descriptions of the magnetohydrodynamic behaviour of a perfectly conducting fluid. The reason is that magnetic effects on the individual particle motion have to be taken into account in the latter case. For electrons in a medium of finite conductivity this magnetic influence is known as the Hall effect.

## 2. ELEMENTARY DERIVATION OF THE HALL EFFECT

In magnetohydrodynamics an extensively used approximation of the generalized Ohm's law is given by the equation

$$\underline{\mathbf{j}} = \sigma \left( \underline{\mathbf{E}} + \underline{\mathbf{V}} \times \underline{\mathbf{B}} \right) \tag{1}$$

where <u>j</u>,  $\sigma$  and <u>E</u> denote the current density, the conductivity and the electrostatic field strength respectively. <u>V</u> is the local mass velocity of the conducting medium, permeated by a magnetic field <u>B</u> which may originate from currents in the fluid and/or external sources. For the simple case of a slightly ionized gas the conductivity is given by the expression

$$\sigma = e^2 n_e \tau / m_e \tag{2}$$

 <sup>\*</sup> In the following the treatment is based upon the assumption of a long collision time for electrons instead of a very small mass, see Eq. (8).

where e and  $m_e$  denote the electron charge and mass,  $n_e$  is the electron density and  $\tau$  is the average time between collisions with gas atoms randomizing the velocity of an electron.

It was pointed out by Alfvén [1] that Eq. (1) together with the assumption of perfect, i.e. infinite conductivity leads to the concept of "frozen in" magnetic lines of force or, explained more distinctly, there will then exist a constancy of magnetic flux through any closed contour moving with the mass velocity. The proof as given in practically all text books on magnetohydrodynamics and plasma physics, is obtained by combining the Maxwell equation

 $\operatorname{curl} \underline{\mathbf{E}} = -\frac{\partial \mathbf{B}}{\partial t}$  (3)

with the condition

$$\underline{\mathbf{E}} + \underline{\mathbf{V}} \times \underline{\mathbf{B}} = 0 \tag{4}$$

Lighthill [6] pointed out that such a derivation is not correct for a fluid where the charge transport is provided by particles which do not move with the mass velocity. For a plasma with the usual electronic conductivity, i.e. negligible ionic motion \* relative to the mass motion, Eq. (1) should instead be written

$$\underline{j} = \sigma \left( \underline{E} + \underline{V} \, \underline{x} \, \underline{B} \right) \tag{5}$$

where  $\underline{V}_{e}$  is a mean value of the electron velocity. Thus in the limit of very high conductivity the magnetic lines of force tend to move with, or be frozen into the electron gas. The mass and electron gas velocities are related to each other as

$$\underline{\mathbf{V}}_{\mathbf{e}} = \underline{\mathbf{V}} + \mathbf{j}/(\mathbf{e}\,\mathbf{n}_{\mathbf{e}}) = \underline{\mathbf{V}} + \underline{\mathbf{V}}' \tag{6}$$

For a highly conducting medium the difference velocity  $\underline{V}'$  is usually taken to be very small even in the case of large current densities because of the abundance of available free charge en. However, it

<sup>\*</sup> Motion of the ions relative to the neutrals can be accounted for here by replacing  $\sigma$  and  $\mu$  by  $\sigma(1+\nu)/(1+\nu\beta^2)$  and  $\mu(1-\nu)/(1+\nu\beta^2)$  respectively where  $\nu$  is the ratio of ionic and electronic mobilities, see ref. [4].

should not be inferred that V' = 0 would generally be a reasonable approximation. Combining Eqs. (5) and (6) it is found

$$\underline{j} = \sigma \left( \underline{E} + \underline{V} \times \underline{B} \right) + \frac{\sigma}{e n_e} \left( \underline{j} \times \underline{B} \right)$$
(7)

where the relative magnitude compared to unity of the last so-called Hall term is determined by the Hall parameter  $\beta$  which is the product of the electron mobility  $\mu$  and the magnetic field strength,

$$\beta = \mu B, \quad \mu = \sigma/(e n_e) = e \tau/m_e \tag{8}$$

 $\beta$  is usually interpreted as the average number of Larmor gyrations which an electron performs during the time  $2\pi\tau$ . For a magnetic field of 1 Wb/m<sup>2</sup>  $\beta$  is of the order unity in a noble gas of atmospheric particle density and room temperature.

## 3. THE HYDROMAGNETIC EQUATIONS

It is assumed that there is no electrical excess charge and that all external forces acting on the fluid can be derived from potentials. The equation of motion for a perfect fluid then takes the form, (see e.g. ref. [5]).

$$\rho \frac{\partial V}{\partial t} + \rho (\underline{V} \cdot \underline{\nabla}) \underline{V} = \underline{j} \times \underline{B} - \operatorname{grad} p + \rho \vee \Delta \underline{V} - \operatorname{grad} \phi$$
(9)

where  $\rho$  and p denote mass density and pressure,  $\nu$  is the kinematic viscosity and  $\phi$  is the potential of the external forces. In the hydromagnetic approximation the generalized Ohm's law is given by the equation, (see e.g. ref. [5])

$$\underline{j} = \sigma (\underline{E} + \underline{V} \times \underline{B}) + \mu \underline{j} \times \underline{B} - \frac{1}{e n_e} \operatorname{grad} p_e$$
(10)

where  $p_e$  is the electron gas pressure. The two basic equations (9) and (10) are supplemented by the pertinent Maxwell equations, one given by Eq. (3) and the remaining three expressed as

$$\operatorname{curl} \underline{B} = \mu_{0} \underline{j} \tag{11}$$

$$\operatorname{div} \underline{j} = 0 \tag{12}$$

$$\operatorname{div} \underline{B} = 0 \tag{13}$$

where  $\mu_{o}$  is the permeability of free space.

In the following we will make the strongly simplifying assumption that both the electron density and mobility are constant and therefore, by Eq. (8), also the conductivity. Further, the fluid is taken to be incompressible implying

$$\operatorname{div} \underline{V} = 0 \tag{14}$$

The second term of Eq. (9) is rewritten by the use of the vector identity

$$(\underline{V} \cdot \underline{\nabla})\underline{V} = \operatorname{grad} V^2/2 - \underline{V} \times \operatorname{curl} \underline{V}$$
(15)

Applying the operator curl to Eqs. (9) and (10) it is then found

$$\rho \frac{\partial}{\partial t} (\operatorname{curl} \underline{V}) - \rho \operatorname{curl} (\underline{V} \times \operatorname{curl} \underline{V}) = \frac{1}{\mu_0} \operatorname{curl} [(\operatorname{curl} \underline{B}) \times \underline{B}] +$$

$$+ \rho \vee \Delta (\operatorname{curl} \underline{V}) \qquad (9a)$$

$$- \frac{1}{\mu_0} \Delta \underline{B} = -\sigma \frac{\partial \underline{B}}{\partial t} + \sigma \operatorname{curl} (\underline{V} \times \underline{B}) + \frac{\mu}{\mu_0} \operatorname{curl} [(\operatorname{curl} \underline{B}) \times \underline{B}] \qquad (10a)$$

# 4. ESTIMATION OF THE RELATIVE MAGNITUDES OF THE TERMS IN THE BASIC EQUATIONS

Let L and  $V_0$  denote a length and a velocity characteristic for a situation where the equations (9a) and (10a) are expected to apply. The vector operators and the time can then be written in dimensionless form

$$\operatorname{curl}^{*} = \operatorname{L}\operatorname{curl}, \quad \Delta^{*} = \operatorname{L}^{2}\Delta, \quad t^{*} = t \operatorname{V}_{0}/\operatorname{L}, \quad (16)$$

and the two variables <u>V</u> and <u>B</u> are preferably normalized with respect to the velocity V<sub>0</sub> and the Alfvén wave velocity V<sub>A</sub> =  $B/(\mu_0 \rho)^{1/2}$  in the following way

$$\underline{\mathbf{v}} = \underline{\mathbf{V}} / \mathbf{V}_{o}, \quad \underline{\mathbf{v}}_{A} = \underline{\mathbf{B}} / (\mathbf{V}_{o}^{2} \mu_{o} \rho)^{1/2}$$
(17)

The two basic equations then become

$$\frac{\partial}{\partial t^{*}} (\operatorname{curl}^{*} \underline{v}) - \operatorname{curl}^{*} (\underline{v} \times \operatorname{curl}^{*} \underline{v}) = \operatorname{curl}^{*} [(\operatorname{curl}^{*} \underline{v}_{A}) \times \underline{v}_{A}] + \\ + \operatorname{R}_{e}^{-1} \Delta^{*} (\operatorname{curl}^{*} \underline{v}) \qquad (9b) \\ - \operatorname{R}_{m}^{-1} \Delta^{*} \underline{v}_{A} = - \frac{\partial \underline{v}_{A}}{\partial t^{*}} + \operatorname{curl}^{*} (\underline{v} \times \underline{v}_{A}) + \\ + \operatorname{R}^{-1} \operatorname{curl}^{*} [(\operatorname{curl}^{*} \underline{v}_{A}) \times \underline{v}_{A}] \qquad (10b)$$

Three dimensionless numbers appear here. Two of them are familiar, the Reynolds number

$$R_e = V_o L / v$$
,

and the magnetic Reynolds number

$$R_m = \mu_o \sigma V_o L$$

.

The third one seems to have been given little attention. It may be called the Hall effect interaction parameter

$$R = L \frac{\sigma}{\mu} \left(\frac{\mu_{o}}{\rho}\right)^{1/2} = L e n_{e} \left(\frac{\mu_{o}}{\rho}\right)^{1/2}$$
(18)

It is well known that a pronounced magnetohydrodynamic behaviour of a conducting fluid requires both  $R_e$  and  $R_m$  to be large, i.e. the viscous effects should be small and the fluid motion should be strongly affected by the magnetic fields induced by the currents in the fluid. In the following we will accordingly assume that terms multiplied by  $R_e^{-1}$  and  $R_m^{-1}$  are negligible, however, this does not imply that the Hall effect term can also be neglected. By taking the ratio

$$R^{-1}/R_{m}^{-1} = \beta V_{0}/V_{A}$$
(19)

a comparison is obtained between the magnitude of the Hall effect term and the magnetic diffusion term, the latter taken here to be negligible. It is obvious that there can exist conditions when this ratio attains a large value, i.e. the Hall effect influence must in general be taken into consideration even for a highly conducting fluid.

## 5. SOLUTIONS TO THE TIME-INDEPENDENT EQUATIONS

It is assumed that there are steady-state conditions, i.e. no explicit time dependence. Further we take the numbers  $R_e$  and  $R_m$  to be very large. Eqs. (9a) and (10a) then become

$$\mu_{o} \rho \operatorname{curl}(\underline{V} \times \operatorname{curl} \underline{V}) + \operatorname{curl}[(\operatorname{curl} \underline{B}) \times \underline{B}] = 0$$
(9c)

$$\mu_{o} \operatorname{curl} \left( \underline{V} \times \underline{B} \right) + \frac{\mu}{\sigma} \operatorname{curl} \left[ (\operatorname{curl} \underline{B}) \times \underline{B} \right] = 0$$
(10c)

For  $\mu = 0$ , i.e. no Hall effect, both equations are satisfied by the relation

$$\underline{\mathbf{V}} = \underline{\mathbf{B}} / (\mu_{o} \rho)^{1/2}$$
<sup>(20)</sup>

which expresses typical features of Alfvén wave propagation in a dissipationless medium, see ref. [1]. However, perfect conductivity in a fluid with a <u>finite</u> charge carrier density implies perfect mobility and hence an infinite Hall effect. Eqs. (9c) and (10c) then have an exact solution\* as shown in Appendix I

$$\underline{\mathbf{V}} = -\frac{\mu}{\sigma\mu_{o}}\operatorname{curl}\underline{\mathbf{B}}$$
(21)

$$\underline{B} = \frac{\mu\rho}{\sigma} \operatorname{curl} \underline{V}$$
(22)

<u>B</u> or <u>V</u> can be eliminated in either of these equations and by using Eqs. (13) and (14) a pair of Helmholz vector equations is obtained

$$\Delta \underline{\mathbf{B}} = \frac{\sigma^2 \mu_0}{\mu^2 \rho} \underline{\mathbf{B}}$$
(23)

$$\Delta \underline{\mathbf{V}} = \frac{\sigma^2 \mu_0}{\mu^2 \rho} \underline{\mathbf{V}}$$
(24)

Using Eqs. (8) and (11), Eq. (21) can be expressed as

$$en_{a} \underline{V} + \underline{j} = 0 \tag{25}$$

\* The solution <u>B</u> = 0, <u>V</u>  $\neq$  0 only leads to Eq. (27).

The interpretation of Eq. (25) is simple. Upon a mass motion with velocity <u>V</u> that charge ene which is subject to the Hall effect will experience an induced field <u>V</u> x <u>B</u> and perform a drift motion (<u>V</u> x <u>B</u>) x <u>B</u>/B<sup>2</sup>. As <u>B</u> is perpendicular to <u>V</u>, Eq. (22), the drift velocity is -<u>V</u> and it cancels any motion with the mass. On the other hand, that charge which is not affected by the magnetic field, i.e. the ions, will follow the fluid motion. Seen in the magnetic field frame, to which the electrons are tied, there is an ion current density  $\underline{j} = - \text{en} \frac{V}{e}$ , seen in the mass frame there is an electron current density of the same magnitude but with reversed direction. If both species of charged particles were subject to Hall effect Eq. (25) would still be satisfied, but trivially, because both the current <u>j</u> and the net charge ene would vanish.

Eq. (22) proves that the magnetic field is purely inductive and arises from rotational mass motion. Multiplying Eq. (22) vectorially with  $\underline{V}$  and applying the identity Eq. (15) it is found

$$\rho(\underline{V} \cdot \underline{\nabla})\underline{V} = \rho \text{ grad } \frac{\underline{V}^2}{2} - \frac{\sigma}{\mu} (\underline{V} \times \underline{B})$$
(26)

The last term is recognized as the Lorentz force  $j \ge B$  when Eq. (21) is substituted in it. A comparison between Eq. (26) and the non-vis - cous and time-independent form of the equation of motion, Eq. (9), then shows

$$\rho V^2/2 + p + \phi = constant$$
(27)

i.e. the purely hydrodynamic Bernoulli's equation for a flow line applies. This may have been expected from Eq. (25) which proves that the Lorentz force has no component along  $\underline{V}$ .

### 6. HALL EFFECT IN A ROTATING BODY

There has been much speculation on whether the rotation of a massive body will in general give rise to a magnetic moment proportional to the angular momentum, see e.g. ref. [2]. Eq. (22) suggests that Hall effect could cause this, however, Eqs. (21) and (22) cannot be exactly satisfied simultaneously for the case of a rigid rotating body. Instead the velocity distribution has to be found from Eq. (24) and the associated boundary conditions. We take spherical coordinates r,  $\varphi$ ,  $\theta$  and assume rotational symmetry

$$\underline{\mathbf{V}} = \mathbf{V}_{\varphi}(\mathbf{r}, \theta) \hat{\boldsymbol{\varphi}}$$
(28)

The solution of Eq. (24) is given in Appendix II, here it is only noted that the angular momentum

$$M_{a} = 2\rho I, I = \pi \iint V_{\phi}(r, \theta) r^{3} \sin^{2} \theta \, dr d\theta$$
 (29)

will be proportional to the magnetic moment

$$M_{\rm m} = -\frac{\sigma}{\mu} I \tag{30}$$

This result can be explained in the following way: The electrons are tied to the magnetic field and the ions to the material. A mass rotation gives rise to an ion ring current system which in turn creates the magnetic field. Such a dynamo mechanism is not in conflict with the famous Cowling [3] disproof of a steady and rotationally symmetric dynamo because Hall currents were not considered there.

The Hall effect in large astronomical bodies is probably all too weak to explain their magnetic fields by the present mechanism (T G Cowling, private communication).

## 7. SUMMARY AND CONCLUSIONS

It has been shown that infinite conductivity can be given two interpretations, both of which, of course, must be regarded as conditions in the very limit for an actual conducting fluid. The classical one assumes a continuum of charge carriers and it leads to well-known concepts like e.g. "frozen in" magnetic lines of force. On the other hand, infinite conductivity can also be achieved by an infinite mobility of a finite number of charge carriers. The magnetic effects on the individual charge carrier must in that case be taken into account. The equations for mass motion and charge transport then give a definite and entirely different mathematical description, Eqs. (21) and (22), of such a fluid. The borderline between the classical and the present case is given when the ratio Eq. (19) is of the order unity which essentially requires a pronounced Hall effect. Both under laboratory and astrophysical conditions this ratio can easily exceed unity very much and thus favour the description as given here.

### REFERENCES

- ALFVÉN H, On the existance of electromagnetic-hydrodynamic waves. Ark. Mat. Astr. Fys. 29B (1942) No. 2;
- 2. BLACKETT P M S, The magnetic field of massive rotating bodies. Nature 159 (1947) 658-666.
- COWLING T G, The magnetic field of sunspots. Monthly Notices Roy. Astron. Soc. <u>94</u> (1934) 39-48.
- COWLING T G, The electrical conductivity of an ionized gas in a magnetic field, with applications to the solar atmosphere and the ionosphere. Proc. Roy. Soc. A183 (1945) 453-479.
- COWLING T G, Magnetohydrodynamics. Interscience, New York 1957.
- LIGHTHILL M J,
   Studies on magneto-hydrodynamic waves and other anisotropic wave motions.
   Phil. Trans. Roy. Soc. A252 (1960) 397-430.

## APPENDIX I

Insertion of Eq. (21) in Eq. (10c) immediately proves it to be a solution. For the remaining three combinations of Eqs. (21) and (22) with Eqs. (9c) and (10c) the solutions are obtained by using one of the equations

$$\operatorname{curl}\operatorname{curl}\underline{B} + \frac{\sigma^{2}\mu_{o}}{\mu_{\rho}^{2}}\underline{B} = 0$$
(23a)

$$\operatorname{curl}\operatorname{curl}\frac{V}{\mu} + \frac{\sigma^{2}\mu_{o}}{\mu^{2}\rho}\frac{V}{\mu} = 0$$
(24a)

which are simple consequences of applying the operator curl to the solution pair Eqs. (9c) and (10c). E.g. Eq. (9c) combined with Eq. (21) gives

$$\operatorname{curl}\left[\left(\operatorname{curl}\underline{B}\right)\times\left(\frac{\mu^{2}\rho}{\sigma^{2}\mu_{0}}\operatorname{curl}\operatorname{curl}\underline{B}+\underline{B}\right)\right]=0$$

#### APPENDIX II

In the rotationally symmetric case Eq. (24) has only a  $\phi\mbox{-component}$  nent

$$\frac{1}{r^{2}} \frac{\partial}{\partial r} \left( r^{2} \frac{\partial V_{\varphi}}{\partial r} \right) + \frac{1}{r^{2} \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial V_{\varphi}}{\partial \theta} \right) - \frac{1}{r^{2} \sin^{2} \theta} V_{\varphi} =$$
$$= a^{2} V_{\varphi}, \ a^{2} = \frac{\sigma^{2} \mu_{\varphi}}{\mu^{2} \rho}$$
(31)

Separation of variables, separation constant  $k^2$ , gives

$$V_{\varphi} = r P(r) \sin \theta Q(\theta)$$
(32)

$$Q'' + 3 \cot \theta Q' + k^2 Q = 0$$
 (33)

$$P'' + \frac{4}{r}P' - (a^2 + \frac{k^2}{r^2})P = 0$$
(34)

Eq. (33) is transformed as

.

$$(s^{2} - 1)Q''(s) + 4sQ' - k^{2}Q = 0, s = \cos\theta$$
 (35)

and it has Gegenbauer polynomials as the general solution. If  $V_{\odot} = 0$  for r = 0, Eq. (34) is satisfied by hyperbolic Bessel functions

$$P = r^{-3/2} J_{\nu}(i a r), \quad i = (-1)^{1/2}, \quad \nu^{2} = 9/4 + k^{2}$$
(36)

If  $V_{\varphi}(R_{o}, \theta) = R_{o}\omega_{o}\sin\theta$ , i.e. there is a rigidly rotating spherical shell at  $r = R_{o}$ , the solution becomes

$$k^{2}=0$$
, Q=1,  $V_{\varphi}=R_{o}^{3/2}\omega_{o}J_{3/2}^{-1}(iaR_{o})r^{-1/2}J_{3/2}(iar)\sin\theta$  (37)

and for  $aR_0 = R >> 1$  (see Eq. (18)) the velocity distribution is simply the same as that in a rigid body

$$V_{\varphi} = r \omega_{o} \sin\theta \tag{38}$$

1-189. (See the back cover earlier reports.)

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