COMITATO NAZIONALE ENERGIA NUCLEARE

CENTRO DI CALCOLO

Doc.CEC(74) 1

ONE GROUP NEUTRON FLUX AT A POINT IN A CYLINDRICAL REACTOR CELL CALCULATED BY MONTE CARLO

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BOLOGNA - VIA MAZZINI, 2

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Abstract

Mean values of the neutron flux over material regions and the neutron flux at space points in a cylindrical annular cell (one group model) have been calculated by Monte Carlo. The results are compared with those obtained by an improved collision probability method.

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1. INTRODUCTION

Sufficiently accurate techniques for design purposes are considered important for estimations of thermal absorption rates in a reactor cell. The real reactor cell has local inhomogeneities so severe that the usual diffusion theory approximations are no longer valid. On the other side we need a solution to a good degree of accuracy with a reasonable cost of computer time. Therefore it is very important to develop methods and compare their results with those of well established methods.

In the present work two different Monte Carlo calculations are performed. First, mean values of the thermal neutron flux over material regions in a cylindrical annular reactor cell are calculated using the track length estimator. Collision probability method results [1] are compared with these ones. Second, neutron flux at space points is calculated.

The estimation by Monte Carlo of the reaction rates at points or in small portions of phase space has become more and more attractive in recent years. Since the idea is a few years old there are only few papers in this connection [2,3]. A pos

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sible approach is based on the reciprocity theorem and was already used to calculate the thermal neutron flux "at a point" by an estimation of average value of adjoint flux in a small volume surrounding the space point [4].
Using the linarity of the neutron transport equation it is possible to express the neutron flux by Green's function, obtaining the neutron flux at a point by Monte Carlo.
The collision probability results are in excellent agreement with those obtained by Monte Carlo.

2. CELL MODEL

The cell is assumed to be infinite in extent in z-dimension and cylindrically symmetric so that the model is one dimensional spatially and we need to compute the flux $\phi(r)$.

To compare Monte Carlo results with those obtained by first collision method, we assume the same boundary conditions as in [1], i.e.; 1) neutrons crossing the boundary from inside have probability α (the albedo of the boundary) of returning back; 2) the angular distribution of the neutrons entering the cell from the boundary is assumed to be uniform.

We consider two types of interaction of a neutron with a nucleus: elastic scattering and absorption. The relative probabilities for each event are specified by the cross section for the process in question. All nuclei are assumed to scatter isotropically in the laboratory system.

The neutron source is assumed isotropic and constant in each of the N regions with given intensity S_i $(i=1,\dots,N)$ The probability p_i to start in the region i, with area A_i, is thus:

$$P_i = \frac{S_i A_i}{\sum_{i=1}^{N} S_i A_i} \tag{1}$$

Let α be the known albedo, w the probability of first escape for a neutron born inside the cell, and q the probability of first escape for a neutron entering the cell from the boundary. Then the boundary condition may be taken into account as an additional surface source, whose intensity s per neutron born inside is clearly

$$s = w \cdot \frac{\alpha}{1 - \alpha q} \tag{2}$$

The two probabilities w, q will be estimated by two distinct Monte Carlo calculations.

The quantity that we need to calculate is

$$\phi_k = \int_{r_k}^{r_{k+1}} \phi(r)dr \qquad (3)$$

where r_k, r_{k+1} are the radii limiting the k-th region.

3. TRACK LENGTH ESTIMATOR

The total neutron flux $\phi(r)$ is by definition the sum of the path lengths traveled per second by all neutrons contained within the unit volume about $\vec{r}$. Thus one may write for the mean flux φ in a region due to one source neutron/sec

$$\varphi \cong \frac{1}{n} \sum_{i=1}^{n} \sum_{j=1}^{n_i} w_{ij} \cdot t_{ij} \qquad (4)$$

where n is the number of neutron histories

n_i is the number of free-paths in the region during neutron history i

w_{ij} is the current neutron weight

t_{ij} is the j-th free path of the neutron in the region during history i

Using estimator (4) both the flux due to real neutron source, φ_v, and the flux due to unit surface source, φ_s, are estimated.

For the i-th region it will be, for the linearity of the transport equation,

$$\varphi_i = \varphi_{v_i} + s \cdot \varphi_{s_i} \qquad (5)$$

and the mean value of the flux per unit volume

$$\phi_i \cong \frac{\varphi_i}{A_i} \sum_{k=1}^{N} S_k A_k \tag{6}$$

4. FLUX AT A POINT

Estimator (4) is very inefficient to compute the flux in certain small part of the volume. Therefore we need a different estimator in this case.

Using the reciprocity relation [2],valid for anisotropic scattering and free boundaries,

$$G(r \to r') = G(r' \to r) \tag{7}$$

and the superposition theorem, the flux at a point r is

$$\varphi(r) = \int G(r \to r')\ S(r')dr' \tag{8}$$

where $G(r \to r')$ is the scalar flux at the point r' due to a unit isotropic source at a point r, and $S(r)$ is a given neutron source .

In our case the contributions to the flux $\phi(r)$ are given by the source over the cell and by the surface source and we can write

$$\phi(r) = \phi_V(r) + \phi_S(r) \tag{9}$$

To compute $\phi_V(r)$ let us denote by n_i the number of collision in the space volume of the cell ΔV_i about r_i due to n original neutrons selected from unit isotropic source at the point r . Then the mean value of $G_v(r \to r_i)$ in the region i is:

$$G_{v_i} \cong \frac{n_i}{n \, \Delta V_i \, \Sigma_{t_i}} \tag{10}$$

where Σ_{t_i} is the total macroscopic cross section

G_{v_i} is computed with the free boundary condition on the cell boundary.

Therefore we find the component ϕ_v of the flux as

$$\phi_v(r) \cong \sum_{i=1}^{N} \frac{n_i S_i}{n \, \Sigma_{t_i}} \tag{11}$$

or, using the weights,

$$\phi_v(r) \cong \frac{1}{n} \sum_{i=1}^{N} \frac{S_i}{\Sigma_{t_i}} \sum_{j=1}^{n_i} w_{i_j} \tag{12}$$

where w_{i_j} is the neutron weight, or using the track lengths

$$\phi_v(r) \cong \frac{1}{n} \sum_{i=1}^{n} \sum_{j=1}^{m_i} w_{i_j} d_{i_j} S_i \tag{13}$$

where d_{i_j} is the free path length between two successive collisions and m_i is the number of collision during a neutron history (boundary crossing is considered as a collision in the formula above).

To compute the surface source contribution we again apply the reciprocity theorem, obtaining

$$\phi_s(r) = \int G(r \to r_s) s dr_s$$

Where the intensity s has been calculated as described in Section 2 and r_s is a point of the surface. Since $G(r \rightarrow r_s)$ is the number of neutron crossing normally the boundary in r_s per neutron born in r, we can estimate $\phi_s(r)$ from n histo ries as follows

$$\phi_s(r) \cong \frac{s}{n} \sum_{1}^{n} {}_i \frac{w_i}{\cos \vartheta_i}$$

where w_i is the weight of the outgoing neutron and ϑ_i the angle (in the space) between the direction of motion and the normal to the boundary.

5. RESULTS

In Table I are given results of average fluxes and form factors from both Monte Carlo and collision probability method calculation. There are big differences in the case of strong flux variation (region 5) or in large region (region 9).

Results are obtained with 10.000 history and standard deviation is always below 5%.

In Table II are given results of fluxes at a point. We note the eccelent agreement of these results. All values are calculated with 10.000 histories and standard deviation is less than 4% .

Table I

			Mean Flux		Form Factor	
Region	Material	Radius (cm)	Monte Carlo	Ref.(1)	Monte Carlo	Ref.(1)
1	D_2O	1.050	142.74	142.77	1.132	1.1036
2	Al	1.200	139.26	139.59	1.105	1.1079
3	D_2O	1.450	137.17	137.30	1.088	1.1061
4	Al	1.550	129.27	132.29	1.025	1.0226
5	U	1.750	126.03	129.36	1.0	1.0
6	Al	1.850	137.91	140.37	1.094	1.085
7	D_2O	2.050	143.063	146.41	1.139	1.1317
8	Al	2.150	146.50	150.57	1.162	1.164
9	D_2O	7.3344	172.52	176.68	1.368	1.366

Table II

	Flux at a Point			Flux at a point	
Radius r(cm)	Monte Carlo	Ref.(1)	Radius r(cm)	Monte Carlo	Ref.(1)
0.0	144.38	144.42	1.800	140.47	140.38
0.840	142.08	142.31	1.892	144.59	144.23
1.125	139.90	139.59	2.008	148.41	148.47
1.252	138.21	138.68	2.100	150.29	150.58
1.397	135.28	136.08	2.284	154.76	154.35
1.500	132.67	132.29	2.832	163.55	163.18
1.564	128.94	128.33	3.718	171.37	171.67
1.616	127.72	127.21	4.789	177.50	177.57
1.684	128.85	129.34	5.860	181.07	180.68
1.736	133.57	134.11	6.748	181.59	181.65

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