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复数 Lorenz-Haken 系统中的超混沌同步及其控制

SYNCHRONIZING AND CONTROLLING HYPERCHAOS IN COMPLEX LORENZ-HAKEN SYSTEMS

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方锦清：中国原子能科学研究院研究员，1964年毕业于清华大学物理系。

Fang Jinqing: Professor of China Institute of Atomic Energy. Graduated from Physics Department, Tsinghua University in 1964.

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方锦清
（中国原子能科学研究院，北京）

摘　要

以描写失谐单模激光特性的复数 Lorenz-Haken 系统及其高阶级联系统作为第一个典型例子，首先实现了存在驱动-响应关系类型的超混沌同步，并采用间歇正比于所有系统主变量反馈控制法，实现了超混沌的有效稳定控制。引入的思想方法及概念可以拓广到其他超混沌系统的同步及其控制。指出了混沌同步、超混沌同步及其控制可能的实际应用前景，诸如在激光、等离子体、电子学、密码学、通讯、化学和生物系统等领域中的可能应用潜力。

SYNCHRONIZING AND CONTROLLING HYPERCHAOS IN COMPLEX LORENZ-HAKEN SYSTEMS

Fang Jinqing
(CHINA INSTITUTE OF ATOMIC ENERGY, BEIJING)

ABSTRACT

Synchronizing hyperchaos is realized by the drive-respone relationship in the complex Lorenz-Haken system and its higher-order cascading systems for the first time. Controlling hyperchaos is achieved by the intermittent proportional feedback to all of the drive (master) system variables. The complex Lorenz-Haken system describes the detuned single-mode laser and is taken as a typical example of hyperchaotic synchronization to clarify our ideas and results. The ideas and concepts could be extended to some nonlinear dynamical systems and have prospects for potential applications, for example, to laser, electronics, plasma, cryptography, communication, chemical and biological systems and so on.

INTRODUCTION

There has been great interest in the controlling chaos in recent years [1,2]. Because it is great possibility of application of chaos control that has inspired much theoretical and experimental work [3~11]. There are two kinds of strategies for controlling chaos: one is for time discrete control and the other for time-continuous control. For example, the OGY method proposed by Ott, Grebogi and Yorke [3] and some improved methods [6] only deal with the Poincare map, the changes of some accessible system parameter are discrete in the time, using only small time-dependent perturbations. Self-controlling feedback proposed by K. Pyragas [7] can realize a time-continuous control. In the latter, the stabilization of unstable period orbits (UPO) embedded in a chaotic attractor is achieved either by combined feedback with the use of a specially designed external oscillator, or by delayed self-controlling feedback.

From the other point of view, there are two ways for controlling chaos: a feedback control and a nonfeedback one. The former takes a given UPO as the goal of the controlling. The desired form is achieved when the controlling input approaches to zero or is very small. On the contrary, the latter is not related to a certain UPO. The aim of controlling is to suppress chaos or to generate a new form of dynamics. Thus the controlling input does not vanish as the desired form is realized under the control.

Another fascinating discovery in the context of chaos control is the synchronization (SYNC) [9~13]. Some kinds of the phenomena of the SYNC have been observed. The scheme of chaotic SYNC developed by Pecora and Carroll [9~11] requires that a chaotic system exists, from which one can separate a stable subsystem, which is duplicated as a stable slave subsystem with only negative Lyapunov exponents. When a chaotic master (drive) system and a stable slave (respone) subsystem are linked with a common drive signal, the two systems may display synchronized chaos. In the scheme, nonchaotic (such as periodic limit cycles) SYNC is also found [12,13]. The other kind of SYNC is the scheme for the SYNC of coupled, chaotic, nonlinear oscillators. More recently, Roy and Thornbury have observed experimental SYNC of the chaotic intensity fluctuations of two Nd: YAG lasers when one or both the lasers are driven by periodic modulation of their pump beams [14]. Sugawara and coworkers also experimentally demonstrate that two chaotic passive Q-switched lasers can be syschronized by modulating the saturable absorber in the cavity of one laser with the output of the other laser [15]. Another hind of the SYNC [16] is that a pair of chaotic systems to the same noise may undergo a transition at large enough noise amplitude and follow almost identical orbits with complete insensitivith to initial condition. It is shown that a pair of generic systems in the same potential evolving to equilibrium through standard Langevin dynamiccs with the same noise collapse into the same orbits at long times. The SYNC phenomena above may bring many interesting possibilities in practical applications. For example, they make possible a variety of interesting technologies for laser, eletronics, communications, plasma, cryptography and so on.

As is known, the present of more than one positive Lyapunov exponents in a given dynamical system has been called hyperchaos [17~19], which is much more interesting in many fields of nature and laboratory. However, Pecora and Carroll specially emphasis that the SYNC of chaotic systems can be achieved only if the conditional Lyapunov exponents (λ) of the slave subsystem are all negative [9~11]. Vieira and his co-workers also stated that the SYNC is lost when the hyperchaos appears [12].

We now address the challenging questions: Could not synchronization of hyperchaos be achieved really ? How can one realize controlling hyperchaos? These are open and challenging topics in the field of chaos control and applications. So one of motivation for our work is to study how to realize synchronizing and controlling hyperchaos in high-dimensional nonlinear dynamical system and its cascading systems.

In this paper, we will take the complex Lorenz-Haken system (CLHS) and its high-dimensional cascading systems (HDCS) as the first example of synchronizing and controlling hyperchaotic system. Another interest for our work is to a great extent motivated by anologies of laser dynamics with chaotic dynamics in the other fields of nature since one has benefited a great deal from the discovery of the anology between the perfectly tuned single mode laser and the well-known real Lorenz-Haken equation. However, until recently there has been a few work on chaos control for the CLHS which anologies with detuned single-mode laser. The field of laser research has also benefited from other advances in chaos control. Chaos control has been already applied to increase the power of laser in experiments. The team of Ira B. Schwartz and his cooperater developed a method known as tracking method [20]. The method has been applied to both chaotic circuits and lasers with astounding results. For example, they can maintain control over a much wider power range and increase the output power by a factor of 15.

This paper is organized as follows. The definition of synchronization and high dimensional cascading systems are described in section 2. The CLHS and hyperchaotic motion are given in section 3. A stability of the SYNC are analyzed by some methods in section 4. Synchronizing and controlling hyperchaos are shown in the sections 5~6, respectively. The discussion and conclusion are made finally.

1 DEFINITION OF SYNCHRONIZATION AND HIGHER-ORDER CASCADING SYNCHRONIZED SYSTEMS

Let us consider a n-dimentional autonomous nonlinear dynamical system that may be divided into n parts:

$$dx_1/dt = f_1(x_1, x_2, \ldots, x_n) \tag{1.1}$$

$$dx_2/dt = f_2(x_1, x_2, \ldots, x_n) \tag{1.2}$$

$$dx_3/dt = f_3(x_1, x_2, \ldots, x_n) \tag{1.3}$$

$$\vdots$$

$$\vdots$$

$$dx_n/dt = f_n(x_1, x_2, \ldots, x_n) \tag{1.4}$$

We can rewrite them as in general nonautonomous form of

$$dX/dt = f(t,X) \tag{1.5}$$

where X, f are vectors:

$$X = \{x(t)_1, x(t)_2, \ldots, x(t)_n\}^T$$

$$f = \{ f_1(t, X), \ldots, f_n(t,X) \}^T$$

We call Eqs. (1.1) ~ (1.4) or Eq. (1.5) the master system or the drive one by Pecaro and Carroll. In what follows, we will restrict ourselves to those cases in which there exists a unique solution f (t, X) satifies the Lipschitz condition),

$$X = X(t) = X(t;\, t_0, X_0) \tag{1.6}$$

Eq. (1.6) satisfies Eq. (1.5) and the initial conditions

$$X\ (t_0; t_0, X_0) = X_0 \tag{1.7}$$

Les us define the synchronization followed by He and Vaidya [13]. Consider two dynamic systems: one is the (1.5) and the other is the primed one but indentical, $dX'/dt = f\ (t,\ X')$, where X, $X' \subset R^n$. Let the solutions of the systems be given by $X\ (t;\ t_0,\ X_0)$ and $X'\ (t;\ t_0,\ X'_0)$, respectively. We say that $f(t, X')$ synchronizes with $f(t, X')$ if there exists a subset of R^n, denoted by $D(t_0)$ such that X_0, $X'_0 \subset D\ (t_0)$ implies

$$\|X(t;\, t_0, X_0) - X'(t;\, t_0, X'_0)\| \to 0 \text{ as } t \to \infty \tag{1.8}$$

The SYNC is defined as global if $D(t_0)$ spans the whole space, i. e., $D\ (t_0) = R^n$. It is defined as local if $D(t_0)$ is a proper subset of R^n. $D\ (t_0)$ is known as the region of synchronization.

In general, the system can be divided into two subsystems arbitrarily: the master (driving) and the slave (driven) subsystems, i. e., $X = X_m + X_s$. Subscript m and s denote the master and the slave system, respectively. The master system simply consists of these two interacting subsystems. The slave system is quite similar, except that a part of it totally driven by the master system. We can write them as follows.

The master system is given by

$$dX_m/dt = h(t, X_m, X_s), \qquad dX_s/dt = g(t, t, X_m, X_s) \tag{1.9}$$

and the slave system is given by

$$d\mathbf{X}'_n/dt = \mathbf{g}(t,\mathbf{X}_m,\mathbf{X}'_n) \quad (1.10)$$

In general, arbitrary variable of (x_1, x_2,... ,x_n) can be chosen for the driving signal only if the other remaining dynamical variables satisfy certain conditions, such as Pecrao-Carroll SYNC condition. Eq. (1.5) may be divided into several subsystems, such as (1.1), (1.2), (1.3), and so on. The successive slave (driven) systems consisting of the other remaining dynamical variables are then duplicated. The number of the cascade-order is equal to the number of the slave (driven) systems. In each cascade one variable is driving one, the others are duplicated as the slave (driven) system. Fig. 1 shows the two kinds of high-dimensional cascading ststems (HDCS), i. e., (a) series form and (b) parallel form. In Fig. 1 the CLHS is taken as the master system (see next section). One primed, double primed and so on denote the first-order , the second-order and higher order cascading, respectively. We consider parallel cascading form driven by x_1 and x_2 respectivehy, in this paper.

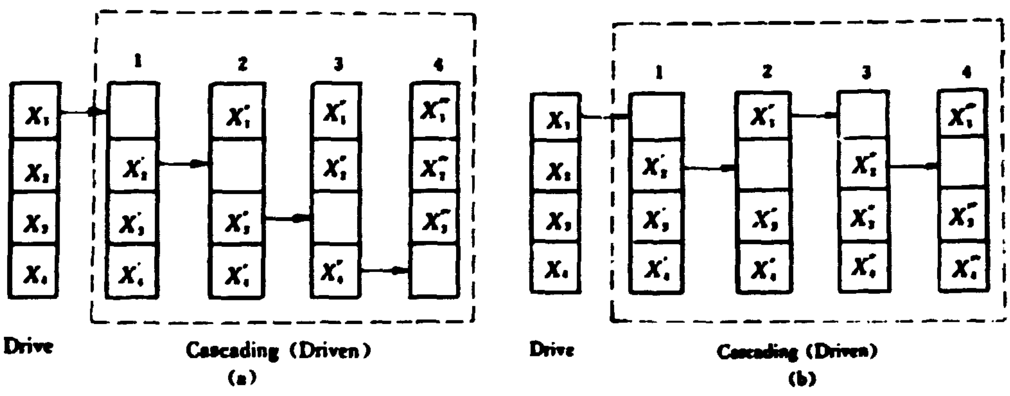

Fig. 1 Two kinds of high-dimensional cascading synchronized systems (a) Series form and (b) Parallel form.

2 COMPLEX LORENZ-HAKEN SYSTEM

The Complex Lorenz-Haken system (CLHE) has been derived from the equations describing a ring laser consisting of an ensemble of two-level homogeneously broadened atoms as follows:

$$dX/dt = -kX + kY \tag{2.a}$$

$$dY/dt = -aY + (r - Z)X \tag{2.b}$$

$$dZ/dt = -bZ + \frac{1}{2}(X^*Y + XY^*) \tag{2.c}$$

Where X^* and Y^* are now complex quantities. Complex parameter $r = r_1 + ir_2$ and $a = 1 - ie$ arise due to the weak dispersive effect. To facilitate the analysis, using a set of real variables through the following transformation:

$$X = x_1\exp(i\varphi) \tag{2.d}$$

$$Y = (x_2 + ix_3)\exp(i\varphi) \tag{2.e}$$

$$Z = x_4 \tag{2.f}$$

We have the resulting equation (for detail see Ref.[21]):

$$dx_1/dt = -kx_1 + kx_2 \tag{2.1}$$

$$dx_2/dt = r_1x_1 - x_2 - ex_3 + kx_3^2/x_1 - x_1x_4 \tag{2.2}$$

$$dx_3/dt = r_2x_1 + ex_2 - x_3 - kx_2x_3/x_1 \tag{2.3}$$

$$dx_4/dt = -bx_4 + x_1x_2 \tag{2.4}$$

with $\dot{\varphi} = \frac{kx_3}{x_1}$ defining a generalized frequency of the laser field which is generally time dependent. Here, k, r_1, r_2 and b are parameters associated with laser system. Eqs. (2.1) ~ (2.4) is transformed from the CLHE (2.a) ~ (2.c) and is an anology with detuned signal mode laser and will be taken as the drive (master) system here.

One of the system variables above may be chosen to be a drive signal in the right parameter space if the Lyapunov exponents (λ) of the driven (slave) subsystem are less than that of the driving variable. Eqs. (2.1) ~ (2.4), is now taken as a good example to clarify the HDCS. Any one of the above variables, (x, x_2, x_3, x_4), in general, may be chosen for the drive signal and the slave system consisting of the three remaining dynamical variables is then reproduced. Thus the system is divided into several subsystems. They can be combined to set up two types of projects of the HDCS, as shown in Fig. 1.

The above types of the HDCS can be synchonized for the right paramerter space although the subsystems and their duplicates are initially set up to produce different outputs. Cascading subsystems began at different initial conditions caught up to each other and synchronized soon, and maintain it as time marches on. The resuls will be shown in the section 5~6.

We numerically find that there are several kinds of driving variables which may realized the SYNC, such as nonchaotic, chaotic, intermitent chaotic and hyperchaotic signal. Our goal in this paper is to realize the SYNC and control of hyperchaos. The others will be given in elsewhese.

3 ANALYSIS OF STABILITY AND SYNCHRONIZATION FOR THE CLHE AND HDCS

3.1 Analysis of Stability and Hyperchaotic Motion

A stability analysis of the CLHE has been undertaken (for detail see Ref. [21]). Some analytical conclusions associated with the SYNC are given here brifly. The CLHE is essentially analogical only to single-mode laser with detuning. For r_2 is not equal to e, the CLHE's has trivial and nontrivial stationary solution which is exact periodic and corresponds to detuned stationary solution. For trival solution, it will be destabilized at $r_1 = r_{1c}$, where

$$r_{1c} = 1 + (e + r_2)(e - kr_2)/(1 + k^2) \tag{3.1.1}$$

For nontrivial stationary state, the only possible for the instability to occur is the Hopf bifurcation. The condition for this bifurcation is

$$a_1(a_3 a_2 - a_1) = a_3^2 a_0 \tag{3.1.2}$$

where a_1, a_2, and a_3 and below a_0 are associated with the parameters in the CLHE. Ning and Haken shift the origin of the phase space to nontrivial stationary state and expand around the nontriveil state up to the third order in the variables [21]. They obtain eigenvalues λ_i ($i = 1, 2, 3, 4$) at the critical point

$$\lambda_1 = \lambda_2^* = i\Omega_c \tag{3.1.3}$$

$$\lambda_{3,4} = -\frac{a_3}{2} \pm \frac{1}{2}(a_3^2 - \frac{4a_4}{\Omega_c^2})^{0.5} \quad (3.1.4)$$

Here Ω_c is the frequency at the critical point and $\Omega_c^2 = [a_1/a_3]^{0.5}$.

For arbitrary relaxation constants and detuning, all analytical and exact results of the second threshold of detuned single-mode laser are obtained. The main conclusion for stability of the CLHE are as follows. (1) In the good-cavity laser $k < b + 1$, there is no instability, i. e., the lasers always operate stably and will oscillate forever. (2) In the bad-cavity laser $k > b + 1$, there exists a critical value of the pumping parameter A_c. When the pumping parameter $A > A_c$, the laser lose stability through a Hopf bifurcation. The Hopf bifurcation generally means an addition of a new frequency to the system.

The conclusions above provide not only a complete understanding of the second threshold of detuned single mode laser but also the right parameter space for cur studies of the SYNC for the hyperchaos. We shall mainly consider the case of $k > b + 1$. All discussion are at two typical system parameters: (a) $r_2 = -1.5$, $b = 1.2$, and (b) $r_2 = -0.5$, $b = 0.5$. They have the same parameters: $k = 6$, $r_1 = 91.0$, $\epsilon = 2.5$.

Let us firstly demonstrate the hyperchaos existting in the CLHE by two typical projections of the hyperchaotic strange attractor, as shown in Fig. 2. Fig. 3 gives some typical power spectra and synchronized results.

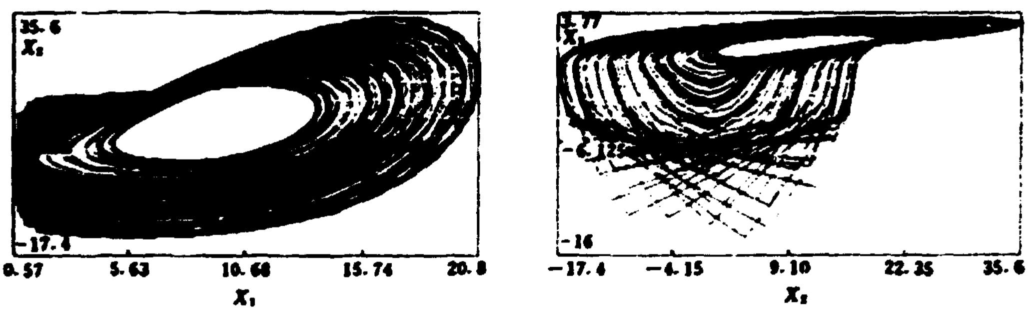

Fig. 2 Projections of the hyperchaotic strange attractor for the CLHE.
Fixed parameters is at the (b) above.

It can be seen from Figs. 2 and 3 that the projections of the Hyperchaotic strange attractor in various phase plans are all of chaos in time series and the power spectra are all of broad frequencies band everywhere. All these features have shown the hyperchaos appears in the CLHE. And it can be understood by studying global Lyapunov exponents further in next section.

3.2 Global Lyapunov Exponents

The best method for observing hyperchaos is to calculate all of the Lyapunov exponents of the global system consisting of the driving and driven systems together. Because He and Vaidya have shown [13] that the conditional Lyapunov exponents as defined by Pecora and Carroll[9~11] are the lyapunov exponennts of the global system. Calculation of the Lyapunov exponents of the global system depends on the driven (slave) subsystems driven by possible driving signals. For the driving signal x_1, we have the slave (driven) subsystem

$$dx'_2dt = r_1x_1 - x'_2 - ex'_3 + kx'^2_3/x_1 - x_1x'_4 \qquad (3.2.1)$$

$$dx'_3dt = r_2x_1 + ex'_2 - x'_3 + kx'_2x'_3/x_1 \qquad (3.2.2)$$

$$dx'_4dt = - bx'_4 + x_1x'_2 \qquad (3.2.3)$$

For the driving signal x_2, the slave subsystem reads

$$dx'_1dt = - kx'_1 - kx_2 \qquad (3.2.4)$$

$$dx'_3/dt = r_2x'_1 + ex_2 - x'_3 - kx_2x'_3/x'_1 \qquad (3.2.5)$$

$$dx'_4/dt = - bx'_4 + x'_1x_2 \qquad (3.2.6)$$

The slave subsystems for the driving signal x_3, x_4 are also obtained. They are ignored here. For the CLHE, there are four kinds of the global systems combinring with the above four possible types of the slave subsystems. For example, the first kind of the global system consists of the CLHE (2.1) ~ (2.4) and eqs. (3.2.1) ~— (3.2.3). The second one consists of the CLHE (2.1) ~ (2.4) and eqs. (3.2.4) ~ (3.2.6), and all that. These equations and their linearized equations can be used to caluculate all of the Lyapunov exponents of the global sys-tems, respectively.

Our algorithms for computing the Lyapunov exponents are mainly used by the decomposition method [22,23] and based on the repeated use of the Gram-

Schmidt reothonormalization (GSR) procedure on the vector frame. The GSR allows the integration of the vector frame for as long as required for spectral convergence. We have made the Fortran code for the ODE's procedure. Time step is taken as $\leqslant 0.001$.

Table. 1 shows the Lyapunov exponents of the global systems. We see from Table. 1 that there are two positive λ's in the global system at least. For the driving signal x_1, we have three positive Lyapunove exponents: $\lambda_1 = 0.02869, \lambda_2 = 0.0000179, \lambda_5 = 0.001298$ and the others are negative. And the driven system is more stable than the driving system because $\lambda_1 > \lambda_5$. Clearly, the above evidences further demonstrate that there exists the hyperchaos in the CLHE and HDCS. The criterion of the hyperchaotic SYNC is satisfied for the driving signals x_1 and x_2. But the SYNC is not achieved for the driving signals x_3 and x_4.

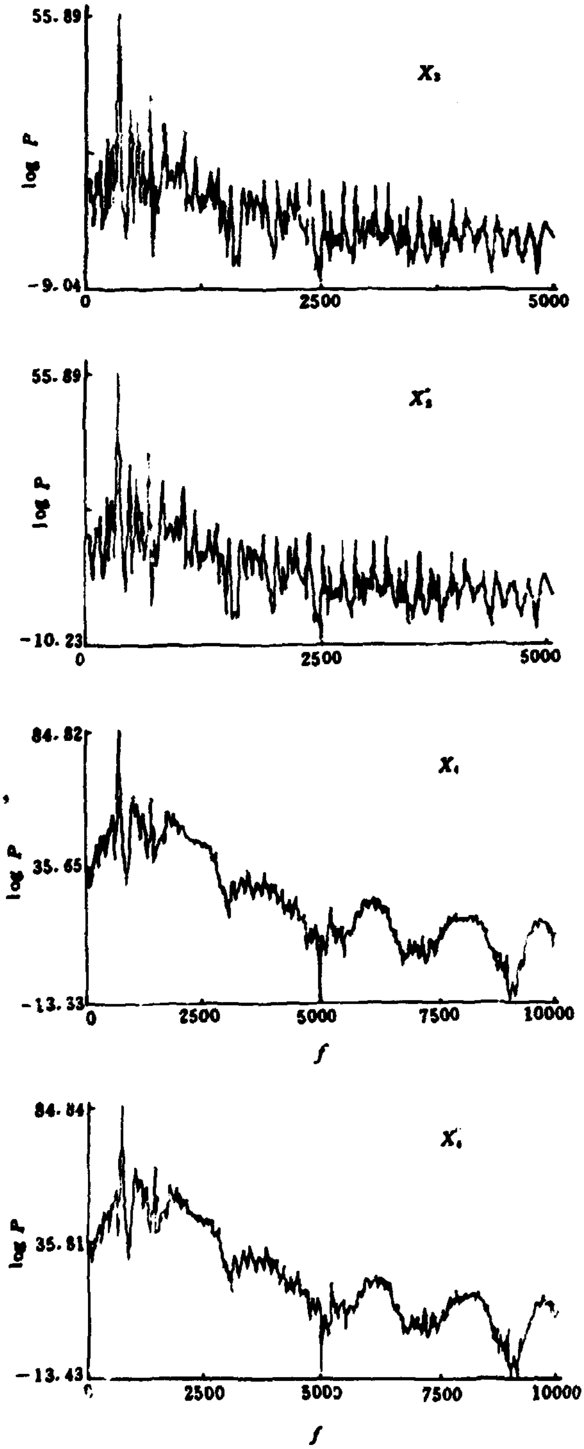

Fig. 3 Typical power spectra for x_3 and x_4 (a) and corresponding synchronized results (b)

Table. 1 The Lyapunov exponents of the global systems

Fixed parameters: $k=6$, $r_1=91.0$, $r_2=-1.5$, $b=1.2$, $e=2.5$.

DV	drive system				driven system		
	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7
x_1	2.869×10^{-2}	1.790×10^{-3}	-2.242×10^{-4}	-7.127×10^{-3}	1.298×10^{-3}	-3.279×10^{-4}	-1.290×10^{-4}
x_2	7.220×10^{-3}	2.450×10^{-4}	1.635×10^{-4}	-1.173×10^{-2}	2.732×10^{-3}	-7.983×10^{-4}	-1.901×10^{-4}
x_3	-1.392×10^{-4}	9.631×10^{-7}	3.440×10^{-8}	-3.197×10^{-5}	8.113×10^{-3}	3.440×10^{-3}	3.010×10^{-3}
x_4	9.302×10^{-2}	3.953×10^{-3}	4.845×10^{-2}	-1.560×10^{-3}	1.234×10^{-2}	3.328×10^{-4}	2.607×10^{-4}

DV denotes driving variable.

4 SYNCHRONIZING HYPERCHAOS

We are in position to see the synchronizing hyperchaos in the CLHE and the HDCS which start with different initial conditions. The definition of the SYNC is followed by Pecora-Carroll and He-Vaidya. The convergent rate of the SYNC is denoted by the difference between the x_i and x_i^j, i. e., defining the quantity of the form $Dx_i = x_i - x_i^j$, as monitoring the SYNC efficiency in phase space, where $i = 1, 2, 3, 4$ and $j = {}', {}'', {}''', \ldots$ denotes the primed numbers corresponding to cascading order. It is the SYNC of the hyperchaos if all of Dx_i go to zero. In Fig. 4 we plot two synchronized convergent curves of Dx_i ($i=1, 4$).

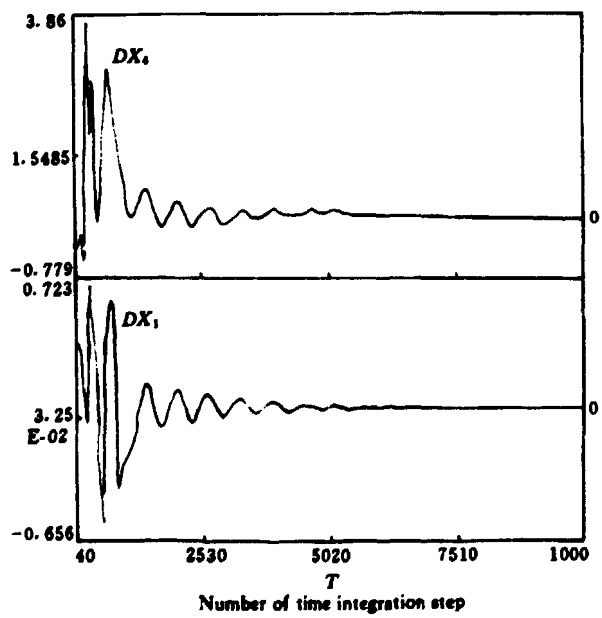

Fig. 4 Two synchronized convergent curves of Dx_i ($i=1, 4$) (a) and corresponding phase phase diagram (b)

Obviously, the SYNC curves converge very quickly to zero or less than absolute precision we specified, say, about 10^{-6}, which shows the SYNC is achieved very well when the time step numbers go to 8 000~10 000 (the maximum is 40 000~120 000). The synchronized phase diagram $x_i - x''_i$ is a linear line. Fig. 3 (b) has already shown the synchronized power spectra for high-order cascading time series, which are in good agreement with the power spactra of the master system.

We can conclude so far that the synchronizing hyperchaos in the CLHE and the high order cascading systems are realized successfully.

5 CONTROLLING HYPERCHAOS

We now focus on controlling hyperchaos in a high-dimentional (HD) synchronized hyperchaotic system (HCS). Controlling hyperchaos in the HD-HCS can be realized by applying a intermittent proportinal to all of the master system variables (IPMSV). This algorithm is similar to the occasional proportinal feedback[4] and to proportional pulses in the system variabes [8]. We combine synchronizing with controlling hyperchaos to realize a new dynamics. This is our subject in this section.

Fig. 5 gives block diagram of controlling hyperchaos by the IPMSV method. The main features of the IPMSV method are described as follows. Firstly, to more specific, we choose an interval time τ, i. e., the number of integration step ΔI. Secondly, x_i in the driving (master) system is replaced by $x_i(1+G)$ at the control activation moment, where $i = 1, 2, 3, 4$; That is, the master system become the form of

$$\mathrm{d}x_i/\mathrm{d}t = f[(1+G)x_i] \tag{5.1}$$

where f represents the function of the right side of the driving (master) equations, say, Eqs. (2.1) ~ (2.4), $i = 1, 2, 3, 4$. Clearly, there are two main adjustable parameters, one is the feedback intensity factor G, the other the step numbers ΔI or time τ elapsed between injection and withdrawal. New desired dynamical behavior mainly depends on the G and ΔI. The IPMSV method does not look for and vary any accessible master system parameters but only rather change the system variables. It is of flexible to change two parameters G and ΔI. There exists a certain relation between them associated

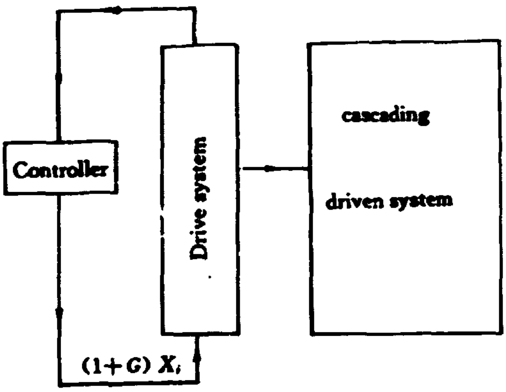

Fig. 5 Block diagram of controlling hyperchaos by the IP-MSV method for the HD—HCS

with dynamical properties of the master system (such as period). We now choose G and ΔI are two different constants (either positive or negative) so that the method works very well. We observed that the full variables are governed very well by applying the IPMSV during control periods, simultaneously. But it does not work very well if applying a IP to some slave system variables.

All discussion below for controlling hyperchaos is at the fixed parameters above and the other one: $k = 6$, $r_1 = 91.0$, $r_2 = -0.5$, $b = 0.5$, $e = 2.5$.

The convergence process of the synchronization and control have three stages: SYNC-CONTROL-SYNC, as shown in Fig. 6. The arrow indicates the control activation moment. To more specific, the left of the arrow is the first stage in which the SYNC is achieved before control on. Between the right of the arrow and the dashed line is the control stage in which controlling hyperchaos is realized. The convergence rate is also denoted by the difference Dx_i between the x_i and slave variables x'_i where $i = 1, 2$. The common point is that all of Dx_i is tend to zero during the SYNC stage. In the control stage if there is some regular oscillation it is instructor of some desired pattern (e. g. period 2 in Fig. 6). The SYNC is realized again in the right of the dashed line when controlling off.

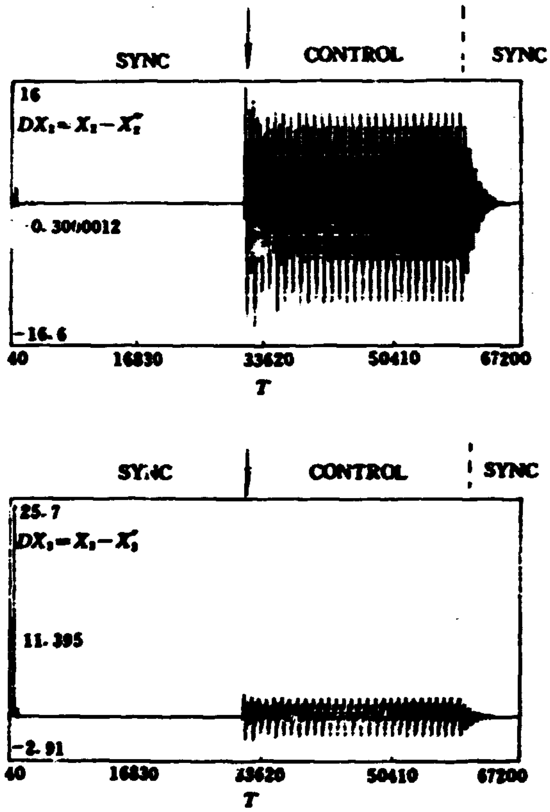

Fig. 6 Synchronizing and controlling processes of the hyperchaos for the HDCS during the SYNC-CONTROL-SYNC stages

Difference Dx_i between the x_i and x_i''' versus time the step numbers, where $i = 1, 2$. Fixed system parameters; $k = 6$, $r_1 = -1.5$, $r_2 = 1.2$, $b = 2.5$, $e = 2.5$, control parameters; $G = -0.2$ and $\Delta I = 1000$.

In the Fig. 7 we obtain the stabilized period 2 for time series of the x_2 and primed x'_2, x''_2 at the same system parameters as the Fig. 6. For different control parameters, such as $G = -0.35$ but different $\Delta I = 500, 700$ and 750, the double-period to chaos are obtained. They are igonored here.

We have also tested and investigated some similar results of synchronizing and controlling hyperchaos for the other values of parameter in the hyperchaotic region as well. For example, stabilized period-3, the steady state, bistability and three stable states are also obtained for another fixed parameters $k = 6$, $r_1 = 91.0$, $r_2 = -0.5$ and $b = 0.5$, $e = 2.5$ but different control parameters by the IPMSV method, as shown in Fig. 8 (a) ~ (d), respectively.

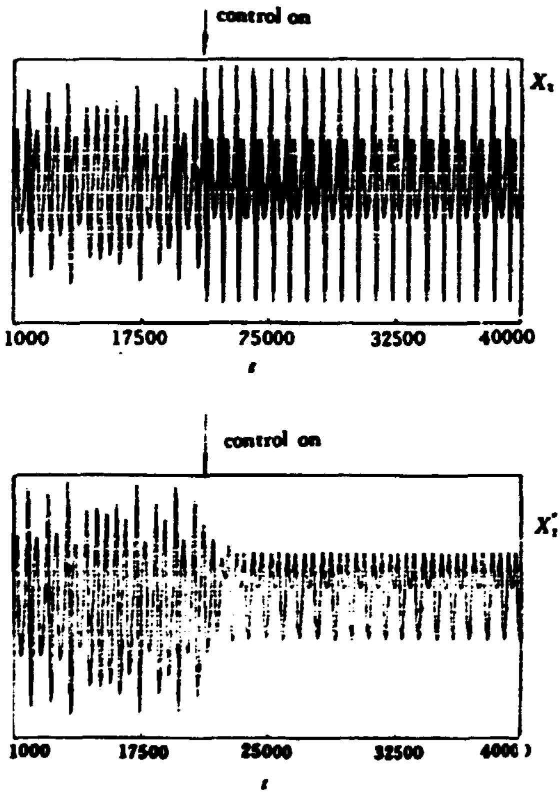

Fig. 7 Stabilized period-2 for x_1 and x^*_1 is obtained for the same system and control parameters as in Fig. 6

The results above show that new dynamical behavior of controlling hyperchaos depends on the system parameters and the critical values of G and ΔI as well. The steady state, bistability, tristability, period, quasiperiod and chaos can also be observed in our system by the IPMSV method. That means that the higher order cascading synchronized systems may become a generator or transformer of various desired form which is a function of G and ΔI.

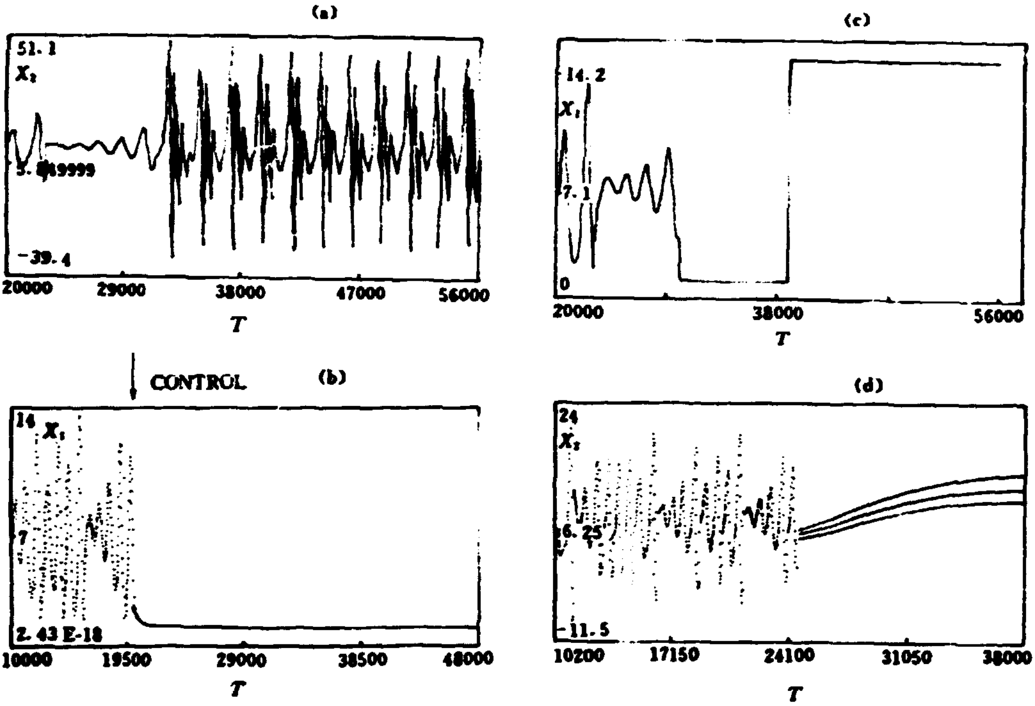

Fig. 8 Stabilized period-3 (a), the steady state (b), bistability (c) and tristability (the three stable states) (d) are obtained for the fixed system parameters: $k = 6$, $r_1 = 91.0$, $r_2 = -0.5$, $b = 0.5$, and $e = 2.5$, but different control parameters: (a) $G = -0.35$ and $\Delta I = 2200$. (b) $G = 0.4$ and $\Delta I = 600$. (c) $G = -0.35$ and $\Delta I = 20$. (d) $G = -0.45$ and $\Delta I = 30$.

6 DISCUSSION AND CONCLUSION

One of interesting problems is that: Why the hyperchaos is synchronized so well? The reasons are analyzed preliminarily as follows [24,25]. First, the CLHE and its cascading systems satisfy the criterion we proposed, i. e. the master system can synchronize with the slave system because the slave system is more stable than the master (driving) system. This can be seen from Table. 1. In other words, the hyperchaotic SYNC may be achieved only if the largest Lyapunov exponent of the master (driving) system is greater than that of the slave (driven) system even if there exist more than to one positive Lyapunov exponents in the global system. Second, more recently, Roy and Thornbury have observed experimental SYNC of the chaotic intensity fluctuations of two chaotic Nd:YAG lasers [14]. Sugawara and co-workers also experimentally demonstrate that two chaotic passive Q-switched lasers can be syschronized [15]. Their results show that the SYNC can be realized between two chaotic systems which settle on slightly different strange attractor without couplings. In a matter of fact, such global system consisting of two chaotic lasers is the hyperchaotic system since there exist two positive λ's. The λ's of laser 1 is just greater than that of laser 2. Such experimental fact has indirectly demonstrated the criterion about hyperchaotic SYNC is correct. But it is to be proved theoretically further.

Third, it is worthwhile to note that He-Vaidya theorem on the chaotic SYNC may be extended to the hyperchaotic SYNC [13]. The theorem tell us that the slave system synchronizes with the master system if and only if there exists a subset $D(t_0) \subset R^n$ such that when the initial conditions of the nondriven part of the slave system fall in $D(t_0)$, the solution of the slave system are asymptotically stable. The initial conditions for our system have just satisfied the requirement of theorem (see Ref. [13] in detail), i.e. our initial conditions have just fallen in the $D(t_0)$. The results in this paper are held at least for the CLHE and may be extended to some hyperchaotic systems, specially for slightly different hyperchaotic systems.

In conclusion, we have demonstrated that the synchronizing and controlling hyperchaos are realized both in the systems (2.1)~(2.4) and in the higher order synchronized hyperchoatic systems, successfully. It is shown that The IPMSV method can effectively control and stabilize the new desired dynamical form in the hyperchaotic system and the HDCS. The results could be extended to some nonlinear dynamical system destabilizing by the Hopf bifurcation. It can create a new dynamical behavior, such as the period-doubling bifurcation to chaos. Every cascaded system may be a richs of patterns at different parameter G and ΔI. It is also used for the hyperchaotic or chaotic filter and the others. Some concepts and results above may be most interesting and promissing for applications.

Finally, we would like to mention that the coupled kind of hyperchaotic synchronization for the Rossler system and the forced coupled van der Pol systems are also realized. Those results will be reported in elsewhere.

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