homogeneity and λ should approach unity. If a is very small λ_0 will be equal to unity and all the higher λ_n will be zero. Equation 24 then reduces to:

$$\lambda = 1 + (\alpha^2/2)\, \lambda_1/(\lambda - \lambda_1) \tag{25}$$

From this we see that λ approaches unity as a approaches zero. It is furthermore clear that if α approaches zero, Eq. 25 will again hold and λ must again approach unity so that we have verified that Eq. 24 has the correct limiting behavior.

We now wish to work out an approximate procedure for calculating λ, which will be reasonably accurate and simple for an arbitrary function $\mu(\underline{x})$. If the wave vector $\underline{k}$ is real, Eq. 9 can be arranged to have a Hermitian kernel. We multiply each side of Eq. 9 by $\sqrt{\mu(\underline{x})}$ and rewrite it as follows:

$$\lambda \phi_k(\underline{x}) \sqrt{\mu(\underline{x})} = (1/4\pi) \int d\underline{x}' \frac{e^{-|\underline{x}-\underline{x}'|}}{|\underline{x} - \underline{x}'|^2} \times e^{i\underline{k}\cdot(\underline{x}'-\underline{x})} \sqrt{\mu(\underline{x})\mu(\underline{x}')}\ \phi_k(\underline{x}') \sqrt{\mu(\underline{x}')} \tag{26}$$

This equation can be derived from a simple variational principle and λ can be written as the maximum of the following expression:

$$U = \frac{\int d\underline{x} \int d\underline{x}'\ \phi_k^*(\underline{x})\ \mu(\underline{x})\ (1/4\pi) \dfrac{e^{-|\underline{x}-\underline{x}'|}}{|\underline{x} - \underline{x}'|^2}\ e^{i\underline{k}\cdot(\underline{x}'-\underline{x})}\ \phi_k(\underline{x}')\ \mu(\underline{x})}{\int d\underline{x}\ \phi_k^*(\underline{x})\ \phi_k(\underline{x})\ \mu(\underline{x})} \tag{27}$$

The maximum will be reached when ϕ_k is an actual solution of Eq. 9. If the variation of $\mu(\underline{x})$ is not too violent, $\phi_k(x)$ will be approximately constant. We, therefore, place, $\phi_k(\underline{x})$ and $\phi_k^*(\underline{x})$ equal to unity and investigate the agreement between the value of U thus obtained and the correct value of λ. It is, of course, clear that the value of U thus obtained will always be lower than the correct value of λ. We write λ for this approximate value and obtain:

$$\lambda = \frac{\int d\underline{x} \int d\underline{x}'\ (1/4\pi) \dfrac{e^{-|\underline{x}-\underline{x}'|}}{|\underline{x} - \underline{x}'|^2}\ e^{i\underline{k}\cdot(\underline{x}'-\underline{x})}\ \mu(\underline{x})\ \mu(\underline{x}')}{\int d\underline{x}\ \mu(\underline{x})} \tag{28}$$