

In curvilinear coordinates  $q^1, \dots, q^n$  the Schrödinger equation has the form (1).<sup>11</sup>

$$H\Psi = \alpha_1 \Psi, \quad (1)$$

where  $H$  is the Hamiltonian operator,  $\alpha_1$  the total energy, and  $\Psi$  the wave function of the entire system.

$$H = \frac{-\hbar^2}{2} \sum_{s,t=1}^n \frac{1}{g^{\frac{1}{2}}} \frac{\partial}{\partial q^s} g^{\frac{1}{2}} g^{st} \frac{\partial}{\partial q^t} + U \quad (2)$$

The  $q^s$  are generalized coordinates, and  $U$  is the potential energy;  $g^{st}$  is a contravariant tensor<sup>12</sup> conjugate to the metric tensor  $g_{st}$  appearing in the line element  $ds$  in mass-weighted space;  $g$  is the determinant of the  $g_{st}$ .

$$g^{st} = \sum_{i=1}^n \frac{1}{m^i} \frac{\partial q^s}{\partial x^i} \frac{\partial q^t}{\partial x^i}; \quad g_{st} = \sum_{i=1}^n m^i \frac{\partial x^i}{\partial q^s} \frac{\partial x^i}{\partial q^t} \quad (3)$$

$$ds^2 = \sum_{s,t=1}^n g_{st} dq^s dq^t; \quad \sum_{r=1}^n g_{sr} g^{rt} = \delta_s^t \quad (4)$$

where  $x^i$  is a Cartesian coordinate of an atom of mass  $m^i$ ; the coordinates of the  $k$ 'th atom are given by  $i = 3k, 3k + 1, 3k + 2$ . Both  $g_{st}$  and  $g^{st}$  are symmetric tensors.