The Seesaw Mechanism at Low Energies

The seesaw mechanism for neutrino masses defines a new scale of nature given by M, the mass associated with the heavy right-handed neutrino ν_e^c. Since M is postulated to be very large, well above the energies accessible through experiment, it is interesting that the "effective" neutrino mass operator in (11) approximates the seesaw terms in (15) at energies below M. To show this, we consider the effective operator

$$\frac{1}{M_{\text{effective}}}(h^0\nu_e - h^+e)^2 \ .$$

When the Higgs vacuum expectation value is accounted for, this operator yields the nonrenormalizable mass term in diagram (a) and a Majorana mass given by

$$\mu_\nu = \frac{\mathrm{v}^2}{M_{\text{effective}}}$$

In the seesaw mechanism, the light neutrino acquires its mass through the exchange of the heavy neutrino, as shown in diagram (b). Diagram (b), which is approximated by diagram (a) at energies below Mc^2, is a renormalizable mass term that involves both Dirac and Majorana masses. It yields a neutrino mass

$$\mu_{\text{light}} = \frac{{m_{\nu_e}}^2}{M} \quad \text{with} \quad m_{\nu_e} \equiv \lambda_{\nu_e}\frac{\mathrm{v}}{\sqrt{2}} \ .$$

Equating the values for μ_ν and μ_{light}, we obtain the relation between M and $M_{\text{effective}}$:

$$\frac{1}{M_{\text{effective}}} = \frac{(\lambda_{\nu_e})^2}{2M} \ .$$

At energies below M_W, the mass of the W boson, a similar type of relationship exists between Fermi's "effective" theory shown in diagram (c) and the W-boson exchange processes shown in diagram (d). The exchange processes are defined by the gauge theory of the charged-current weak interactions. Fermi's theory is a nonrenormalizable current-current interaction of the form

$$\mathcal{L}_{\text{Fermi}} = \frac{G_F}{\sqrt{2}}{J^{\mu}_W}^{\dagger} J_\mu^W \ ,$$

where the weak current for the neutrino-electron doublet is given by

$$J_W^\mu = 2{\nu_e}^\dagger\,\overline{\sigma}^\mu e \ \text{ and } \overline{\sigma}^\mu = (1, -\sigma^i) \ ,$$

and the Fermi constant G_F defines the strength of the effective interaction in diagram (c), as well as a new mass/energy scale of nature. The experimentally observed value is $G_F = 1.66 \times 10^{-5}\ \text{GeV}^{-2}$. Equating the low-energy limit of diagram (c) with that of diagram (d) yields the formula

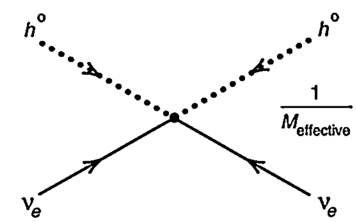
$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} \ ,$$

where g is the weak isospin coupling constant in the charged-current weak Lagrangian given by

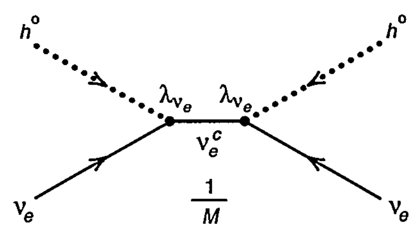
$$\mathcal{L}_{\text{weak}} = -M_W^2 W^{\mu +} W_\mu^- + \frac{g}{2\sqrt{2}} W_\mu^+ J^\mu{}_W + \frac{g}{2\sqrt{2}} W_\mu^- {J^\mu{}_W}^\dagger \ .$$

This Lagrangian neglects the kinetic term for the W, which is a valid approximation at energies much less than the W boson mass.

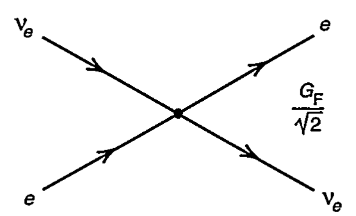
(a) Effective neutrino mass term



(b) Seesaw mass term for the light neutrino



(c) Fermi's current-current interaction



(d) Weak charged-current gauge interaction