III. SYMMETRY BREAKING AND MASS FORMULAS

We can contemplate four kinds of symmetry breaking within our context.

1. Breaking which destroys non-γ_5 $SU(3)_1 \times SU(3)_2$, reducing it to simple SU(3).

2. The Gell-Mann-Okubo breaking of simple SU(3).

3. Spontaneous breaking of γ_5 SU(3) groups and possibly non-γ_5 groups.

4. Non-spontaneous breaking which violates γ_5 symmetry and gives zerons finite masses.

The last one is a rather obscure problem at the moment, and we will not go into it in this paper. We start with the first kind.

1. Lifting of degeneracy. The pattern of breakdown of $SU(3)_1 \times SU(3)_2$ is similar to that due to spin-orbit coupling for angular momentum. A multiplet (ℓ_1, ℓ_2) of product rotation group $O(3)_1 \times O(3)_2$ will split, as the coupling between $O(3)_1$ and $O(3)_2$ is turned on, into irreducible representations of an O(3) characterized by the total angular momentum $(\underset{\sim}{\ell}_1 + \underset{\sim}{\ell}_2)^2$. In the present case, the generators $F_1^i = G_L^i + G_R^i$, $F_2^i = H_L^i + H_R^i$ correspond to $\underset{\sim}{\ell}_1$ and $\underset{\sim}{\ell}_2$. In contrast to O(3), however, two scalars (Casimir operators) may be constructed out of them. These are

$$F^2 = \sum_{i=1}^{8} F^i F^i ,$$

$$G^3 = \sum_{i,j,k} d_{ijk} F^i F^j F^k , \qquad F^i = F_1^i + F_2^i \tag{3.1}$$