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Kinetics of Pressurized Water Reactors with Hot or Cold Moderators

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Summary:

The set of neutron kinetic equations developed in this report permits the use of long integration steps during stepwise integration. Thermal relations which describe the transfer of heat from fuel to coolant are derived. The influence upon the kinetic behavior of the reactor of a number of parameters is studied. A comparison of the kinetic properties of the hot and cold moderators is given.

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1. Introduction

In this report the kinetic properties of different reactor systems are evaluated mainly from considerations of the magnitude of the transients from reactivity perturbations and the stability with locked control rods. A triangular reactivity perturbation is assumed for initiating the transient. The perturbation has a height of 300 pcm, a 10 second rise time and a 10 second fall time.

It is difficult to appraise the inherent control characteristics of reactor systems and such an evaluation will not be attempted here. With control rods, activated by mechanisms which are slow moving, yet reliable and fail safe, it might be possible to control all competitive heavy-water reactors irrespective of their inherent safety characteristics.

Three different moderator arrangements are investigated. The first system is called t y p e A. One coolant circuit is used and the coolant in the reactor first passes the cooling channels of the fuel elements and then acts as moderator.

The second system is called t y p e B. One cooling circuit is used, but the coolant first passes the moderator and then the cooling channels.

The third system is called t y p e C. In this system, two cooling circuits are used. The cooling channels are connected to the main cooling circuit. The moderator is cooled by a separate circuit and heat is exchanged between the circuits only by conduction through the cooling channel walls.

2. Definitions of symbols

Symbols used in par 5 and 9 are not defined below but defined in connection with their application in equations. Also some symbols defined below are redefined in these paragraphs.

t	time (in general, from the beginning of the disturbance)
c_i	number of precursors of i^{th} delayed neutron group in a certain cubic centimeter of the reactor
λ_i	decay constant of precursor of i^{th} delayed neutron group
β_i	fractional yield of precursor of i^{th} delayed neutron group
n	number of neutrons in a certain cubic centimeter of the reactor
k	effective neutron multiplication constant
l_0	time between generations of prompt neutrons
P	total reactor power (thermal)
P_0	P at the stationary state before disturbance
P'	$= P - P_0$ (deviation in reactor power)
n_0	n at the stationary state before disturbance
C_i	$= (P_0/n_0)\, c_i$
C_i'	$= l_0\, C_i$
β	total fractional yield of precursors of delayed neutrons
$F(t)$	see eq. (5)
$S(t)$	see eq. (6)
k_0	k after a reactivity step
K_0	$= k_0 - 1$
S_0	$S(t)$ at the stationary state before disturbance
τ_{pj}	prompt jump time constant
H_0	characteristic thermal conductance of fuel = (total power in fuel)/(average temperature rise in fuel)

C_f thermal capacitance of fuel

p_f fraction of nuclear power liberated in fuel

cQ heat transport per degree by the main coolant circuit

cM_m thermal capacitance of moderator

p_1 power distribution factor for 1^{st} and 3^{rd} sections of mathematical model of moderator

p_2 power distribution factor for 2^{nd} section of mathematical model of moderator

f_f reactivity influence factor for fuel temperature

f_1 reactivity influence factor for temperature of 1^{st} and 3^{rd} sections of mathematical model of moderator

f_2 reactivity influence factor for temperature of 2^{nd} section of mathematical model of moderator

α_f fuel temperature reactivity coefficient

α_c cooling channel temperature reactivity coefficient

α_m moderator temperature reactivity coefficient

K_f contribution to reactivity from fuel temperature deviation

K_c contribution to reactivity from cooling channel temperature deviation

K_m contribution to reactivity from moderator temperature deviation

K_d external contribution to reactivity (from, e.g., control rods)

H_m thermal conductance between cooling channels and moderator for type C reactors

cQ_m heat transport per degree by the moderator cooling circuit for type C reactors

k_u exit residue fraction for main heat exchangers

C_s thermal capacitance of secondary of main heat exchangers

k_{um} exit residue fraction for moderator cooler for type C reactors

U_f fuel average temperature deviation

T_c temperature deviation of cooling channel entrance

U_c temperature deviation of cooling channel exit

T_m temperature deviation of moderator entrance

U_m temperature deviation of moderator exit

U_1 exit temperature deviation of 1^{st} section of mathematical model of moderator

U_2 exit temperature deviation of 2^{nd} section of mathematical model of moderator

$T_{d\ in}$ entrance temperature deviation of mixing volume representing transport delay of reactor entrance side

$U_{d\ in}$ exit temperature deviation of mixing volume representing transport delay of reactor entrance side

$T_{d\ out}$ entrance temperature deviation of mixing volume representing transport delay of reactor exit side

$U_{d\ out}$ exit temperature deviation of mixing volume representing transport delay of reactor exit side

T_p entrance temperature deviation of main heat exchanger primary

U_p exit temperature deviation of main heat exchanger primary

U_s temperature deviation of secondary of main heat exchanger

P_s deviation in power load on heat exchanger secondary

ϕ thermal neutron flux

dV volume element

$\vec{r}$ space coordinate

$U_f(\vec{r})$ local temperature of fuel

3. Neutron kinetics

Starting from the normal space-independent neutron equations, an approximately equivalent system of equations will be derived. In this system long steps can be used when integrating by the Runge-Kutta-Gill method.

The reader is referred to par 2 for the definitions of the symbols. For the emitters of delayed neutrons the following equations are valid:

$$\frac{dc_i}{dt} = - \lambda_i c_i + \beta_i \frac{kn}{l_0} \quad ; \quad i = 1, 2, \ldots, 6. \tag{1}$$

For the number of neutrons one has:

$$\frac{dn}{dt} = \frac{1}{l_0} \left[(1 - \beta) kn - n \right] + \sum_{i=1}^{6} \lambda_i c_i \tag{2}$$

Equations (1) and (2) are multiplied by P_0/n_0 and one obtains:

$$\frac{dC_i}{dt} = - \lambda_i C_i + \beta_i \frac{kP}{l_0} \quad ; \quad i = 1, 2, \ldots, 6. \tag{3}$$

$$\frac{dP}{dt} = \frac{1}{l_0} \left[(1 - \beta) kP - P \right] + \sum_{i=1}^{6} \lambda_i C_i \tag{4}$$

Now the variables in eq. (4) are regarded as functions of the time only. To simplify the notation we introduce

$$F(t) = \int_0^t \frac{1}{l_0} \left[1 - (1-\beta) k(t') \right] dt' \tag{5}$$

and

$$S(t) = \Sigma \lambda_i \, C_i \, (t) \tag{6}$$

The solution of eq. (4) is then:

$$P = e^{-F(t)}(P_0 + \int_0^t S(t') \, e^{F(t')} dt') \tag{7}$$

We investigate the solutions for a reactivity step; $k(t) = k_0 = 1 + K_0$, $0 < t$; when $S(t)$ is equal to its constant value S_0 for the stationary initial state. We introduce the promt jump time constant for this case

$$\tau_{pj} = \frac{1_0}{1 - (1 - \beta) \, k_0} = \frac{1_0}{\beta} \, \frac{1}{1 - (K_0/\beta) + \beta(K_0/\beta)} \tag{8}$$

Using the relation between S_0 and P_0 one obtains for the power:

$$P = \frac{1}{1 - (K_0/\beta) + \beta(K_0/\beta)} \, P_0 - (\frac{1}{1 - (K_0/\beta) + \beta(K_0/\beta)} - 1) \, \exp(-\frac{t}{\tau_{pj}}) \tag{9}$$

τ_{pj} is small if $K_0 \ll \beta$. E.g. $1_0 = 0.001$ sec and $\beta = 0.5$ % gives $\tau_{pj} = 0.2$ sec. This shows that the power rapidly approaches the value described by the first term of the right member of eq. (9).

When the reactivity is small and changing rather slowly, the power approximately follows the analogous expression to eq. (9):

$$P = \frac{1_0}{1 - (1 - \beta) \, k(t)} \, S(t) \tag{10}$$

We introduce $C_i = C_i'/1_0$ in eq. (3) and obtain as a substitute for eqs. (3) and (4) the following eqs. (11) and (12).

$$\frac{dC'_i}{dt} = -\lambda_i C'_i + \beta_i kP \tag{11}$$

$$P = \frac{\Sigma \lambda_i C'_i}{1 - (1 - \beta) k} \tag{12}$$

The equations are independent of l_0 and the accuracy in their description of the kinetic behavior increases with decreasing l_0. The system therefore is specially suited for reactors with small l_0, as fast reactors and also light-water reactors. The derivation of the equations confirms the known fact, that for reactivities occuring during circumstances other than very extraordinary, the neutron lifetime has small influence on the kinetic behavior of the reactor.

To make possible long intervals in the step-integration, the two "fastest" groups of delayed neutron precursors are not represented. Therefore, these two "fastest" groups are treated as prompt neutrons in the calculations presented in the following paragraphs and only the four "slower" groups are included in the calculations. A reactivity disturbance considered a 9.67 second step has been used. If one uses eqs. (3) and (4) a 0.1 second step must be used. A substantial saving in computer time is realized from using eqs. (11) and (12).

4. Thermal relations

The transients investigated in this paper are rather slow. The heat transport from the fuel to the cooling channels therefore is described as from an isothermal mass, connected with the cooling channels through a thermal conductance. In the appendix (par 9) equations are derived, which can be adapted to arbitrarily fast transients. The equation for the fuel heat balance will be:

$$C_f \frac{dU_f}{dt} = H_0 (U_c - U_f) + p_f P' \tag{13}$$

It is common to treat the cooling channels as a mixing volume when writing the heat balance equation. This is a considerable approximation in relation to the true circumstances. We use the following also approximate equation, which will not describe the conditions much worse than would a mixing-volume equation.

$$cQ\,(U_c - T_c) = H_0\,(U_f - U_c) \tag{14}$$

As basis for the equations for the moderator a model with three series-connected mixing volumes is used. One obtains:

$$\frac{1}{3}\,cM_m\,\frac{dU_1}{dt} = cQ\,(T_m - U_1) + p_1\,(1 - p_f)\,P' \tag{15}$$

$$\frac{1}{3}\,cM_m\,\frac{dU_2}{dt} = cQ\,(U_1 - U_2) + p_2\,(1 - p_f)\,P' \tag{16}$$

$$\frac{1}{3}\,cM_m\,\frac{dU_m}{dt} = cQ\,(U_2 - U_m) + p_1\,(1 - p_f)\,P' \tag{17}$$

The power distribution factors are calculated by the following expression:

$$p_i = \frac{\iiint\limits_{mod_i} \phi\, dV}{\iiint\limits_{mod_{tot}} \phi\, dV}\;; \quad i = 1,\ 2. \tag{18}$$

For the heat exchanger primary circuit, the following equation is valid:

$$U_p = k_u T_p + (1 - k_u)\,U_s \tag{19}$$

For the heat exchanger secondary circuit, we have:

$$C_s \frac{dU_s}{dt} = c\Omega\,(1-k_u)(T_p - U_s) - P_s \tag{20}$$

The transport delays between the reactor and the heat exchanger are represented by mixing volumes, which gives:

$$\frac{dU_{d\,in}}{dt} = \frac{1}{\tau_d}\,(T_{d\,in} - U_{d\,in}) \tag{21}$$

$$\frac{dU_{d\,out}}{dt} = \frac{1}{\tau_d}\,(T_{d\,out} - U_{d\,out}) \tag{22}$$

The neutron multiplication constant is divided according to the following equations:

$$k = 1 + K_d + K_f + K_c + K_m \tag{23}$$

$$K_f = f_f\, a_f\, U_f \tag{24}$$

$$K_c = a_c\, U_c \tag{25}$$

$$K_m = a_m\,(f_1 U_1 + f_2 U_2 + f_1 U_m) \tag{26}$$

For the factors for the influence on the reactivity one has:

$$f_f = \frac{\iiint\limits_{core} dV \;\iiint\limits_{core} \phi^3 dV}{\iiint\limits_{core} \phi\, dV \;\iiint\limits_{core} \phi^2 dV} \tag{27}$$

$$f_i = \frac{\iiint\limits_{mod_i} \phi^2 dV}{\iiint\limits_{mod_{tot}} \phi^2 dV} \quad ; \quad i = 1,\ 2. \tag{28}$$

For the J_0-cos flux distribution chopped at 0.2 we found $f_f = 1.55$ and this value has been used in the calculations. In the equations above, the fuel temperature reactivity coefficient corresponds to the same temperature increase in all parts of the fuel. The definition is described by the following formula:

$$a_f = \lim_{\Delta \to 0} \frac{k(U_f(\vec{r}) + \Delta) - k(U_f(\vec{r}))}{\Delta} \tag{29}$$

The equations for type C reactors are somewhat different. There is heat conduction from the cooling channels to the moderator. The equations for the cooling channels will be:

$$(cQ + H_0 + H_m)\ U_c = H_0 U_f + H_m U_m + cQ\ T_c \tag{30}$$

The moderator cooling circuit must handle the power liberated in the moderator and the heat losses from the cooling channels to the moderator. The moderator coolant flow Q_m therefore is small in comparison to the main coolant flow Q. The flow conditions in the moderator probably do not correspond to three series-connected mixing volumes. One mixed volume is likely to give a better representation. Therefore we use the equation:

$$cM_m \frac{dU_m}{dt} = cQ_m\ (T_m - U_m) + H_m\ (U_c - U_m) + (1-p_f)\ P' \tag{31}$$

A satisfactory description of the moderator cooler is given by the equation:

$$T_m = k_{um} U_m \tag{32}$$

The moderator temperature influence on the reactivity for type C is expressed by:

$$K_m = a_m U_m \tag{33}$$

The situation of different parts in the cooling circuit gives some equations which have not yet been presented. The connection between the transport delay volumes and the heat exchanger gives:

$$T_p = U_{d\ out} \tag{34}$$

$$T_{d\ in} = U_p \tag{35}$$

For type A reactor systems the following eqs. (36) - (38) are valid:

$$T_c = U_{d\ in} \tag{36}$$

$$T_m = U_c \tag{37}$$

$$T_{d\ out} = U_m \tag{38}$$

Instead of the three foregoing equations, the following eqs. (39) - (41) applies to systems of type B.

$$T_m = U_{d\ in} \tag{39}$$

$$T_c = U_m \tag{40}$$

$$T_{d\ out} = U_c \tag{41}$$

Eqs. (30) - (33) are valid for type C reactors. The two additional eqs. (42) and (43) must also be used for this type.

$$T_c = U_{d\ in} \tag{42}$$

$$T_{d\ out} = U_c \tag{43}$$

5. Basic reactor specifications

At stationary conditions the following eq. (44) is valid between the average fuel temperature U_f, the maximum fuel temperature U_{fcc}, the cooling channel temperature U_c and the thermal neutron flux distribution factor d_{ft}. The equation is somewhat approximate, among other things due to the temperature dependence of the fuel thermal conductivity and the temperature drop between the fuel surface and the coolant. The fuel elements are assumed to consist of cylindrical rods.

$$U_f - U_c = \frac{U_{fcc} - U_c}{2d_{ft}} \tag{44}$$

We apply our definition of the characteristic thermal conductance and obtain with the power P_f in the fuel:

$$H_0 = \frac{P_f}{U_f - U_c} = \frac{2d_{ft}P_f}{U_{fcc} - U_c} \tag{45}$$

Uranium oxide is considered as the normal fuel. We assume $U_{fcc} = 1700\ ^{\circ}C$ at nominal power $P_0 = 390$ MW. Further, we have

$U_c = 230\ ^\circ C$ and $d_{ft} = 2.50$ and find from eq. (45) $H_0 = 1.25\ MW/^\circ C$. This value is used as the normal characteristic thermal conductance between fuel and coolant.

The abundance of the delayed neutron precursors has been determined in connection with burn-up calculations for future reactors. The abundances of the four "slower" precursors are given in table 1.

Table 1. Delayed neutron precursor abundance

i (prec. no.)	1	2	3	4	
λ_i, 1/sec	0.0124	0.0305	0.1114	0.3013	β
0 % burn-up, β_i	0.0002	0.0014	0.0013	0.0026	0.0055
100 % burn-up, β_i	0.0001	0.0009	0.0008	0.0015	0.0033

The specifications necessary to determine the investigated kinetic properties are given in table 2. The influence of a number of parameters on the kinetic behavior has been examined. These parameters are marked by a star in the table.

Table 2. Basic reactor specifications

Item	Symbol	Units	Value
Characteristic thermal conductance between fuel and coolant	H_0	MW/°C	1.25 *
Fuel thermal capacitance	C_f	MWsec/°C	9
Fraction of nuclear power liberated in fuel	p_f		0.94
Heat transport by the main coolant circuit	$c\Omega$	MW/°C	15.5
Moderator thermal capacitance	cM_m	MWsec/°C	265
Exit residue fraction for main heat exchangers	k_u		0.286
Thermal capacitance of secondary of main heat exchangers	C_s	MWsec/°C	500 *
Transport delay from reactor to main heat exchangers	τ_d a)	sec	5
Fuel reactivity coefficient	α_f	pcm/°C	-2 *
Coolant channel reactivity coefficient	α_c	pcm/°C	+1.6
Moderator reactivity coefficient	α_m b)	pcm/°C	-30 *
Reactor power (thermal)	P_0 c)	MW	390
Thermal conductance between cooling channels and moderator for type C	H_m	MW/°C	0.2
Heat transport by moderator cooling circuit for type C	$c\Omega_m$	MW/°C	1
Exit residue fraction for moderator cooler for type C	k_{um}		0.25

* Are changed in the calculations. See table 3.

a) Should also include half of the heat exchanger primary transport delay.

b) The value noted is for zero burn-up. The coefficient probably increases to positive values with the burn-up.

c) Properly, type C should have higher power to compensate for the losses dissipated through the moderator cooler.

6. Specification of cases investigated

The initial intention was to examine the inherent stability of the reactors and their reaction to reactivity disturbances. The dynamic characteristics are poorer when the moderator reactivity coefficient changes in the positive direction. The transient also becomes higher with a decreasing fraction of delayed neutrons. The fraction of delayed neutrons decreases with increasing burn-up. Because of the cited circumstances, we study mainly cases with neutron data for a reactor which has undergone irradiation and positive values of the moderator reactivity coefficient. The cases investigated are specified in the following table.

Table 3. Specification of cases investigated a)

Case b)			1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Parameter	Units	Value															
β	%	0.55	x														
		0.33		x	x	x	x	x	x	x	x	x	x	x	x	x	x
α_f	pcm/°C	-2	x	x	x	x	x	x	x	x	x	x					x
		-1											x	x	x	x	
α_m	pcm/°C	-30	x	x					x	x							
		0			x												
		5				x							x		x		
		10					x				x	x		x		x	x
		30						x									
H_0	MW/°C	1.25	x	x	x	x	x	x					x	x			x
		2.5							x		x				x	x	
		5								x		x					
C_s	MWsec/°C	250															x
		500	x	x	x	x	x	x	x	x	x	x	x	x	x	x	

a) The disturbance in all cases is a triangular reactivity perturbation as described in par 1. (Height 300 pcm, rise time 10 seconds and fall time 10 seconds.)

b) All cases are calculated for the three reactor types.

7. Results

The calculations have been made on the Ferranti Mercury Computer of AB Atomenergi. The codes have been written by the author in the Manchester Mercury Autocode System. The computing time was about two minutes per case. The behavior of the reactors in most cases was investigated for the 200 seconds following the disturbance.

It is not feasible to report the results in detail in this paper. The reactor power deviation P' is exemplified by the cases 2 and 6, which are pictured in figs. 1 and 2 at the end of the paragraph. These curves are representative of the general appearance of the power function in the different cases. As will be evident from table 4, the height of the maximum at t = 10 sec varies much among the different cases.

In most of the cases the reactor systems are not inherently stable. The instability manifests itself as a disturbance initiated transient of exponential type.

In the following tables 4 - 6 is noted the power deviation P' at t = 10 sec, the first maximum in the fuel temperature deviation U_f and the period $((dP/dt)/P)$ for the unstable cases. The periods noted are approximated from the total power about one minute after the beginning of the disturbance.

Table 4. Power deviation P' after 10 seconds in MW

Case	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Type A	154	159	198	206	214	250	197	242	314	490	420	448	608	674	214
Type B	178	191	198	200	201	206	264	376	283	414	399	404	556	565	202
Type C	176	190	196	198	199	204	258	359	274	388	384	388	516	524	198

Table 5. First maximum of deviation in fuel mean temperature U_f in °C

Case	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Type A	68	72	86	89	92	105	62	51	91	94	175	185	174	194	92
Type B	77	83	86	87	87	89	77	72	83	69	168	170	160	163	87
Type C	77	82	85	86	86	88	75	68	80	74	162	163	149	152	86

Table 6. Period ((dP/dt)/P) after about one minute in the unstable cases

Case		5	6	9	10	11	12	13	14	15
	A	2.6 h	4.1 min	13 min	2.4 min	46 min	7.8 min	6.5 min	1.8 min	1.3 h
Type	B	4 h	21 min	42 min	12 min	1.4 h	25 min	19 min	6.6 min	2.2 h
	C	24 h	3 h	> 24 h	1.8 h	10 h	1.9 h	1.4 h	19 min	15 h

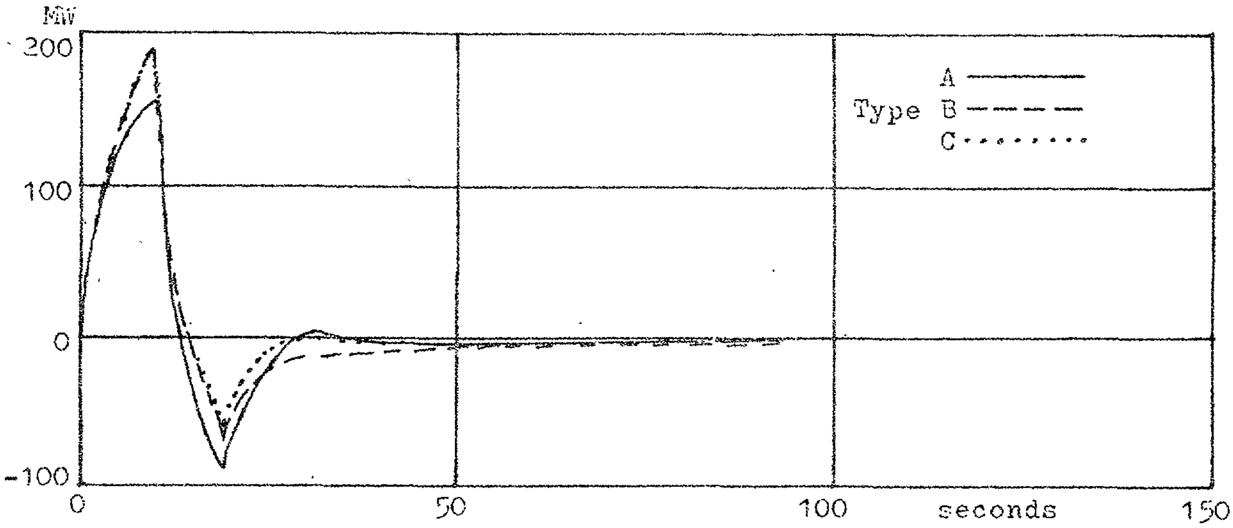

Fig 1. Power deviation P' in case 2

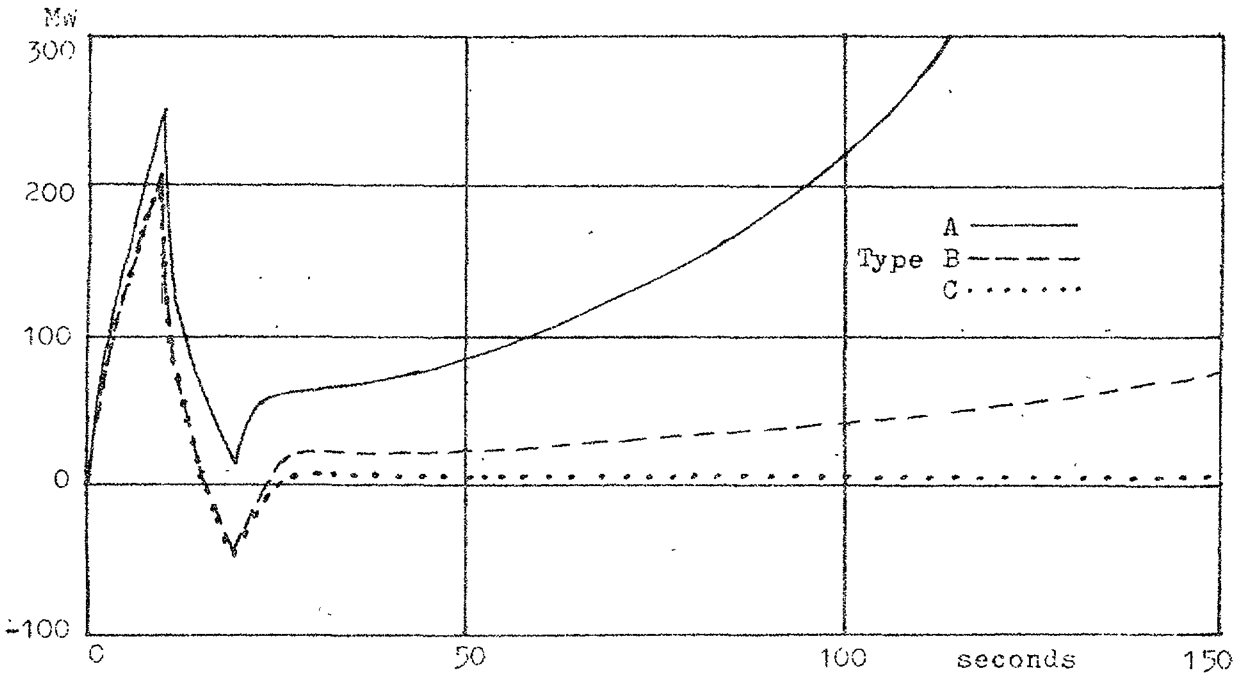

Fig 2. Power deviation P' in case 6

8. Discussion

The influence of β, the fraction of delayed neutrons upon the transient behavior, is found by comparing cases 1 and 2. From tables 4 and 5 (and from the curves, which are not reproduced in this paper) one finds that the influence is small. During the initial portion of a reactivity disturbance the excess reactivity is compensated by the negative effect of the fuel temperature increase. In this connection, the magnitude of the negative fuel temperature reactivity coefficient and the characteristic thermal conductance of the fuel are fundamental. A change in β has a similar effect for the three reactor types.

The moderator reactivity coefficient α_m varies much with the burn-up. With a negativa α_m we find a damping of the transient for type A in comparison to the conditions for types B and C. For positive α_m the case is just the reverse. The systems have turned unstable for $\alpha_m = 10$ pcm/°C, but with the base values for the other parameters the periods are long. As one expects, the periods decrease with increasing α_m. The period for type A is much smaller than for type B and C.

The importance of the negative reactivity coefficient of the fuel, α_f, is seen when comparing cases 4, 11, 5 and 12. A halving of the magnitude of α_f entails a doubling of the power and temperature transients. In the unstable cases the periods become shorter when the magnitude of α_f decreases.

The characteristic thermal conductance of the fuel, H_0, has a strong influence on the kinetic behavior. The conditions for different values of α_m and α_f is shown by a comparison of cases 2, 7, 8, 5, 9, 10, 11, 13, 12 and 14. H_0 is seen to be an important parameter. The base value corresponds to oxide fuel and the highest value corresponds to metallic fuel. The power transients are strongly influenced by H_0. The fuel temperature transients are influenced much less. The coolant temperature transients are similar to the power transients and therefore depend strongly on H_0. The stability is very dependent on H_0 and a big H_0 in connection with a α_f of small magnitude and a positive α_m can for type A result in an unpleasantly short period in the increasing transient. For type B, and especially for type C, the transients increase with substantially longer periods.

The heat capacitance of the secondary of the heat exchangers, C_s, influences the transients from reactivity disturbances only after a long time. In the unstable cases C_s influences the stability rather much, as can be seen by comparing cases 5 and 15.

It is advantageous for the kinetic properties of heavy water reactors that the moderator is kept separated from the reactor power cooling system as this leads to a substantial decrease of the influence on reactor transient behavior of a positive moderator temperature reactivity coefficient. In this regard type C is the best reactor and type B better than type A. At normal operation, when one is regulating the reactors by control rods, the differences between the three types will turn out to be small.

APPENDIX

9. The accurate representation of the fuel heat transport

The parameters of a mathematical model for the temperature distribution and heat flow in a cylinder will be derived under the following conditions (i) - (vi).

/(i) The temperature is the same over the cylinder surface. /
/(ii) The thermal conductivity and specific heat are constant. /
/(iii) The power is uniformly distributed in the cylinder. /
/(iv) In the mathematical model the fuel shall be radially divided in equal-mass parts. /
/(v) In the mathematical model every fuel part shall be treated as having a common temperature. /
/(vi) At stationary conditions, the temperatures of every part of the mathematical model should be equal to the true average temperature of the corresponding part in the cylinder. /

The quantities introduced in the derivation are defined below:

x	radial coordinate. Cylinder surface at $x = 1$
P	power liberated in cylinder with radius x
T	temperature at radius x
V	volume of cylinder with radius x
P_0	total power in cylinder
T_0	average temperature of cylinder
V_0	total volume of cylinder
H_0	characteristic thermal conductance. (See eq. (46))
C_0	total thermal capacitance of cylinder
i	index for i^{th} part in mathematical model
n	number of parts in mathematical model
T_i	average temperature of i^{th} part
P_i	heat flow from i^{th} part
H_i	thermal conductance between i^{th} and $(i+1)^{th}$ part
C_i	thermal capacitance of i^{th} part

For a reactor, we have defined the characteristic thermal conductance of the fuel as: (total power in fuel)/(average temperature rise in fuel). Correspondingly we use the following definition for the cylinder:

$$H_0 = P_0/T_0 \tag{46}$$

According to the assumptions, the temperature at stationary conditions will be parabolically distributed with the center temperature rise equal to two times the average temperature rise. One has for the power, temperature, and volume, the following three equations:

$$P = P_0 x^2 \tag{47}$$

$$T = 2T_0 (1 - x^2) \tag{48}$$

$$V = V_0 x^2 \tag{49}$$

For the average temperature of the i^{th} part one obtains:

$$T_i = \frac{2T_0}{1/n} \int_a^b (1 - x^2)\, 2x\, dx \tag{50}$$

$$a = ((i-1)/n)^{1/2} \tag{51}$$

$$b = (i/n)^{1/2} \tag{52}$$

After performing the simple integration one obtains:

$$T_i = nT_0 (2x^2 - x^4) \Big|_a^b = T_0 (2 - 2i/n + 1/n) \tag{53}$$

The temperature difference between adjacent parts is:

$$T_i - T_{i+1} = \frac{2}{n} T_0 \quad ; \quad i \neq n. \tag{54}$$

For the power the following equation is valid:

$$P_i = \frac{i}{n} P_0 \tag{55}$$

To satisfy condition (vi), we write for the thermal conductance:

$$H_i = \frac{P_i}{T_i - T_{i+1}} = \frac{i}{2} H_0 \ ; \quad i \neq n. \tag{56}$$

For the thermal conductance from the last part, one has:

$$H_n = \frac{P_0}{T_n - 0} = nH_0 \tag{57}$$

For the thermal capacitance, one has for every part:

$$C_i = \frac{1}{n} C_0 \tag{58}$$

To test the mathematical model, the solution from a model with three parts has been compared with the exact solution for a power step. The difference between the exact and model solutions was found to be small and indicated that a division in three parts should be sufficient even for rather violent power transients.

Temperature dependence in thermal conductivity and specific heat is easily considered by introducing the appropriate temperature dependence in H_i and C_i.

In space-independent calculations the thermal flux distribution is similar to the initial distribution. The power has the same distribution as the thermal flux. If the reactor uses cylindrical fuel rods, it is therefore possible to divide the fuel radially as described above. If the coolant temperature rise is of the same magnitude as the temperature rise in the fuel, it might be appropriate to use a somewhat more complicated model for describing the fuel and coolant thermal conditions.

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