It is possible, of course, to investigate more precisely the shape of the energy versus fragment separation curve near $r = 0$ if some specific model is assumed. Considering the Bohr liquid drop model, we assume that the original nucleus is a sphere and then calculate the change in energy for a small deformation. Let us further assume that the sphere, in beginning to split, deforms in a very simple manner, namely, it stretches slightly in one direction and flattens our perpendicularly to this direction, thus becoming an ellipsoid. If we assume that the sphere does not change its volume on becoming an ellipsoid, and this is reasonable in view of the fact that all nuclei tend to maintain the same density of nuclear particles, the change in the energy of the nucleus upon deformation will be due to only two of the the five factors discussed in the last section. First, the *surface energy* will tend to *increase* with deformation because more surface will be exposed. Second, the *electrostatic energy* will *decrease* because the repelling charges will be effectively separated to some extent. Thus we have at least two energies changing in opposite ways with deformation of a spherical nucleus. The surface or capillary energy is proportional to the surface area or $A^{2/3}$, and the electrostatic energy is proportional to $Z^2A^{1/3}$, which is $\sim A^{5/3}$. The latter energy becomes more important for heavy nuclei so that for heavy nuclei it is likely that the energy of a nucleus tends to decrease with deformation, making a spherical nucleus unstable. The opposite is true for light nuclei. From this picture, it is in heavy nuclei that we would expect fission.

Let us investigate in some detail the change of energy of a spherical nucleus upon distortion. The surface energy is proportional to the surface area, which for an ellipsoid is:

$$2\pi b^2 + 2\pi \frac{a^2 b}{\sqrt{a^2 - b^2}} \arccos (b/a)$$

a and b are the semimajor and semiminor axes, respectively. (Note that for $b = a$ this reduces to $4\pi b^2$, since $\arccos (b/a) \simeq \arctan (\sqrt{a^2 - b^2}/b \simeq \sqrt{a^2 - b^2}/b$.) The electrostatic energy of a charge distributed throughout the volume of an ellipsoid can be shown to be:

$$\frac{3}{10} \frac{Z^2 e^2}{\sqrt{a^2 - b^2}} \log_e \frac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}}$$

(This reduces to $(3/5)(Z^2/r)$ for $b = a = r$.) Now consider a sphere of original radius R. If it is stretched in one direction, then $a = R(1 + \varepsilon)$. The minor axis b changes so as to keep the volume of the sphere constant, i.e., $(4\pi/3\, ab^2 = (4\pi/3)R^3$, from which $b = R/\sqrt{1 + \varepsilon}$. Substituting for a and b in the two energy expressions above and developing the results in powers of ε, we find the electrostatic energy to be:

$$\frac{3}{5} \frac{Z^2 e^2}{R} \left[1 - \frac{1}{5}\varepsilon^2 + \ldots\right]$$

and the surface energy to be:

$$4\pi R^2 \left[1 + \frac{2}{5}\varepsilon^2 + \ldots\right]$$

It is to be noted that the first terms in each of these equations are simply the electrostatic energy and surface area, respectively, of the undistorted sphere, while the second terms are the corrections for distortion. The correction term for the electrostatic energy is negative, indicating a decrease with distortion; that for the surface area is positive, corresponding to an increase in area. Using the proper coefficients for these energies from the formula for M(A,Z), equation (8-8), the excess in energy of the ellipsoid over the sphere is:

$$\varepsilon^2 \frac{2}{5}\left[(0.014\, A^{2/3}) - \frac{1}{5}(0.000627) \frac{Z^2}{A^{1/3}}\right]$$

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