This formula can be used for calculation of the binding energies of neutrons to isotopes of uranium. This information will be very closely connected with the ability of slow neutrons to fission these various isotopes, as we shall see. Let us calculate the binding energy of a neutron to U^{235}.

$$\begin{array}{rll} U^{235}: & M = 235.11240 & \text{[from equation (8-8)]} \\ \text{Neutron:} & M = \underline{\;\;1.00893\;\;} & \\ \text{Sum} & \;\;\; = 236.12133 & \\ U^{236}: & -M = \underline{-236.11401} & \text{[also equation (8-8)]} \end{array}$$

$$\text{Binding energy} = 0.00732 \text{ mass units or } 6.81 \text{ Mev}$$

Similarly the binding energies of neutrons to U^{236}, U^{237}, and U^{238} would be 5.51, 6.56, and 5.31 Mev, respectively. The alternation of the magnitudes of the binding energies comes from the factor δ. This alternation is superposed on the regular variation of M(A,Z) with A and Z given by the other five terms of equation (8-8).

Additional examples of this type are given in the problems at the end of the chapter.

8.2 THE FISSION PROCESS – ENERGY CONSIDERATIONS

The packing fraction curve (see Section 3.2 and Figure 15) shows that in the region of uranium, the packing fraction is of the order 0.0006, whereas for middle-weight nuclei it is of the order −0.0007. This implies that the heavy nuclei are not energetically stable against breaking into two middle-sized nuclei. Examining this more closely, we see that the energy that would be released in such a splitting is $M(A,Z) - 2M(A/2, Z/2)$. If this is positive, the splitting is energetically possible. This difference can be written in terms of the packing fractions:

$$A\left[\frac{M(A,Z)-A}{A} - \frac{M(\frac{A}{2},\frac{Z}{2}) - \frac{A}{2}}{\frac{A}{2}}\right] \tag{8-9}$$

or A times the difference in the packing fractions. Thus, when the difference between the packing fractions is positive, then fission is energetically possible. It is to be noted that the packing fraction difference does not give the energy released in a fission process. In Figure 49, the curve of N versus Z is shown (see also Figure 12). The transition from P to Q on the diagram results in an energy release proportional to the packing fraction difference. Actually in fission the end state is on the curve of stable isotopes at point R in Figure 49. Since R is at a lower mass point, the energy release in fission is *greater* than that given in equation (8-9). For example, if A is 240, then A/2 is 120; Z_A is 93.74, equation (8-7), for A = 240, and $Z_A/2$ is then 46.87. Using A/2 = 120 and the formula for Z_A gives the stable Z_{120} as 51.15, or about 4 units from $Z_A/2$. (That is $Z_A/2 < Z_{A/2}$.) This means that about four beta particles will be emitted per fragment after fission.

From the packing fraction curves, it appears that fission is exoergic for all nuclei with A greater than 100. Why, then, is fission such a rare process? Consider a nucleus that breaks into two fission fragments. Plot the energy of the nucleus, i.e., the fragments, as a function of the distance between the two parts. At infinite separation, the energy is taken to be zero. When the fragments are combined (r = 0), we know from measurements that the energy is about 200 Mev or greater. What about points between r = 0 and infinity? Up to a distance of the order of the diameter of the fragments, it is the Coulomb energy between the particles alone that contributes to the energy between particles, since that is the only force acting between the fragments. This energy is $(Ze/2)^2/r$. When r is less than the diameter of the fragments, the energy must change in such a way that it becomes the fission energy (200 Mev) at r = 0. If $(Ze/2)^2/r$ is smaller than, equal to, or greater than the fission energy at r = diameter of the fragments, then there are three corresponding transition curves, Figure 50, which can

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