PAULI PRINCIPLE AND PION SCATTERING

by

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ABSTRACT

It is pointed out that if the Pauli principle is taken into account in the discussion of pion scattering by complex nuclei (as it ought, of course, to be) some rather implausible consequences of some earlier treatments of this problem can be avoided.

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The scattering of pions by complex nuclei is commonly treated by the Kisslinger potential.[1] In momentum space, this has the form

$$\langle \vec{k}' | V | \vec{k} \rangle = a_0(\omega) + a_1(\omega)\, \vec{k} \cdot \vec{k}' \ , \qquad (1)$$

where k, k' are the initial and final momenta of the pion inside the nucleus, $\omega$ is the energy of the pion, and $a_0 a_1$ are the s- and p-state $\pi$-nucleon scattering amplitudes. The form (1) is very natural; in particular, the second term reflects the p-wave character. As is well known, in the energy range from about 50 to 400 MeV the p-wave scattering dominates. The momentum space amplitude is then translated into a potential in coordinate space, and the Schrödinger equation is solved in this potential.

Auerbach, Fleming, and Sternheim,[2] among others, have used the Kisslinger potential to calculate the scattering of $\pi^+$ and $\pi^-$ by various nuclei, and found good agreement with experiment. They introduce the new amplitudes

$$b_0 = -2(2\pi)^3 E_\pi a_0 / k_0^2 \ ,$$

$$b_1 = -2(2\pi)^3 E_\pi a_1 \ , \qquad (2)$$

where $k_0$ is the pion momentum in vacuum and $E_\pi = \omega$, its energy. They discuss how $b_0$ and $b_1$ can be calculated from pion-nucleon phase shifts. In this calculation, they take into account the difference between laboratory and center-of-mass momentum for $\pi N$ scattering. Fortunately, the real parts of $b_0$ and $b_1$ do not depend strongly on $\omega$, at least as long as $E_\pi$ is well below the $\pi N$ resonance (180 MeV). For $b_1$ this is very plausible because the main momentum dependence of the p-wave amplitude is contained in the factor $\vec{k} \cdot \vec{k}'$ in (1); for $b_0$ it follows from the fact that $a_0$ is essentially zero for $p_0 = 0$.

The theoretical values of $b_0$, $b_1$, i.e., those derived from $\pi N$ scattering, are at 80 MeV

$$b_0 = -0.75 + 0.34\, i \qquad F^3 \ ,$$

$$b_1 = 6.5 + 1.8\, i \qquad F^3 \ . \qquad (3)$$

The positive sign in the real part of $b_1$ signifies attraction, the negative $b_0$ repulsion. The dimension is fermi$^3$ for both amplitudes.

If we apply the "Schrödinger" equation of Auerbach et al. to a region of constant nuclear density[3] $\rho$, we obtain

$$-(1 - b_1\rho)\nabla^2\psi = k_0^2(1 + b_0\rho)\psi \ , \qquad (4)$$

where $\psi$ is the pion wave function, and $p_0$ its momentum in free space. Introducing the momentum k in the nuclear medium, we have