Mathematical and numerical modeling of the AquaBuOY wave energy converter

A. Wacher and K. Nielsen

December 18, 2008

1 Introduction

Due to pressing energy demands in the world, harnessing alternative or renewable energy is crucial. This is a time when the world is anxious to find cost effective ways to harness energy in a clean and sustainable way. This topic has been under investigation generally for decades and there are several forms of renewable energy currently under investigation around the world, including wave, wind, solar and nuclear power to name but a few. Amongst the options available to the world, wave energy is of particular interest to the authors and has been under investigation since the late seventies, for example see [2]. The history of wave energy research is complex and has not always been continuous due to lack of funding, however research has persisted and continues to be a challenging problem.

Here we present a model of a wave energy device called AquaBuOY. The mathematical modeling of wave energy devices utilizes mathematical modeling techniques commonly used in ocean research. Generally speaking, the modeling of wave energy devices can be rather broad with regards to the details involved which includes but is not limited to the modeling of the mechanics, hydrodynamics and materials, even for a particular wave energy device such as the AquaBuOY. The scope of this paper is by no means to cover all of the aspects of modeling such a device. However the the scope of this paper is to present the model of the vertical dynamics of the AquaBuOY which is used to obtain a prediction of the absorbed power of the device. The main model described here is used as a tool to optimize the wave energy extraction from a particular device. This was and continues to be of particular interest to the company Finavera Renewables Inc. in the design of their wave energy device. The first steps of the research and development are to physically build larger scales of the improved device. This process of course can become very expensive and time consuming and as with many technologies the use of Industrial Applied Mathematics can prove to be useful to Finavera in order to optimize their design over several variables and under different conditions. For example predicting the power output of the same device placed on sites with different wave profiles. The aim of the work presented here was to validate the proposed design of the buoy and hose-pump system in order to then validate the results on a life size buoy device (the AquaBuOY 2.0 deployed off of the USA's Oregon coast in 2007) and to continue optimizing the system in order to produce a maximal amount of energy per buoy. The results from the numerical model discussed in this paper compared very well to the actual buoy design at a tenth scale and half scale (deployed in 2007 off of the coast of Oregon, USA). However the results are extensive and are not discussed in this paper. There will be a paper to follow this publication with the main results geared towards the ocean energy community. However, we will show some results from the numerical model of a theoretical full scale buoy in both regular and irregular wave regimes as examples of the results that are used in the optimization process. All of the inputs required to obtain these results will not be provided however in order to respect the interests of Finavera Renewables Inc. Despite this, the authors hope that the modeling techniques and processes discussed within this paper will be useful in the future research of its readers. For the interested reader see related papers [5] abd [7].

2 Mathematical model

The AquaBuOY represents a combination of the Hose-Pump and the IPS buoy (both developed in Sweden).

The Hose-Pump is a steel armored rubber hose with two check valves. When the hose is elongated by the wave force its volume is decreased, the pressure inside the hose increases, and water is flowing through one of the check valves into a connecting hose. When the wave falls back, the volume increases and the pressure decreases and the water is then sucked into the hose from the sea.

The basic IPS (otherwise known as the IPS-OWEC) system is a circular or oval buoy with diameter and weight adapted to the predominant wave situation at the place of location. Energy transfer from the motion of ocean waves takes place by converting the vertical component of wave kinetic energy into pressurized water flow. Pumped water is directed into a conversion system consisting of a Pelton turbine driving a conventional electrical generator.

The 'AquaBuOY' is modeled as a two-body system including a float connected to a submerged mass via a power take off system.

- The float displacement, velocity and acceleration (measured from its initial values) are defined as the variables z_1, $\dot{z}_1$ and $\ddot{z}_1$.
- The piston displacement, velocity and acceleration (measured from its initial values) are defined as the variables z_2, $\dot{z}_2$ and $\ddot{z}_2$.
- We begin the discussion of the model with the linear second order ordinary differential equations of motion for the vertical motion of the two bodies. This can be expressed as follows:

$$
\begin{aligned}
&(M_1 + a)\ddot{z}_1 + b\dot{z}_1 + S_{hs}z_1 + b_2(\dot{z}_1 - \dot{z}_2) + c(z_1 - z_2) = F_w(t) \\
&M_2\ddot{z}_2 = b_2(\dot{z}_1 - \dot{z}_2) + c(z_1 - z_2)
\end{aligned}
\tag{1}
$$

Technical note: This formulation of the problem ignores the hydro dynamic forces on the mass of water in the tube and ignores any friction and drag between the water and the hull.

These equations can be solved analytically in the frequency domain. In this paper we show how these equations are solved in the time domain by solving the differential equations. The time varying model construction described in this manuscript was implemented in Matlab and validated against Simulink irregular wave model results as well as regular wave analytical solutions (benchmark test cases) implemented in Mathcad (though these results are not included here).

For further background reading on ocean waves see [4] and for a good reference about general wave energy extraction modeling see [3].

2.1 Rearrangement of equations in (1)

In order to solve the system (1) using the Matlab solver packages we need to convert the second order differential equations into a system of first order differential equations. The following 'trick' is a standard technique covered in general ordinary differential equations courses in university. We introduce the variables $v_1 = \dot{z_1}$ and $v_2 = \dot{z_2}$. The second order system of ordinary differential equations (1) can then be rearranged into the following system of first order ordinary differential equations as:

$$\begin{aligned} \dot{z_1} &= v_1 \\ \dot{z_2} &= v_2 \\ \dot{v_1} &= \tfrac{F_w(t) - bv_1 - S_{hs}z_1 - b_2(v_1 - v_2) - c(z_1 - z_2)}{M_1 + a} \\ \dot{v_2} &= \tfrac{b_2(v_1 - v_2) + c(z_1 - z_2)}{M_2} \end{aligned} \tag{2}$$

Let $\mathbf{y} = (z_1, z_2, v_1, v_2)^T$ then $\dot{\mathbf{y}} = (\dot{z}_1, \dot{z}_2, \dot{v}_1, \dot{v}_2)^T$ so that the above system is written in matrix form as

$$M\dot{y} = f(y) \tag{3}$$

where M is the identity matrix,

$$f(y) = \begin{bmatrix} v_1 \\ v_2 \\ \tfrac{F_w(t) - bv_1 - S_{hs}z_1 - b_2(v_1 - v_2) - c(z_1 - z_2)}{M_1 + a} \\ \tfrac{b_2(v_1 - v_2) + c(z_1 - z_2)}{M_2} \end{bmatrix} \tag{4}$$

and the initial conditions at time $t = 0$ are

$$\begin{aligned} z_1(0) &= 0, \\ z_2(0) &= 0, \\ v_1(0) &= 0, \\ v_2(0) &= 0. \end{aligned} \tag{5}$$

The system defined by (3) together with the given initial conditions (5) form the system of equations which are solved for the unknown displacements and velocities as functions of time using a program written in Matlab. The following are definitions of the variables found in the system (1), (2) or in (4):

2.2 Variables involved

Definition	Label	Units
Period (for regular waves)	T	$[s]$
Wave height (for regular waves)	H	$[m]$
Average wave period (for irregular waves)	T_z	$[s]$
Significant wave height (for irregular waves)	H_s	$[m]$
Wave exciting force	$F_w(t)$	$[N]$
Water density	ρ	$[kg/m^3]$
Acceleration due to gravity	$g = 9.81$	$[m/s^2]$
Float diameter	D	$[m]$
Float draft	h_f	$[m]$
Float water plane area	$A_w = (\pi/4)D^2$	$[m^2]$
Structural mass of float	$M_1 = A_w h_f \rho$	$[kg]$
Hydrostatic stiffness parameter	$S_{hs} = A_w \rho g$	$[N/m]$
Tube length	L_t	$[m]$
Tube diameter	D_t	$[m]$
Mass of water in accelerator tube	$M_2 = (\pi/4) L_t \rho D_t^2$	$[kg]$
Added mass of water for float	a	$[kg]$
Hydro dynamic damping of float	b	$[Ns/m]$
PTO spring stiffness	c	$[N/m]$
Time	t	$[s]$
Linear damping of relative motion	$b_2 = cd\sqrt{S_{hs} M_1}$	$[Ns/m]$
Linear damping ratio	cd	no units
Non-linear Coulomb damping (see eqn. (27))	F_{fric}	$[N]$

The variables a and b are produced for the AquaBuOY shape using a hydro dynamic software package called Wamit (there is also an equivalent software called Aqua which was used in earlier numerical validation). For the given shape of the buoy, and a given array of periods T, Wamit produces corresponding arrays for the values of a, b and the excitation force F (at unit amplitude). See equation (11) or (19) where F is used. The given data is non-dimensional and scaled. These values are used for both regular and irregular waves.

The variables c for the spring stiffness and b_2 or $Fric$ for the damping between the float and piston are variables within the context of this report. In practice, in order to obtain the linear damping b_2 from a physical model, one graphs the Force versus the velocity and the slope of that is b_2. In order to obtain the damping coefficient F_{fric} from a physical model, one graphs Force versus extension (which is a trapezoidal graph) and takes the difference between the upper line and the lower line, divided by two which would give F_{fric}.

2.3 Retardation function and average added mass

From theory we know we can replace the added mass a and the term $b\dot{z}_1 = bv_1$ in (1) or in (4) by the average added mass and the term bv_1 by the following integral:

$$\int_{-\infty}^{t} h(t-\tau)v(\tau)\mathrm{d}\tau \tag{6}$$

where the retardation function $h(\tau)$ (see [1] and [8]) is defined as

$$h(\tau) = \frac{2}{\pi}\int_{0}^{\infty} b(\omega)cos(\omega\tau)\mathrm{d}\omega \tag{7}$$

This replacement is valid for both regular waves and irregular waves, however it is mostly useful for irregular waves since at each time the irregular wave exciting force (see equation (19)) is composed from different wave periods which means that we would need to interpolate the corresponding a and b at each step in time. However with the added mass replaced by its average value (since the added mass does not change very much) and the term bv_1 replaced by the integral in (6) (which is approximated by the sum in (8)), these values need not be calculated (interpolated from Wamit data for a given set of wave periods) at each time step. The integral in (6) is approximated using the standard quadrature rule (numerical rule for integral approximations) called Trapezoidal rule, taught in general university Calculus courses, as follows

$$\int_{-\infty}^{t} h(t-\tau)v_1(\tau)\mathrm{d}\tau = \sum_{n=1}^{N}(h(n\mathrm{d}t)v_1(t-n\mathrm{d}t)\mathrm{d}t) + \frac{1}{2}h(0)v_1(t)\mathrm{d}t, \tag{8}$$

noting that only the first N ($N = 40$ say) terms are included in the sum since the trailing terms of the retardation function are close to zero.

The added mass is calculated as

$$a_\infty = a(\omega) + \frac{1}{\omega}\int_0^\infty h(\tau) sin(\omega\tau)\mathrm{d}\tau, \tag{9}$$

and since this value doesn't have much variation within the frequencies we are interested in $\omega = 0.5$ and $\omega = 2$, the average value is used. Due to these replacements this means that the right hand side (4) of the system (3) is rewritten as

$$f(y) = \begin{bmatrix} v_1 \\ v_2 \\ \frac{F_w(t) - (\sum_{n=1}^{N}(h(n\mathrm{d}t)v_1(t-n\mathrm{d}t)\mathrm{d}t) + \frac{1}{2}h(0)v_1(t)\mathrm{d}t) - S_{hs}z_1 - b_2(v_1-v_2) - c(z_1-z_2)}{M_1 + a_\infty} \\ \frac{b_2(v_1-v_2)+c(z_1-z_2)}{M_2} \end{bmatrix} \tag{10}$$

2.4 Wave exciting force

The wave exciting force F_w in the system of differential equations (in any of the forms above) is a function of time, both for regular waves and for irregular waves. The theory described in this section can be found in many journals and books in the areas of ocean energy and ship research, however a good reference is [6]. For regular waves (known as monochromatic waves or harmonic waves) the wave exciting force can be calculated analytically for a particular wave amplitude ($A_{wave} = H/2$) and for a particular wave period T or rather for the corresponding frequency $\omega = 2\pi/T$ (discussed in the next section). In the section following that, it is outlined how we obtain a wave exciting force in a random or irregular sea (panchromatic waves) for some average wave period T_z. The latter is a more realistic model of the waves in the ocean since these are waves formed by a composition of waves of different frequencies.

2.4.1 Wave exciting force for regular waves

Assuming monochromatic waves we can define the wave exciting force

$$F_w(t) = A_{wave}F(\omega)cos(\omega t + \gamma) \tag{11}$$

where the wave amplitude is $A_{wave} = H/2$, γ is the phase difference between the wave and the force (we set $\gamma = 0$), and the cyclic frequency $\omega = 2\pi/T$.

2.4.2 Wave exciting force for irregular waves

We now look at a wave super-imposed by many regular waves of different heights and periods, which models the irregular wave patterns in a random or irregular sea. The height of each wave component is guided by the spectrum. The spectrum is a function that defines how much energy is contained in each frequency of the wave. Based on real sea measurements a number of generic spectrum have been proposed to describe how the energy is distributed. In nature one can measure the waves over say 20 minutes and from the measured time series determine what is the average wave period and the significant wave height. To reproduce an irregular wave with the same significant wave height and average period one can use the PM spectrum as defined below.

The wave conditions are described in terms of the significant wave height H_s and the average wave period T_z. The panchromatic (PM) spectrum can be defined from these two parameters as there is a general relation between the average period T_z and the peak period T_p.

$$T_p = 1.4T_z \tag{12}$$

PM-spectrum (H_s and T_p) The spectrum we describe following is the Brechsneider spectrum, but in view of its general similarity in shape compared to the PM spectrum it is referred to as the PM spectrum, named after two researchers Pierson and Moscowitz that derived a spectrum S(f), from numerous studies and measurements at sea as:

$$S(f) = \frac{A}{f^5}\exp(-\frac{B}{f^4}) \tag{13}$$

To generate specified sea conditions two parameters are given to determine the shape of the spectrum:

- The peak period T_p.

- The significant wave height H_s.

The value A is then determined from the desired significant wave height H_s and desired peak frequency $f_p = 1/T_p$ and B depends on the peak frequency alone as:

$$A = \frac{5H_s^2 f_p^4}{16} \tag{14}$$

$$B = \frac{5f_p^4}{4}. \tag{15}$$

The spectrum is then composed of a set of N frequencies, with components f_i. Each frequency is calculated as:

$$f_i = \frac{\triangle f \cdot \mathrm{rand}(1)}{2} + i \cdot \triangle f \tag{16}$$

where $\triangle f = 0.01$, rand(1) is a random number between 0 and 1, and $i = 0, ..., 40$. Using a randomly chosen number rand(1) for each frequency in this way, means that the interval between each frequency is not constant. This means that the time series does not repeat itself as it would with constant steps. The amplitude of each wave component is then given by

$$a_i = \sqrt{2S(f_i)\triangle f}, \tag{17}$$

with a corresponding random phase

$$\phi_i = 2\pi \cdot \mathrm{rand}(1). \tag{18}$$

The force signal is then calculated at any time t by multiplying each wave component with the wave exciting force amplitude (at unit wave amplitude):

$$F_w(t) = \sum_{i=0}^{N} [a_i \sin(2\pi f_i t + \phi_i) F(1/f_i)] \tag{19}$$

2.5 Fluid friction

To include fluid friction we add to the left hand side of the first equation in the system of ODEs (1), the term:

$$F_{ff} = A_w Cd\rho |v_1| v_1 \tag{20}$$

where Cd is a value between 1 and 2. This is equivalent to adding to the right hand side of the third equation in the system (2) or the third component of (4) the term:

$$-\frac{F_{ff}}{M_1 + a} = -\frac{A_w Cd\rho |v_1| v_1}{M_1 + a} \tag{21}$$

And if we are including the retardation function and the average added mass then we add to the third component of (10)

$$-\frac{F_{ff}}{M_1 + a_\infty} = -\frac{A_w Cd\rho |v_1| v_1}{M_1 + a_\infty} \tag{22}$$

2.6 Force and Power

In the case of linear damping, the absorbed power P_{abs} $[Watts]$ in the PTO system is in general the force times the velocity

$$P_{abs} = F_{pto} v_r \tag{23}$$

where the extension velocity (or relative velocity) is given by $v_r = (v_1 - v_2)$, and the force F_{pto} $[N]$ is given by

$$F_{pto} = b_2 v_r + c z_r \tag{24}$$

where the extension $z_r = (z_1 - z_2)$, and so the absorbed power in the PTO is then

$$P_{pto} = b_2 v_r^2 + c z_r v_r. \tag{25}$$

For the case of non-linear damping which characterizes 'AquaBuOY A', we describe the expression for Force and Power in the following section.

2.7 Non-linear damping

The equations discussed so far have been modeling a system with linear damping. That is, the linear damping used at previous steps gives a PTO force of:

$$F_{pto} = b2(v_1 - v_2) + c(z_1 - z_2) \tag{26}$$

however, the model of the hose-pump is not linear and a good approximation can be obtained by combining the linear spring with the Coulomb damping. The Coulomb damping gives a constant force opposing the direction of the motion, this would be

$$F_{pto} = F_{fric} sign(v_1 - v_2) + c(z_1 - z_2) \tag{27}$$

where

$$sign(x) = \left\{ \begin{array}{lll} -1 & if & x < 0 \\ 1 & if & x > 0 \\ 0 & if & x = 0 \end{array} \right\} \tag{28}$$

This means that if the relative velocity becomes zero, the acceleration force from the water in the tube must overcome the PTO force (which then depends on the position of the piston).

To include the non-linear Coulomb damping the only change that needs to be made in any of the equations in (1),(2), (4) or (10) is to replace the term

$$b2(v_1 - v_2) \tag{29}$$

by the term

$$F_{fric} sign(v_1 - v_2). \tag{30}$$

Note: In the Matlab implementation of the model described, the function $sign(x)$ is replaced with $tanh(100x)$ to easily avoid the singularities in the system of ODEs, which result in numerical instabilities, however other techniques can be used to avoid these singularities. The function $tanh(100x)$ satisfies the same conditions as sign(x) however it transitions between 1 and -1 smoothly.

3 Sample results

Figure 1 shows the displacements z_1, z_2 (from the initial positions) for the float and the piston respectively, as well as the relative displacement in a regular wave regime. To obtain these solutions we used the excitation force defined in equation 11. The results shown in figures 1,2 and 3 are obtained using the full scale buoy specifications (not included in this paper) in the regular wave regime with wave period $T = 10.2s$ and wave height $H = 2.5m$. The model which uses the regular wave regime is not a realistic model of ocean waves. However obtaining the solutions in regular wave regimes is very quick and can been useful for the initial stages of optimizing a buoy design. These types of solutions have been used for the validation of the forces as a function of time and extension (as shown in figure 2) of the actual hose pump laboratory experiments). Also the solutions shown in figure 3 are used for approximations of absorbed power assuming the average wave height and period are used as the wave height and period for a regular wave regime.

In figures 4, 5 and 6 we show results of a half scale buoy configuration in an irregular wave regime (see equation 19) with an average wave period $T_z = 5.5s$ and a significant wave height $H_s = 3m$. These solutions are naturally more realistic and compare to the actual buoy data results more accurately. This is of course due to the more realistic wave exciting force which models the irregularity of ocean waves. The solutions shown in figure 4 for the displacements of the buoy, the piston and the relative displacement are very different than those shown in figure 1 and are included here to display what the solutions of motion of the AquaBuOY look like in a more realistic wave regime. The results from this model compared very well with the actual data processed from the half scale AquaBuOY in 2007 as well as the fiftieth scale model of the AquaBuOY tested in Cork, Ireland in 2006 and 2007. The time scales for the solutions in the irregular wave regime is about 10 times the time range for the regular wave regime solutions due to the fact that the amount of time for the running average absorbed power (see figures 3 and 6) to converge to a steady value is tenfold in the irregular wave regime as compared to the regular wave regime. This is one of the reasons that regular wave regime models can be useful initially since the time that is required to compute the solutions is ten times less than that required for solutions in the irregular wave regime. Also interesting to note when comparing the results from a regular and irregular wave regime are the graphs of the forces. In figure 5 it is clear that under more realistic conditions the force (as a function of extension of the hose pump) is not a perfect quadrilateral but rather has some data scattered inside the outline of a quadrilateral. While the results obtained for force in a regular wave regime were used in the earlier stages to compare the hose-pump failure tests the

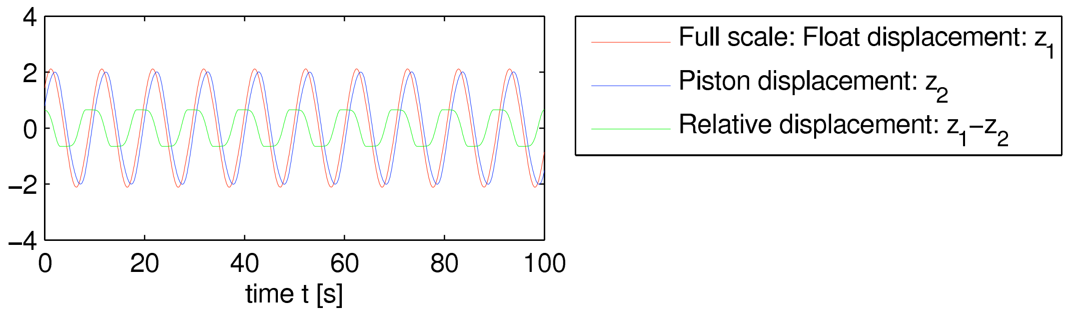

Figure 1: Solutions of displacement in a regular wave regime. Displacement is measured in meters [m].

results obtained in the irregular wave regime were much more realistic and fit much better to the actual hose-pump data.

4 Conclusions

We have introduced a mathematical model of the vertical dynamics of the AquaBuOY's IPS buoy and hose-pump power take off system. The numerical results obtained proved to be very accurate as compared to real life data of Finavera's fiftieth and tenth scales of the AquaBuOY. The numerical implementation of the model is extremely fast for the regular wave regime and nearly real time for the irregular wave regime, however the results in the irregular wave regime are far more accurate than for regular waves. The model and method have proved to be robust, efficient and accurate however future work is recommended in the time integration scheme used to solve the ordinary differential equations in the irregular wave regime as it would be useful for optimization over many variables to make the numerical integration faster.

5 Acknowledgements

The first author would like to thank the CA-OE partners for supporting her training at the Hydraulics and Maritime Research Centre (HMRC), University College Cork. We would like to thank Dr. Dominique Roddier for providing the hydrostatic data for which this model relied on.

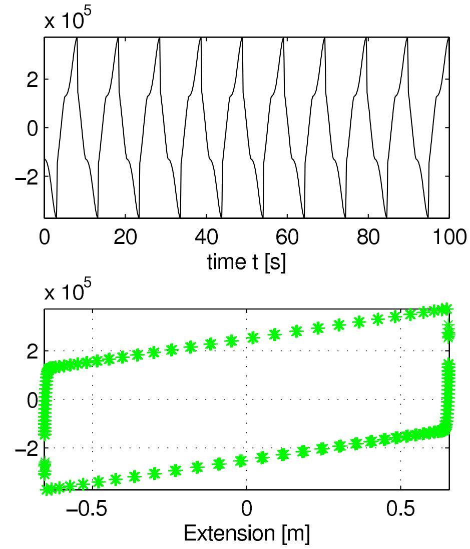

Figure 2: The force in the power take off system is calculated using the solutions of displacement in a regular wave regime. The top graph shows the force as a function of time. The bottom graph shows the force as a function of extension. Force is measured in Newtons [N].

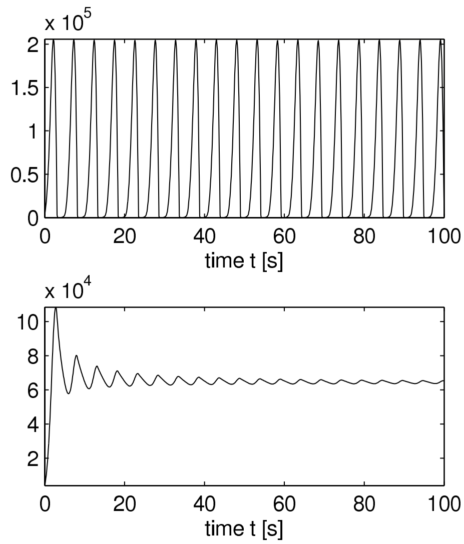

Figure 3: The power in the power take off system is calculated using the solutions of velocity and force in a regular wave regime. The top graph shows the power as a function of time. The bottom graph shows the power as a running average function over time. Power is measured in Watts [W].

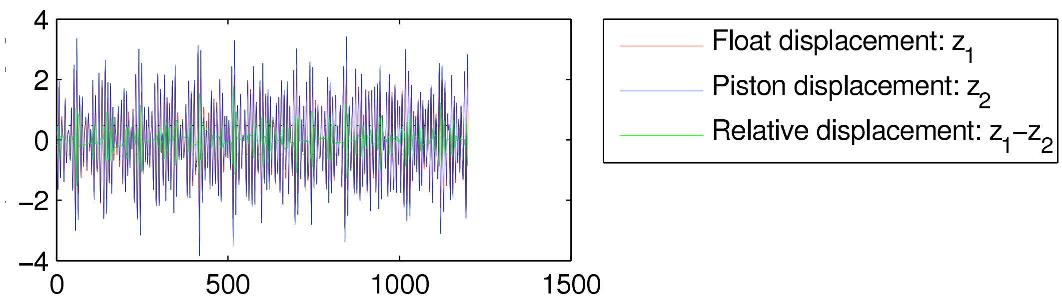

Figure 4: Solutions of displacement in an irregular wave regime. Displacement is measured in meters [m].

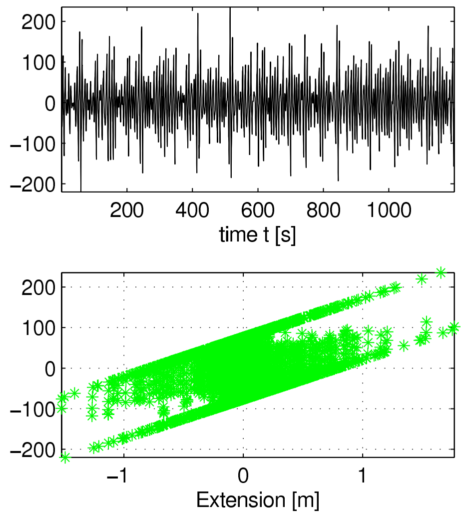

Figure 5: The force in the power take off system is calculated using the solutions of displacement in an irregular wave regime. The top graph shows the force as a function of time. The bottom graph shows the force as a function of extension. Force is measured in Kilonewtons [kN].

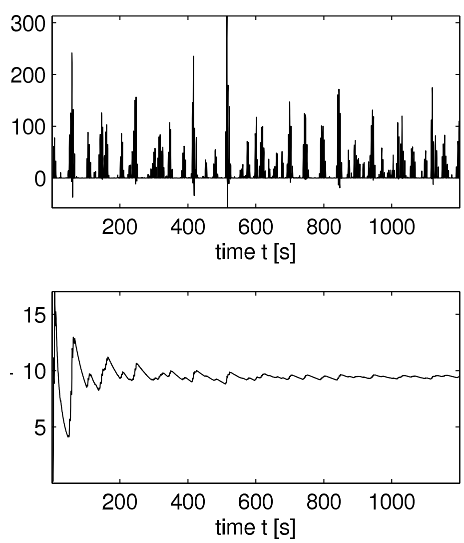

Figure 6: The power in the power take off system is calculated using the solutions of velocity and force in an irregular wave regime. The top graph shows the power as a function of time. The bottom graph shows the power as a running average function over time. Power is measured in Kilowatts [kW].

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