

where $\sqrt{2} g_{\pi NN} \gamma_5$ and $-g_{\pi} q_{\mu}$ represent the coupling of the pion to nucleon and the external field (lepton pair) respectively. Thus if we make the assumption that the original $1/q^2$ term is a hypothetical limit $m_{\pi} \rightarrow 0$ of this pion contribution, we obtain by comparing Eqs. (9) and (11)

$$g_{\pi NN} g_{\pi} = 2m_N G_1(0) G_0 \cos\theta/\sqrt{2} = 2m_N G_A/\sqrt{2} \quad (12)$$

This is the Goldberger-Trieman⁸ relation which relates $g_{\pi NN}$ ($g_{\pi NN}^2/4\pi = 14.6$), G_A ($= 1.18 \times 10^{-5}/m_p^2$) and g_{π} , the $\pi - \mu\nu$ decay constant. Originally it was obtained using simple dynamical assumptions and dispersion relations.

Because in actuality $m_{\pi} \neq 0$, the real axial vector current cannot satisfy $\langle n | \partial_{\mu} a_{\mu} | p \rangle = 0$ (unless $G_1(q^2)$ has the form $[q^2/(q^2+m_{\pi}^2)] G_1'(q^2)$, $G_1'(0) \neq 0$, in which case, however, $G_1(0) = 0$ or $G_A = 0$, in contradiction with β decay. Thus $\partial_{\mu} a_{\mu} \approx 0$ only to the extent that m_{π}^2 can be ignored compared to other parameters such as q^2 or m_p^2 . In this sense it is known as the partially conserved axial vector current (PCAC) hypothesis. To show this situation more clearly, we use here a formulation due to Gell-Mann and Lévy.⁹ Take the isotopic axial vector current $a_{\mu}^i = i\bar{q}\gamma_{\mu}\gamma_5\tau^i q$. Its divergence $\partial_{\mu} a_{\mu}^i$ has the same quantum numbers as the pion field ϕ^i . This is true at least if there are no hidden quantum numbers that can distinguish them. Then we can use $\partial_{\mu} a_{\mu}^i$ as the definition of pion field after proper normalization

$$\partial_{\mu} a_{\mu}^i(x) = C\phi^i(x) \quad (13)$$