3.3 INTERNATIONAL PIPING BENCHMARKS: USE OF SIMPLIFIED CODE PACE 2

by
J. Boyle and J. Spence

This section presents the results for analyses of all three recommended International Piping Benchmark Problems. A simplified inelastic analysis was performed for each problem using the PACE 2 code which, utilizing beam bending theory, models pipes or elbows as a series of beam segments.

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J BOYLE, J SPENCE, University of Strathclyde

Edited by C Blundell, Central Technical Services

Risley Nuclear Power Development Establishment
Risley, Warrington, WA3 6AT

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SUMMARY

This report compares the results obtained using the code PACE 2 with the International Working Group on Fast Reactors (IWGFR) International Piping Benchmark solutions. PACE 2 is designed to analyse systems of pipework using a simplified method which is economical of computer time and hence inexpensive. This low cost is not achieved without some loss of accuracy in the solution, but for most parts of a system this inaccuracy is acceptable and those sections of particular importance may be reanalysed using more precise methods in order to produce a satisfactory analysis of the complete system at reasonable cost.

CONTENTS

1. INTRODUCTION

The principal aim of this report is to provide a comparison between a selection of the IWGFR International Piping Benchmark solutions and results obtained using the inelastic piping code PACE 2, developed by the authors.

This report was produced on the recommendations of the IWGFR Specialists' Meeting on High Temperature Structural Design Technology held in Champion Pennsylvania, USA, during April 1976. The meeting recognised that some kind of international collaboration on piping benchmarks was required and emphasis has been placed in this report on evaluation of recently developed simplified methods for the analysis of the inelastic deformation of complex piping systems in a benchmark context. The aim of this collaboration was to collect a number of problems for which experimental results were available, and to present these to various analysts in IWGFR member countries for solution. The compilation of benchmark problems and resulting solutions was to be undertaken by Oak Ridge National Laboratory, USA, with a view to eventually providing a report summarising the results. Eventually three suitable benchmark problems were identified and released in a uniform format [1-4]. These are:

a. Elevated Temperature Elastic-Plastic-Creep Test of an Elbow subjected to In Plane Moment Loading. Submitted by A IMAZU, R MIURA, K NAKAMURA, T NAGATA and K OKABAYASHI, of the Power Reactor and Nuclear Fuel Development Corporation, Japan.

b. Tests at 593°C (1100°F) on a 101.6 mm (4 in) Schedule - 10 Elbos Pipe Assembly subjected to In-Plane Moment Loading. Submitted by W I GRIFFITH and E C RODABAUGH of Battelle-Columbus Laboratories, USA.

c. Room Temperature Elastic/Plastic Response of a Thin-Walled Elbow subjected to In-Plane Bending Loads. Submitted by D BROUARD, R L ROCHE and B VRILLON of Commisariat a l'Energie Atomique, Centre d'Etudes Nucleaires de Saclay, France.

These will be described in more detail later. It was proposed at the IWGFR meeting that not only the simplified methods but also more detailed finite element calculations should be performed, presumably for comparison purposes. This report will be divided into four sections. The first will describe the benchmark problems themselves and extract solutions, the second will briefly describe the basic assumptions and current development of the program PACE 2. The third and fourth sections will deal with the results of the comparison and discuss their significance.

2. DESCRIPTION OF THE BENCHMARK PROBLEMS

Summaries of the benchmark problems (roughly keeping to the format of the original document (1) give problem description, data on material properties and results) with brief comments.

2.1 Benchmark No 1

Tests were conducted on a 12 in Schedule 20 Type 304 steel elbow assembly by applying in-plane moment loadings at 600°C (1112°F). The test lasted approximately 340 hrs and seven different load levels were used. One end of the assembly was rigidly fixed while a transverse concentrated force was applied at the other end. During the test the free end deflection and the radial cross sectional deformation of a selected cross section of the elbow were measured. A summary of the nominal dimensions of the elbow assembly is given in Fig 1 from Ref (1). In an addendum (2) more detailed information on the cross sectional dimensions of the assembly is supplied. The variations in thickness and radius mentioned therein will be ignored here.

The stress/strain properties for the pipe assembly material Type 304 steel are shown in Table 1. A total deformation Ramberg-Osgood relation is recommended for the elastic-plastic behaviour, with a Dorn total strain equation for the creep strain/time relation using a Garafalo relation for the minimum creep rate/stress dependence.

The results of the test are presented in the form of two (short) tables and three graphs giving

i Assembly free end deflection against time for the entire test with numerical values for the start and end of the first two loading steps.

ii The relationship between load and free end deflection for each loading step.

iii The measured radial displacement of the elbow at a particular point on the elbow for the start and end of each of the first two loading steps, together with the radial displacement distribution round the cross section at the start and end of the third loading step. From the latter data the circumferential strain distribution is calculated approximately using thin curved pipe shell theory. Finally, the relationship between measured free end deflection and calculated strain is given for the first four loading steps.

Although these represent only selected results this test is well documented elsewhere (5-7). In evaluating this benchmark, reference has also been made to these documents. Three aspects of the test are worth mentioning here. Firstly, actual strains recorded during the test are not quoted, rather strains calculated from the radial displacement on the assumptions of pure bending of the elbow and inextensionality of the cross section are given. It is known that strains were measured during the test and that these did not compare well with the calculated strains, possibly due to inaccuracies in the gauges. Secondly, there is also a fair amount of scatter between the free end deflection and calculated strains in the first three loading steps which presumably should show a linear dependence. Thirdly and finally, actual data on the inelastic response of the assembly material is not known. Therefore quoted material relations have been taken from the literature, although the quoted relations have previously been known to yield poor comparison with the experimental results (5-7). It is unfortunate that a total strain relation is recommended for the creep data since comparison cannot be expected to be fair if the time functions were derived from constant loading tests.

2.2 Benchmark No 2

Tests were conducted at 593°C (1100°F) on a 90° elbow assembly meeting ANSI Standard 4 in Schedule 10 specifications by applying in-plane moment loadings. The assembly was manufactured from Type 304 steel supplied by Oak Ridge National Laboratory and designated by its heat treatment number 9T 2796. One end of the assembly was rigidly secured (and considered clamped) while the other (free) end was covered by a rigid frame used to apply the load by way of a weight and pulley system.

The complete assembly was placed inside a thermally insulated duct. During the test the overall deflections of the free end of the assembly were measured, whilst twenty-three strain gauges were placed as critical points. Initially some room temperature proving tests were performed, before the long term creep test at temperature, which consisted of an initial load applied over 295 hrs followed by an increment to a higher load for 44 hrs. A summary of the dimensions of the assembly is given in Fig 2. Also provided in (1) are actual dimensions of the specimen (prior to testing). Whilst some significant variations in the thickness was observed, due to the pipe matching in manufacture, these will be ignored here. Again the material properties data is presented in two parts. The initial elastic-plastic tensile curve is based on a Ramberg Osgood relationship. An equation of creep for constant stress conditions, Table 2 is provided from the actual stress-creep strain-time data, taken from an Oak Ridge report (9). In an addendum (3) a suitable Dorn type total strain creep equation is supplied by the staff of Oak Ridge.

The results of this test are presented in the two tables 5 & 7 of numerical values representing the measured free end deflections and strains, at selected positions. Also given are the results of the room temperature proving test.

The information provided for this benchmark gives a fairly complete picture of the entire test, which again is well documented elsewhere (10, 11). Once more it is difficult to expect agreement with the step loading part of the test, since only material data for constant loading creep tests is supplied.

2.3 Benchmark No 3

A thin-walled, large diameter elbow of ICN 472-SP-304 stainless steel piping assembly consisting of a 180° bend with attached equal length straight tangent pipes was used for these tests, performed entirely at room temperature. The sections of tangent pipe at each end of the elbow allowed suitable extensions for load application in the plane of the bend. One end of the assembly was fixed, whilst a transverse load, permitting application of both opening and closing moments, was applied to the other (free) end. The dimensions of the assembly are given in Fig 3, taking into account the information given in an Addendum (4): the pipe wall thickness was again supplied in some detail and indicated only a trivial variation from the nominal thickness. The loading history of this test was quite complex as is shown in Fig 4 and is described below:

i Initially (load step 0) a small internal pressure was introduced into the assembly giving a slight "opening" of the bend. The elbow was subsequently cycled in the elastic range using opening and closing moments of the same magnitude (load steps 1 to 12).

ii Load steps 13 to 21 consisted of an alternating, but somewhat irregular, amplitude of moment application whilst from load steps 22 to 27 a positive increasing magnitude moment was applied to bring the elbow into the plastic range. The elbow was then closed with a moment comparable to the opening moments applied previously.

iii Load steps 30 to 33 consisted of a positive moment with internal pressure followed by an alternating increasing moment magnitude until instabilities were evident for the closing moments (load steps 34 to 41).

iv The test was continued with increasing opening moments until failure occurred (load steps 42 to 59).

In each load step the deflection of the elbow position of the assembly was measured and the strains associated with one (cross sectional) plane of the elbow were obtained using strain gauges mounted both on the inside and outside surfaces of the specimen.

The uniaxial monotonic elastic/plastic tensile curve for the assembly material is shown in Fig 5. The numerical data is given in Table 3. Rough graphs of the elbow deflections for each loading step are presented, figures whilst the strain distributions around the cross section are given for loading steps 26 and 46. No numerical values are given. Again this test is wellknown (12, 13), although few of the strain results have been presented. It is unfortunate that the cyclic stress strain data for the assembly material is not available. Also it would have been useful, given the amount of data generated, for selected numerical values of the deflection to have been given. Finally, it is noted that the sequence of loads given in Ref 1; Table 2b, Fig 3, is incorrect, viz (12). The correct sequence is given in Fig 4.

2.4 Existing benchmark comparisons

Since each of these benchmarks has been extensively described in the available literature, comparisons with some of the piping codes have been already attempted.

The Japanese test has been compared with the nonlinear finite element code MARC (5-7) and the simplified code PACE 2 (14). Comparison was found to be poor in general, although it is worth pointing out that the PACE 2 results in (14) were obtained using a different elastic/plastic constitutive equation from that eventually used here.

The Battelle test has been compared with the results of ELLAX (10), TRICO (16) and TEDEL (16) although some ad-hoc modifications to the simplified codes ELLAX and TEDEL were necessary to account for end effects. The designation ELLAX here indicates the simple analysis using flexibility coefficients outlined in (15).

Finally the French test has been compared with TRICO (17), TEDEL (17) and PACE 2 (14) with excellent results (although it will be shown here that the results of PACE 2 can be improved).

3. THE INELASTIC PIPING CODE PACE 2

The inelastic piping code PACE 2 (13) is a modified and improved version of the original code PACE (19) developed by the authors for the UKAEA. The performance of its basic simplifications and assumptions has been described elsewhere (14); here these are summarised:

a. At present the program only considers two-anchor, but spatially three dimensional piping systems subject either to concentrated loadings or prescribed thermal displacements. The modification required to incorporate multianchor runs is straightforward although there is a lack of information on the performance of junctions in plasticity and creep.

b. The deformation of the system, assumed composed only of straight or curved sections, is modelled using simple beam bending theory. The influence of cross sectional flattening on the deformations produced by the curved sections, is introduced by way of "flexibility factors". Throughout small deformations are assumed, although it would be possible to include instabilities due to large deformations as described in Refs (14) and (20) but this facility is not yet available. Inelastic deformations are modelled using a global constitutive model (14) derived from the local uniaxial material data using the well established approximation of super-position of (deformation) states.

c. The program can deal with plastic or with creep strains in the system (but not both as in visco-plasticity). It is assumed that the elastic/plastic material relation takes the form of a total deformation equation, with the plastic strains related to stress by a Ramberg-Osgood power law equation. Currently an incremental formulation is being verified but, as yet, is not operational. Similarly the creep material relation is assumed to be of the time-hardening type with the minimum creep strain rate related to stress by a Norton power law. These restrictions are only necessary since the global constitutive equation for the piping system is based on the use of flexibility factors for the inelastic strain derived from power laws. It is possible to use other constitutive equations employing suitable inelastic flexibility factors derived from a best fit power law. (Alternatively the appropriate flexibility factor could be derived within the program for any constitutive relation supplied).

d. The inelastic flexibility factors are approximated in the program using the formula (14)

$$K_n = K_E \left(\frac{D_n}{\pi}\right)^n \alpha^{n-1} \qquad (1)$$

where K_E is the ASME recommended in-plane flexibility factor, n is the exponent in the power law,

$$D_n = \int_0^{2\pi} \sin\phi \quad 1 + \frac{1}{n} \quad d\phi$$

and

$$\alpha = \frac{p}{\lambda q} \qquad \lambda = \frac{Rt}{r^2}$$

where R is the bend radius, t the thickness, r the cross sectional radius and

$p = 1 \qquad q = 1/2$

For upper and lower bound results

$p = 1 \qquad q = 5/3$

$p = \pi/4 \qquad q = 5/3$ respectively.

These flexibility factors were developed on the assumption of pure bending of a pipe bend, and are not immediately applicable to situations where end effects (such as a tangent pipes, flanges, etc) are of importance. In order to include the influence of end effects here the program uses a modification to the elastic flexibility factor and on "reference stress" arguments: If $K_{E,ASME}$ is the ASME recommended flexibility factor, and $K_{E,ESDU}$ are the elastic flexibility factors which include end effects (21) then we define a reduction factor

$$k = \frac{K_{E,\ ESDU}}{K_{E,\ ASME}} \qquad \text{(ii)}$$

In eqn (i) α is a geometrical parameter, independent of the material parameters and can be identified with the "reference stress parameter" for the power law flexibility factor (22, 23, 24). Since a log-log plot of α against the pipe bend parameter λ is linear, end effects could be approximated by a translation of the plot. Then only p is altered and it can be shown that this must be proportional to k in (ii) by applying the reference stress argument to the elastic case. Therefore in (i) K_n is reduced by a factor k^n, ie

$$K_n\Big|_{modified} = k^n K_n \qquad \text{(iii)}$$

This relation is only approximate owing to approximations in the reference stress argument itself and remains to be verified.

e. When evaluating the appropriate stress concentration factors a similar modification is made for the influence of end effects. At present simple formulae for the unmodified factors are not available: the required values are input to the program from a preprocessor. The results are then modified on a basis of the reduction factors for the elastic Stress Concentration Factors (21). It is assumed that a similar reduction can be applied to the power-law SCF's (this assumption being based on the fact that the maximum SCF is approximately linear in the reciprocal of the exponent in the power law (22)). However it should be noted that the results are very sensitive to the reduction factor and at present this can only be estimated from the data given in (21) which only relates to pipe bends in which R/r = 2.

f. The problem of combined loading on piping components for inelastic behaviour is treated in the program by suitable definition of the global constitutive model (although it is acknowledged that there is little information to check this). In the program a global constitutive model for the straight pipe is adopted, while that for the curved pipe is developed from this using "flexibility" or "load amplification" factors (16) derived from single loading cases. The applicability of this approach has been partly proven by comparison with other solutions (25).

g. The program formulates the elastic/plastic behaviour as a system of non-linear algebraic equations, and the creep behaviour as a system of first order ordinary differential equations in time (26). The former (algebraic) can be resolved using initial strain, initial stress or tangent modulus formulations of Picard or Steffenson iteration with continuation. The latter can be resolved using simple Heun integration or more sophisticated Runge-Kutta-Gill or Fehlberg (with error controls) procedures at any integration step. Special Filon integration routines have been adapted for spatial integrations in curved sections, with more common Gaussian formulae for the straight pipe.

The code PACE 2 is in a continuing state of development and improvement. At present it consists of just ove 2,000 lines of FORTRAN statements, although it is currently being simplified into an interactive form in BASIC suitable for implementation on mini-computers. The code was originally written for an ICL 1904S, although it has been successfully implemented on an IBM 360, GE 415, CDC 7600 and H 6060 with practically no alteration and is quite portable. It is modular in format to allow for the easy addition (and deletion) of solution routine segments.

4. DATA PREPARATION

The three benchmarks were prepared for the current version of PACE 2 on the University of Strathclyde ICL 1904S. The total running time for all of the benchmark runs was of the order of 25 seconds, although it should be noted that this machine is relatively slow.

The data for Benchmarks 1 and 3 was transformed into Imperial Units for the run (since the program works best in these units), although the results are presented in their original units. Further details concerning the runs are listed in the following:

4.1 Benchmark No 1

The basic dimensions of this assembly have been taken as,

Nominal mid c/s radius	= 6.14 in
Wall thickness	= 0.26 in
Bend radius	= 18 in
Length of fixed pipe leg	= 47.24 in
Length of free pipe leg	= 68.61 in

It should be pointed out that the free pipe leg has been modelled entirely by a straight pipe of length equivalent to the distance from the end plane of the elbow to the point of application of the load, ignoring the tee junction associated with the air duct and the loading bracket. In fact if these are included (by increasing the stiffness of the leg) the results do not alter significantly.

The elastic material properties have been taken as

Young's modulus = 21.7×10^6 psi

Poisson's ratio = 0.3

and the plastic stress strain relation as

$$\epsilon_p = 2.97 \times 10^{-37} \quad \sigma^{8.045}$$

where σ is in psi and the premultiplying scaling factor has dimensions such that a strain is produced. Since the actual material properties are unknown the creep equation has been taken from the literature as being typical of austenetic steels at 600°C. Then the minimum creep rate is related to the stress by

$$\dot{\epsilon}_{min} = 2.4 \times 10^{-22} \quad \sigma^4$$

where σ is in psi and $\dot{\epsilon}_{min}$ in $(hr)^{-1}$.

The pipe bend parameter λ for the pipe bend is

$$\lambda = 0.122$$

so that the ASME recommended elastic flexibility factor is

$$K_E\Big|_{ASME} = \frac{\sqrt{12(1 - \nu^2)}}{2\lambda} = 13.54$$

The equivalent ESDU elastic flexibility factor, taking into account the effect of the tangent pipes is

$$K_E\Big|_{ESDU} = 10.6$$

so that the reduction factor is

$$k' = 0.783$$

Consequently the reduction factor for plasticity is 0.14, and for creep is 0.38.

4.2 Benchmark No 2

The basic dimensions of this assembly (already in Imperial units) are taken as

Nominal mid o/s radius	=	2.19 in
Thickness	=	0.12 in
Bend radius	=	6 in
Length of fixed pipe leg	=	12.75 in
Length of free pipe leg	=	12.75 in

The elastic material properties have been taken as

Young's modulus = 21.7×10^6 psi

Poisson's ratio = 0.3

and the plastic stress strain relation as

$$\epsilon_p = 3.88 \times 10^{-41} \sigma^{9.3553}$$

where σ is in psi. The relation quoted in (1) is

$$\epsilon_p = 4.59 \times 10^{-41} \sigma^{9.3377}$$

but it is readily verified that the relation used here provides a better fit of the supplied data in Table 2.

The minimum creep rate is related to the stress by

$$\dot{\epsilon}_{min} = 1.02 \times 10^{-25} \quad \sigma^{4.6}$$

where σ is in psi, $\dot{\epsilon}_{min}$ in $(hr)^{-1}$. This relation was found by a weighted least squares fit of a power law to the log transform of the supplied data in Table 2 (and not to the unweighted log-transformed data which can lead to spurious results which don't minimise the residual). The actual least squares solution for the untransformed data is

$$\dot{\epsilon}_{min} = 2.8291 \times 10^{-26} \quad \sigma^{4.6878}$$

In fact the results herein were based on the former equation although the later equation gives essentially the same results. The creep equation used by Griffith and Rodagaugh in (10).

$$\dot{\epsilon}_{min} = 6.579 \times 10^{-21} \quad \sigma^{3.711}$$

was obtained from short term creep data and was consequently not used here.

The pipe bend parameter for this case is

$$\lambda = 0.15$$

so that

$$K_E\Big|_{ASME} = 11.01 \qquad\qquad K_E\Big|_{ESDU} = 8.6$$

and the reduction factor is

$$k = 0.781$$

The equivalent reduction factor for plasticity is then 0.099, and for creep is 0.32.

During the test, displacements δ_1 δ_2 and δ_3 (in) of the end of the attached loading frame were measured. In the program δ_x δ_y (in) and θ (rod) are obtained (Fig 6); these are related to the measured displacements by

$$\delta_1 = 12\theta + \delta_y$$

$$\delta_2 = 12\theta - \delta_y$$

$$\delta_3 = \delta_x - 6.5\theta$$

4.3 Benchmark No 3

The dimensions of this assembly are taken as

Nominal mid c/s radius	= 9.84 in
Wall thickness	= 0.47 in
Bend radius	= 30 in
Length of fixed pipe leg	= 27.17 in
Length of free pipe leg	= 27.17 in

The elastic material properties are taken as

Young's Modulus = 17.7×10^6 psi

Poisson's ratio = 0.3

with the monotonic elastic/plastic stress/strain relation as

$$\epsilon_p = 2.99 \times 10^{-33} \; \sigma^{6.6485}$$

where σ is in psi. This relation was found by a least squares fit of the supplied data (Table 3) to a power law. The pipe bend parameter is

$\lambda = 0.146$

so that

$$K_E\Big|_{ASME} = 11.35 \qquad K_E\Big|_{ESDU} = 10.2$$

and the reduction factor is

$k = 0.9$

and is 0.50 for plasticity.

5. RESULTS AND DISCUSSION

5.1 Results

The results of the benchmark runs are shown in a number of Figures and Tables listed below. As far as possible the format of the original document (1) has been adhered to:

Fig 7 shows the free end deflection in Benchmark No 1 against time for the first four loading steps.

Table 4 gives numerical values for the free end deflection in Benchmark No 1 at the beginning and end of the first two loading steps.

Fig 8 shows the normalised relationship between load and free end deflection for the first four loading steps.

Fig 9 shows the circumferential strain distribution at the beginning of the third loading step.

Fig 10 shows the measured displacements during the creep test in Benchmark No 2 against time.

Fig 11 shows the measured strains during the creep test against time.

Table 5 gives numerical values of the measured displacements in the room temperature test for Benchmark No 2.

Fig 12 shows the elbow load-deflection behaviour for load steps 0 to 28 in Benchmark No 3.

Fig 13 shows the outside circumferential strain distribution for load step 26.

In each case the actual Benchmark results have been shown also for comparison. For completeness results obtained using unmodified flexibility factors are shown:

Table 6 gives a comparison of calculated free end displacement in Benchmark No 1 using modified and unmodified factors.

Table 7 gives a comparison of calculated displacements in Benchmark No 2 using modified and unmodified factors.

Table 8 gives a comparison of calculated elbow load-deflection behaviour for load steps 1 to 18 in Benchmark No 3 using modified and unmodified factors.

5.2 Discussion

Before examining in detail the significance and consequences of these benchmark comparisons we should consider firstly how they should be interpreted. The authors have previously suggested a tri-phase approach to the analysis of piping systems for design purposes (27). Briefly summarised these phases are:

a. The complete system should be analysed using a primary method which should be cheap and efficient yet retain sufficient accuracy to predict the overall behaviour of the system.

b. On a basis of the primary analysis a small loop of interest can be isolated and reanalysed using a secondary method which would provide a more accurate evaluation of the geometrical and material behaviour.

c. Finally, suspected overstressed components could be isolated and a tertiary method, capable of including detailed material and geometrical behaviour, employed.

Examples of primary methods are TEDEL, PACE 2 or PIRAX 2, of secondary methods MARC (element 17) or the Ohtsuba and Watanabe ring elements (28) and of tertiary methods any nonlinear finite element code using shell elements eg MARC, EPACA, TRICO or ANSYS. The results of any benchmark comparison should be interpreted with reference to the above. In particular the results of a primary method should not be compared to those of a secondary method, etc.

Bearing in mind the minimal running time and data preparation the results of the benchmark comparisons with PACE 2 are excellent, in particular for Benchmarks 1 and 3 (although it should be noted that the elastic/plastic behaviour in Figs 8 and 12 are, only represented by a single loading curve. Also the strains derived from Fig 9 and 13 could be improved, given better information of the influence of end effects). The results of Benchmark No 2 are slightly offset by the initial plastic solution (and in the second step greatly offset by the plastic/creep interaction). Other known comparisons with this Benchmark have been equally interesting. For example, Griffith and Rodabaugh (10) obtain acceptable agreement with reduction factors on the flexibility factors, derived from the room temperature experiments, of 0.026 for plasticity and 0.212 for creep (compared to 0.099 and 0.32 here).and Bourrier and Hoffman (16) reported excellent comparisons with TEDEL. It should be noted that the strains obtained here, Fig 11, are much better than those obtained in (10) and (16), in particular the longitudinal strain.

As mentioned in the Introduction, only a selection of the results given in the IWGFR piping benchmarks (1) have been chosen for comparison with PACE 2. As described in section 3, PACE 2 is being extended to include large deflections, plastic/creep interactions by way of an incremental visco-plastic formulation and detailed combined loading situations in particular with internal pressure (about which very little is known about the pipe bend in inelasticity (29)). Therefore those aspects of the Benchmarks which include these phenomena have been omitted from the comparisons. For example, load step 5 onwards in Benchmark No 1 and the second load step in Benchmark No 2 (where deflections δ_1, δ_2 and δ_3 obtained on application of the step are 0.83, 0.57 and 1.32 in respectively, considerably higher than the measured values) have been omitted due to the obvious plastic-creep interaction. Also, in Benchmark No 3 load steps 31 onwards have been ignored due to the influence of internal pressure (from Ref (30) the parameter $Pr^3/M = 0.15$ which, just for the straight pipe, leads to a considerable stiffening).

In conclusion some general remarks on the IWGFR Benchmarks have two possible uses. Firstly, the benchmarks may be used to assess the programs as defined in (1) or secondly, they may be used in the development of a program. The type of benchmark proposed by the IWGFR fall into the first category since they concern programs which are complete and in a final form, eg secondary and tertiary methods using finite element codes developed over a considerable number of years and for which there is a great amount of theoretical backing. However primary methods, based on simplified techniques, are still in a state of development, so that the IWGFR benchmarks are not applicable. Indeed it could be argued that such a comparison would be premature since the benchmarks should be used to further develop the programs. More sensibly one could see the IWGFR benchmarks as a means of developing a theoretical model of the behaviour of piping components (based on finite elements) which adequately predicts the component behaviour and which in turn could be used as a theoretical benchmark for a more complete assessment of the primary methods under loading and environmental conditions different from those used in the actual testing. By definition since the benchmarks have been used in the development of a program, they cannot be used for its assessment (a reason why benchmark comparisons with PIRAX 2 and TEDEL would be unsatisfactory). In short, we need two types of benchmarks, and the IWGFR ones could be used as either. At present the authors think it is wiser to use them for development.

6. REFERENCES

1. OAK RIDGE NATIONAL LABORATORY. International piping benchmark problems published for IAEA, IWGFR November 1977.

2. Addendum 1 March 1978.

3. Addendum 2 April 1978.

4. Addendum 3 June 1978.

5. IMAZU, A and others. PNC internal report March 1976.

6. NAGATA T and IMAZU A. Recent efforts of PNC on creep-fatigue, creep-buckling and thermal ratchetting of the LMFBR primary coolant system components. Specialists meeting on high temperature structural design components. Champion PA 1976. Summary Report pp 171-96 IAEA IWGFR 1976 (IWGFR/11 paper III - 4)

7. IMAZU, A and others. ASME Paper 76-PVP-64.

8. GREENSTREET, W L and others. High temperature structural design methods for LMFBR components. Progress report October 1972 - March 1973. 1973 ORNL-TM-4089.

9. CORUM, J M and WRIGHT, W B (Editors). Pressure vessels and piping: verification and qualification of inelastic analysis computer programs ASME Publ G00088, 1975. pp 99-109.

10. GRIFFITH, W I and RODABAUGH, E C. Report on tests of two four inch sch 10 elbow assemblies 1974. ORNL-Sub-3651-2.

11. GRIFFITH, W I and RODABAUGH, E C. Pressure vessels as reference 9 by CORUM J M and WRIGHT W B.

12. ROCHE, R and others. Phenix power station: 500 mm diameter austenitic steel elbow: interpretation of results of investigation (in French)) CEA Saclay internal report SEMT/74/42 October 1974.

13. VRILLON, B and others. Experimental analysis of piping components of Fast Breeder Reactors. Third International Conference on Structural Mechanics in Reactor Technology, London, 1975 Proceedings Paper F3/4. Bundesanstalt fur Material Prufung, BAM, 1975.

14. SPENCE, J and BOYLE, J T. Simplified methods in pressure vessel analysis 1978. ASME Publ PVP-PB-029.

15. RODABAUGH, E C and WORKMAN, G H. Strain concentrations in an elbow in a piping system under high temperature relaxation conditions. Second International Conference on Pressure Vessel Technology, San Antonio Tex 1973. Proceedings, pp 375-97 ASME 1974.

16. BOURRIER, P and HOFFMAN, A. Plastic creep behaviour of an elbow pipe. Fourth International Conference on Structural Mechanics in Reactor Technology, San Francisco, 1977. Proceedings paper L 6/2. Bundesanstalt fur Material Prufung, BAM, 1978 (also as CEA Saclay Internal Report SEMT/77/34).

17. ROCHE, R and others. Piping systems inelastic analysis: a simplified numerical method. Third International Conference on Pressure Vessel Technology, Tokyo, April 1977. Proceedings pp 133-42.

18. BOYLE, J T and SPENCE, J. Report for the Reactor Group, UKAEA, Con 1R 53345. June 1975.

19. BOYLE, J T and SPENCE, R. PACE, a FORTRAN program for the inelastic analysis of piping systems by simplified methods. Report for the Reactor Group, UKAEA, 1978.

20. HOFFMAN, A and ROCHE, R. CEASEMT system a simple model of the non-linear geometric behaviour of small-bore elbows in the TEDEL program (in French) CEA Saclay internal report SEMT/73/14.

21. ESDU Engineering Services Data Unit, 1975. ESDU 75014.

22. SPENCE, J. _J Mech Eng Sci_ vol 15. 1973 pp 252-265.

23. BOYLE, J T and SPENCE, J. _Int J Pressure Vessels and Piping_ vol 4. 1976 pp 221-252.

24. BOYLE, J T and SPENCE J. ASME Paper PVP-75-88.

25. BOYLE, J T and SPENCE, J. Paper to be given to the _Fifth International Conference on Structural Mechanics in Reactor Technology Berlin 1979_.

26. BOYLE, J T. _Arch Mechanics_ vol 29 1977 pp 229-249.

27. REICH, M and others. Elastic and inelastic methods of piping systems analysis, a preliminary review. 1975. Brookhaven National Lab, Dept of Applied Science Informal Report BNL 19768.

28. OHTSUBO, H and WATANABE, O. _ASME J Pressure Vessel Technology_. May 1977 pp 281-

29. SPENCE, J. _J Strain Anal_ vol 12. 1977 pp 1-7

30. BOYLE, J T and SPENCE, J. _Eng Trans Polish Acad Sci_ vol 22 1974 pp 411-420.

TABLE 1 : Benchmark No.1, Stress/Strain properties

Young's modulus	15,300 kgf/mm^2
Poisson's ratio	0.30
Yield stress	7.0 kgf/mm^2
Stress-plastic strain	$\varepsilon_p = 6.806 \times 10^{-10} \sigma^{8.045}$

Creep equation $\varepsilon_c = \varepsilon_x(1 - e^{-st}) + \varepsilon_t(1 - e^{-rt}) + \dot{\varepsilon}_m t$

$$\text{where } \varepsilon_x = \begin{cases} 0 & \sigma < 4.2 \ kgf/mm^2 \\ -0.257143 + 6.09571 \times 10^{-2}\sigma & \sigma \geq 4.2 \ kgf/mm^2 \end{cases}$$

$$S = 4.233 \times 10^{-3} [\sinh(0.1001\sigma)]^{3.5}$$

$$r = 0.1 \times S$$

$$\dot{\varepsilon}_m = 2.320 \times 10^{-4} [\sinh(6.901 \times 10^{-2}\sigma)]^5$$

$$\varepsilon_t = 5\dot{\varepsilon}_m/r$$

Units strain are percent, units stress kgf/mm^2, units time hours.

TABLE 2 : Benchmark No.2, Stress-Strain properties

Young's modulus 21.7 x 10^6 psi

Poisson's ratio 0.3

Elastic-plastic tensile curve:

$\sigma \times 10^3$ psi	$\varepsilon \times 10^{-2}$
5	0.02
8	0.05
9	0.08
10	0.15
10.5	0.21
11	0.30
11.5	0.43
12	0.62

Stress-minimum creep rate:

$\sigma \times 10^3$ psi	$\dot{\varepsilon}_m \times 10^{-5}, hr^{-1}$
1	0.0001
2	0.0002
3	0.0004
4	0.0005
5	0.0008
6	0.0011
7	0.0018
8	0.0032
9	0.0050
10	0.0150
11	0.0375
12	0.0485
13	0.0650
14	0.082
15	0.099
16	0.13
17	0.18
18	0.26
19	0.325

TABLE 3 : Benchmark No.3. Stress-Strain properties

Young's modulus 191,300 MPa

Poisson's ratio 0.3

Elastic-plastic tensile curve:

σ, MPa	ε %
101	0.052
192	0.123
228.7	0.316
233	0.404
300	2.38
348	5.95

TABLE 4 : Benchmark No.1: Comparison of predicted and measured free end deflection of assembly

STEP	DEFLECTION (mm)			
	Incremental		Accumulated	
	P	M	P	M
First step loading	20.4	22.6	-	-
First step creep	4.0	9.5	24.0	32.1
Second step loading	12.3	15.9	36.3	48.0
Second step creep	11.3	9.5	47.6	57.5

P - predicted
M - measured

TABLE 5 : Benchmark No.2: Comparison of predicted and measured free end deflection of assembly

Load = 7458 lb in

	Displacement		
	δ_3 (in)	δ_Y (in)	θ(rad)
Predicted	0.1571	0.0223	0.0071
Average measured	C.1617	0.0196	0.0075

TABLE 6 : Benchmark No.1: Comparison of modified and unmodified results

Load step	Deflection			
	1	2	3	4
Plastic displacement (mm)				
a) Modified	20.4	32.3	47.8	109.2
b) Unmodified	26.4	65.8	148.3	548.6
Creep rate(mm/hr)				
a) Modified	0.042	0.16	0.28	0.61
b) Unmodified	0.11	0.41	0.75	1.61

TABLE 7: Benchmark No.2: . Comparison of modified and unmodified results

	δ_3	
	Plastic (in)	Creep rate(in/hr)
Modified	0.2819	4.8×10^{-5}
Unmodified	1.03	1.45×10^{-4}

TABLE 3 : Benchmark No.3: Comparison of modified and unmodified results

Load step	Elbow deflection [mm]	
	Modified	Unmodified
1	9.04	10.2
5	9.37	10.6
9	9.63	10.9
14	16.05	19.96
18	12.3	14.42
20	10.2	11.6
22	12.6	14.8
24	17.4	22.1
26	30.7	46.0
28	19.2	25.0

8. FIGURE CAPTIONS

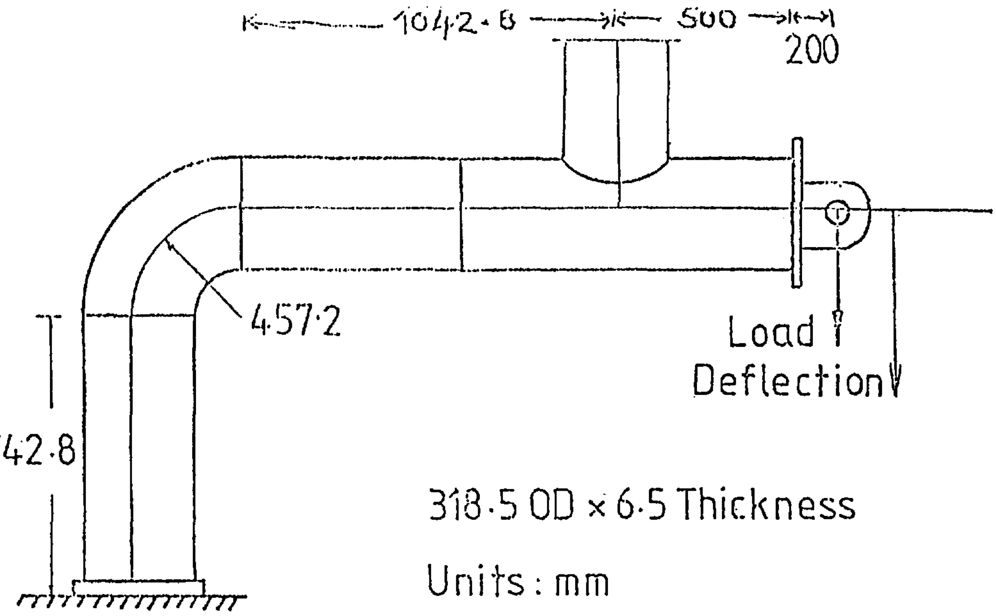
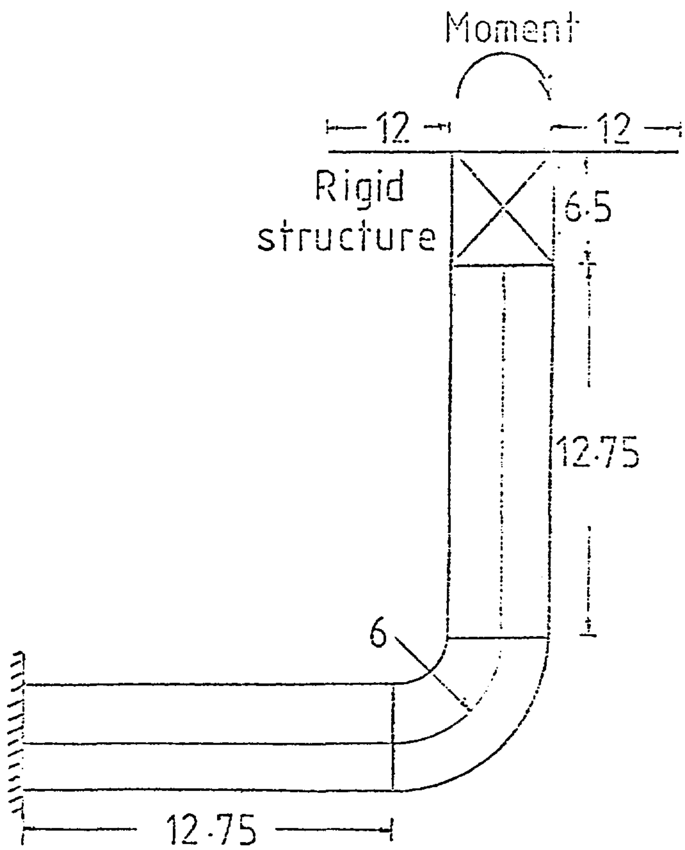


Figure 1

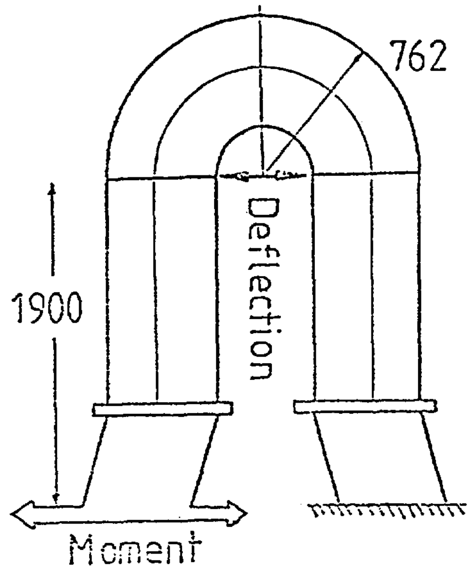
Benchmark No 1 Dimensions



4·5 OD × 0·120 Thickness
Units: in.

Figure 2

Benchmark No 2   Dimensions



507 OD×12 Thickness

Units: mm

Figure 3

Benchmark No 3 Dimensions

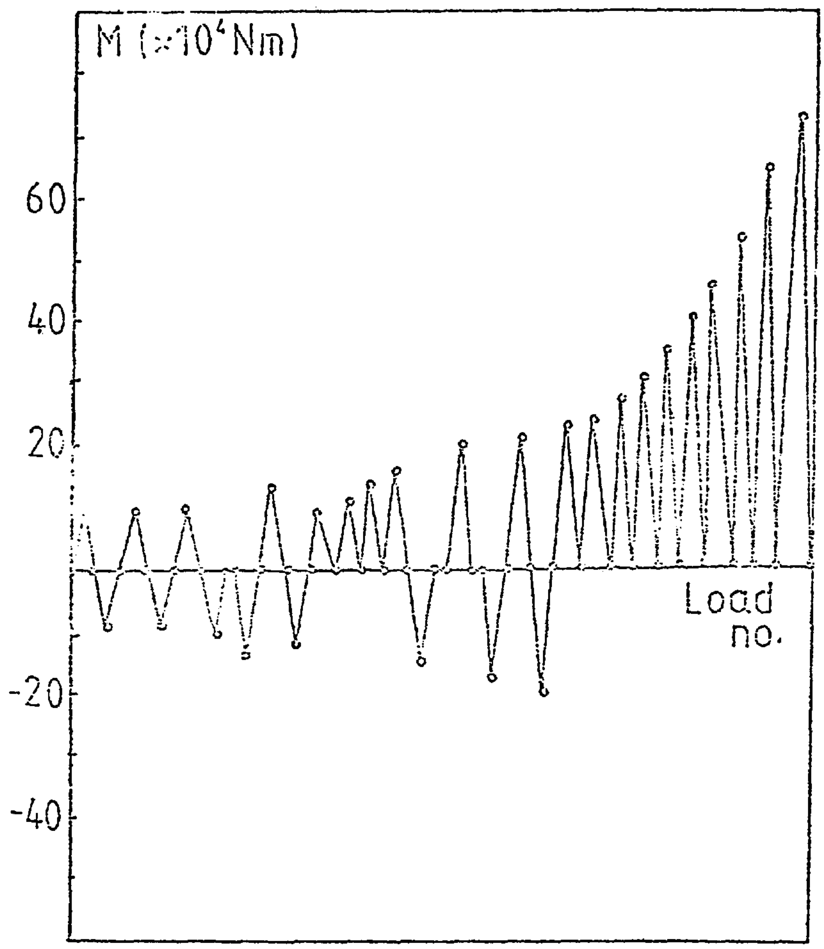


Figure 4

Benchmark No 3 Load History

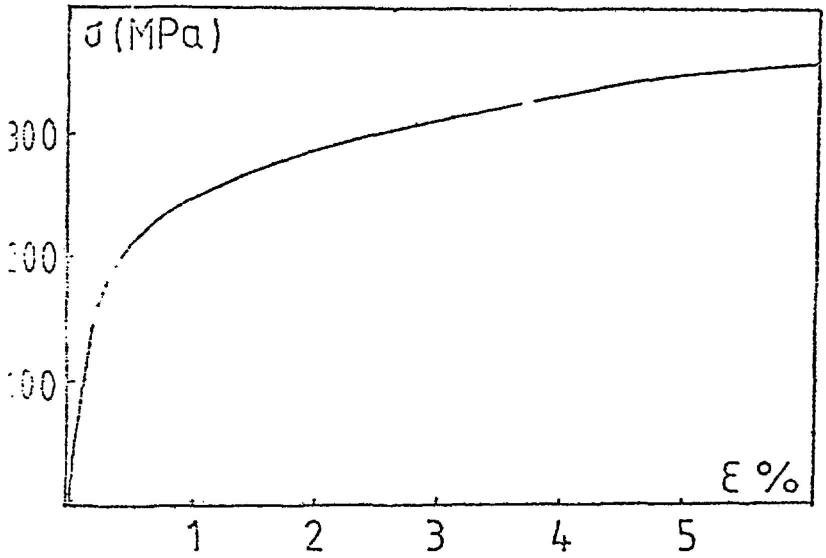


Figure 5

Benchmark No 3

Elastic/Plastic tensile curve

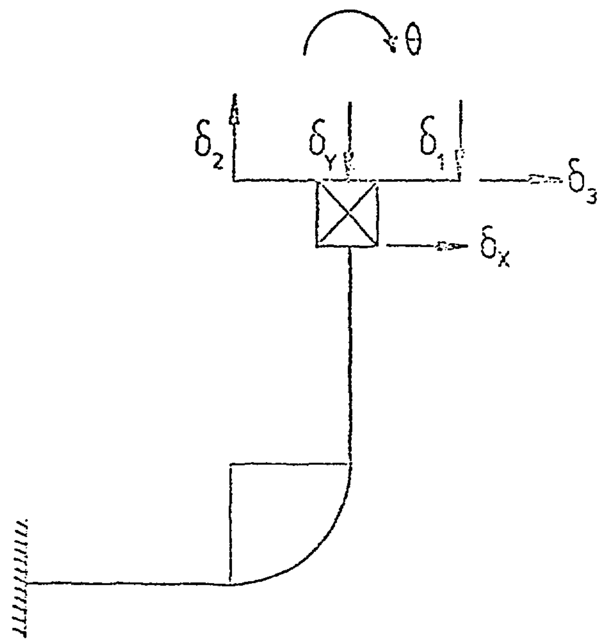


Figure 6

Benchmark No 2

Definition of measured deflections

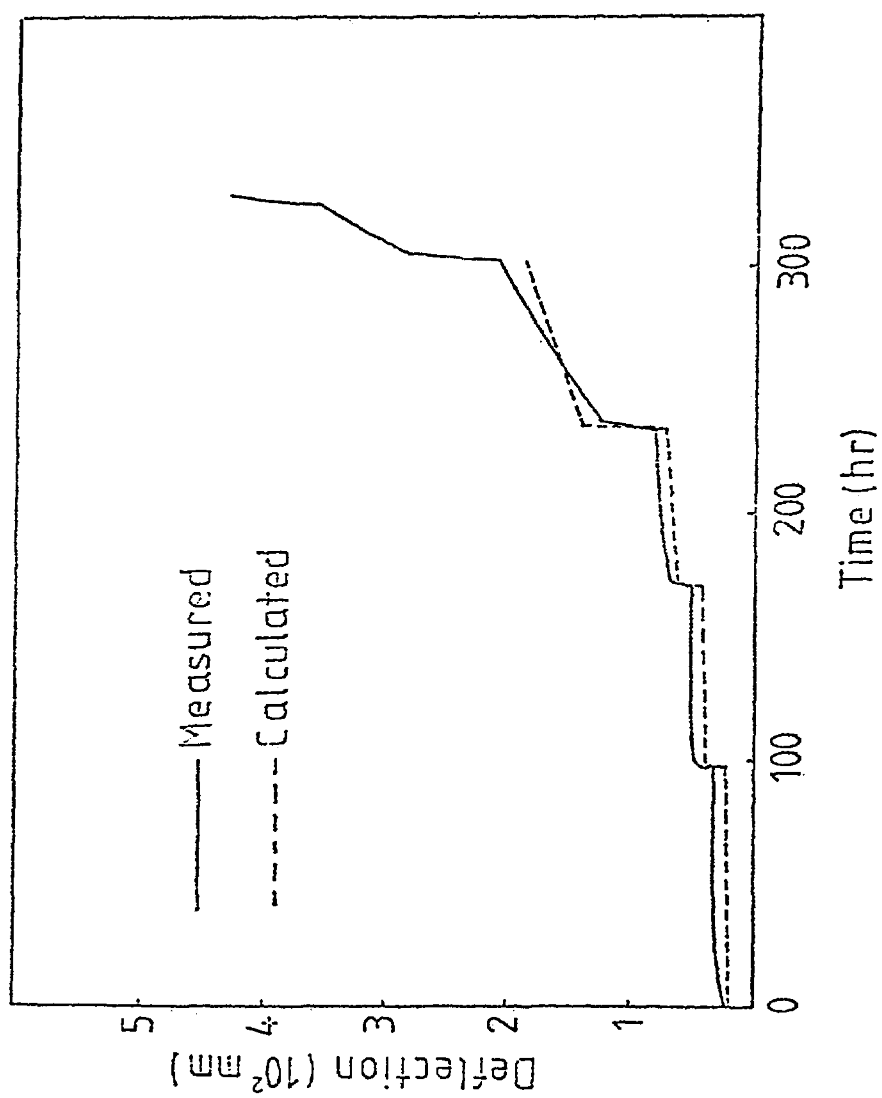


Figure 7

Benchmark No 1

Comparison of calculated and measured free end deflection in time.

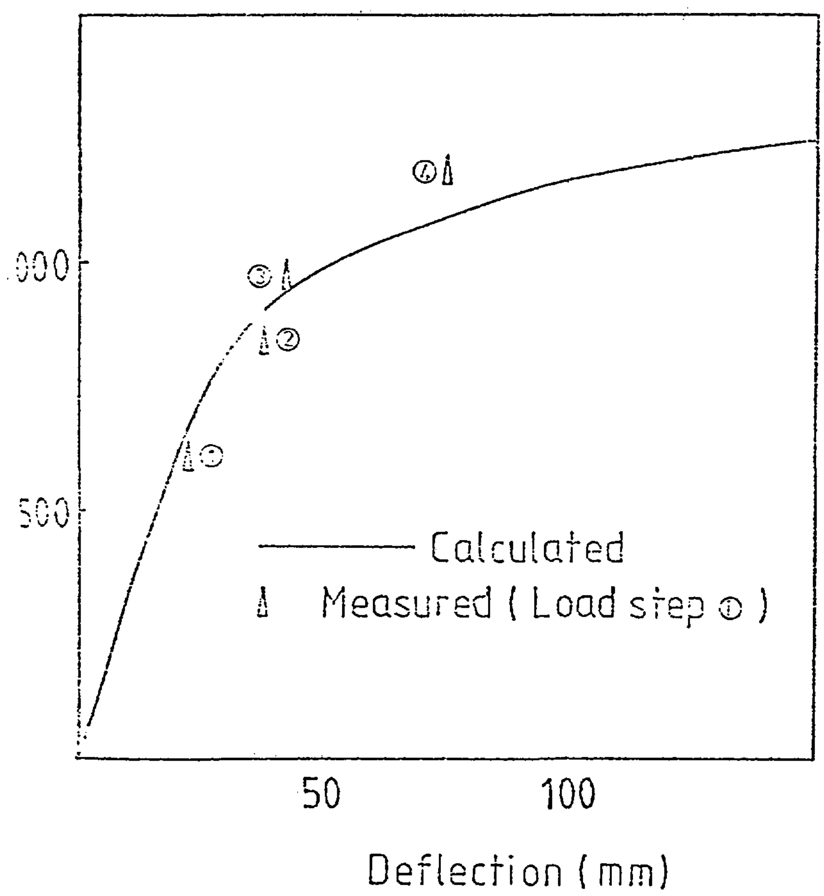


Figure 8

Benchmark No 1

Comparison of calculated and measured instantaneous deflection/load.

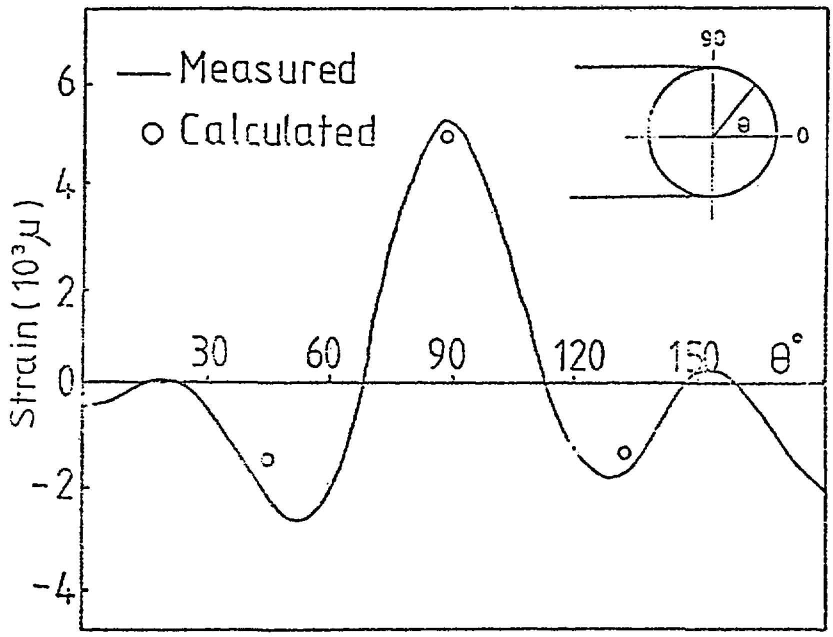


Figure 9

Benchmark No 1

Comparison of calculated and measured circumferential strain at beginning of load step 3

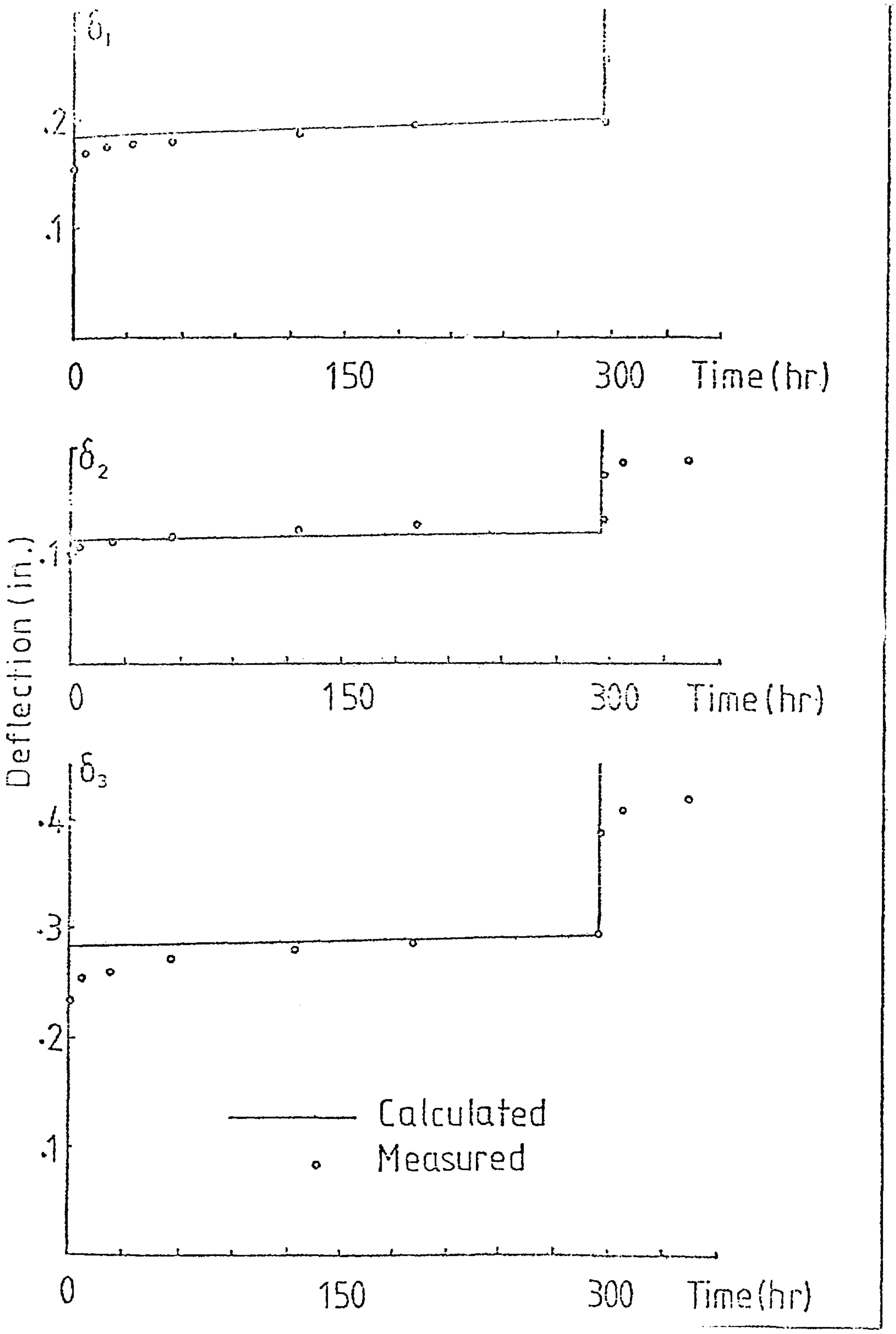


Figure 10

Benchmark No 2

Comparison of calculated and measured deflections in time

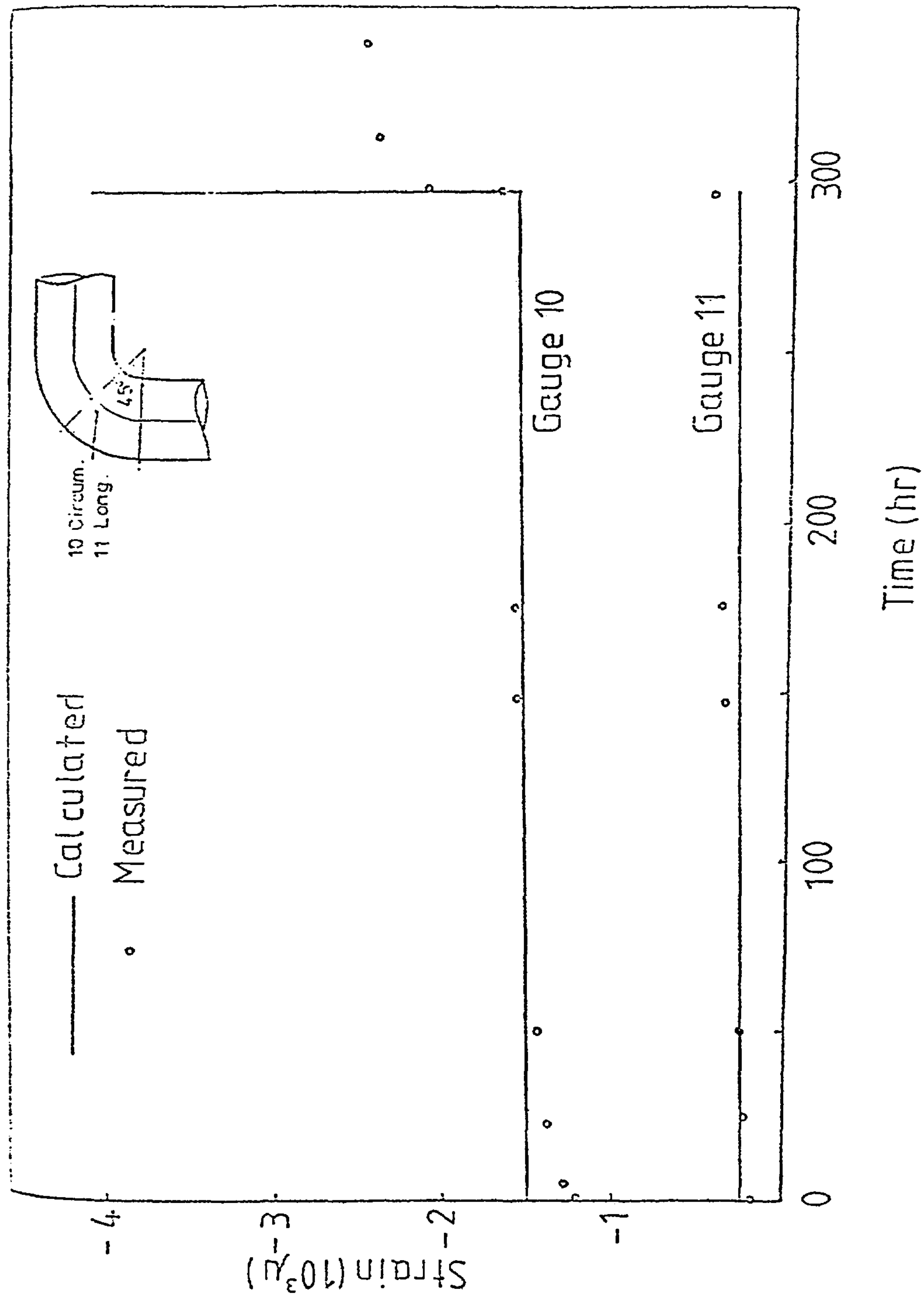


Figure 11

Benchmark No 3

Comparison of calculated and measured elbow deflections

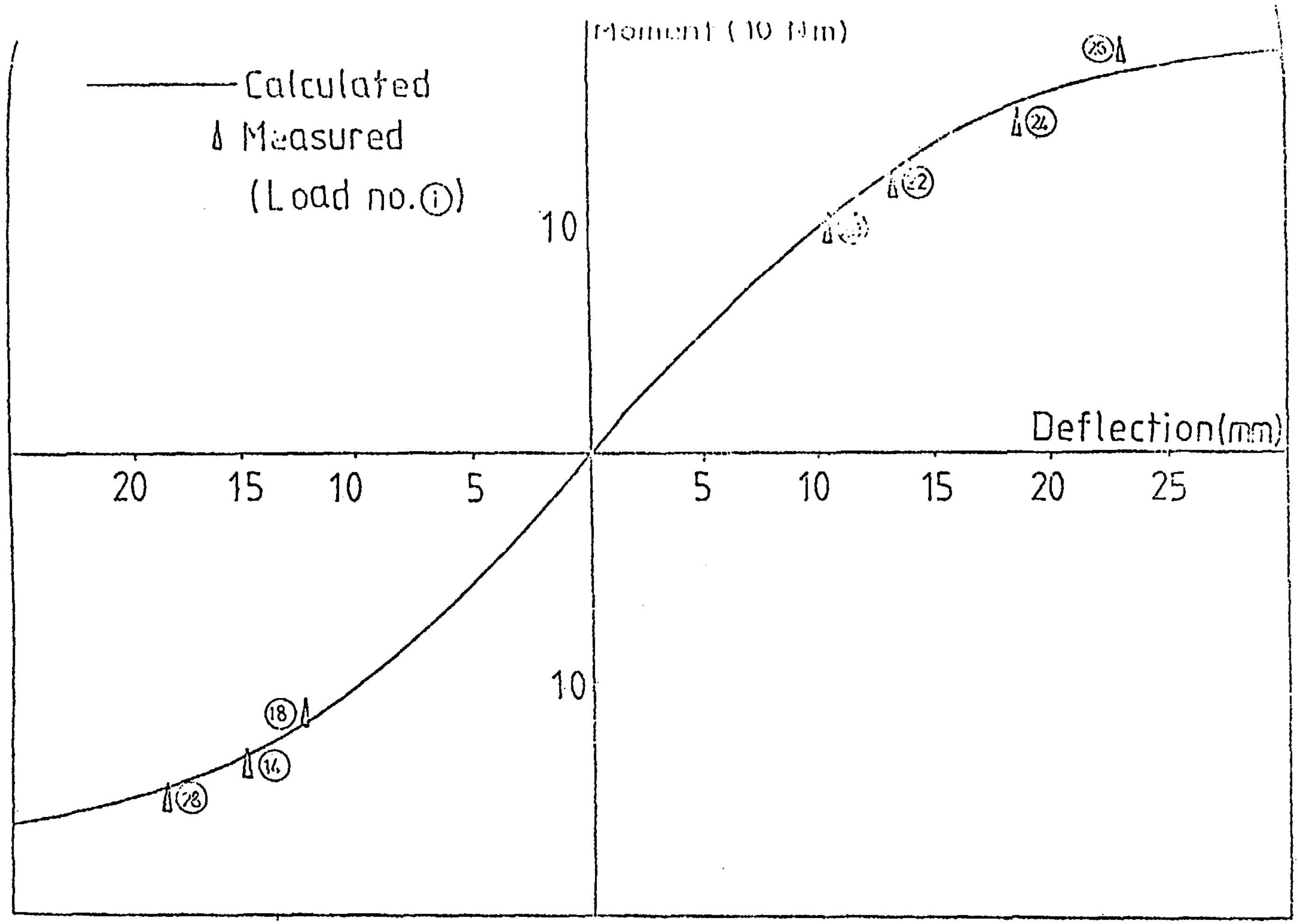


Figure 12

Benchmark No 3

Comparison of calculated and measured elbow deflections

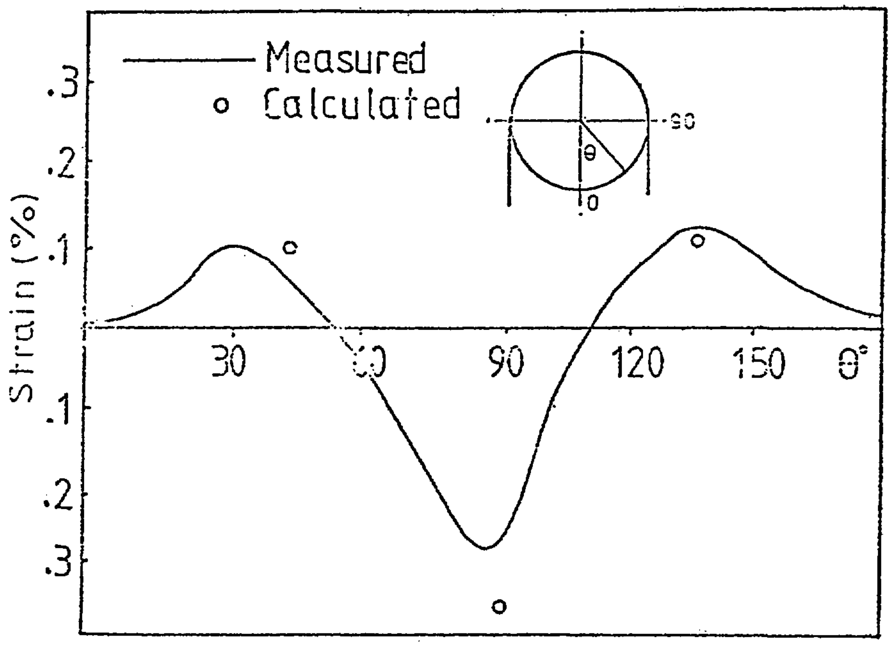


Figure 13

Benchmark No 3

Comparison of calculated and measured circumferential strain for load step 26.