CHAPTER VIII

NUCLEAR FISSION

8.1 THE BINDING ENERGIES OF NUCLEI

Before we proceed to take up the subject of fission, it is necessary to understand the general nature of the forces that hold nuclei together. For this purpose, we should like to find an expression for the nuclear binding energy. As we have seen in Chapter III the nuclear mass is related to the binding energy. The relation is simply:

$$M = (A - Z)M_n + ZM_p - (\text{Binding energy}/c^2) \tag{8-1}$$

with M the nuclear mass, M_n the neutron mass, M_p the proton mass, and A,Z the mass and atomic numbers, respectively. This relationship shows that we may check any conclusions about nuclear binding energies by comparison with nuclear masses.

In the absence of exact knowledge concerning the nuclear forces, the problem of finding the dependence of binding energy on Z and A is a difficult one. We must examine our empirical knowledge about nuclei for implications concerning the nuclear forces or the binding energy. Our empirical knowledge includes:

Nuclear size and constancy of density of nuclear matter.
Tendency of Z to be equal to A/2.
Effectiveness of Coulomb forces in making Z less than A/2.
Rarity of nuclei with even A and odd Z.

Each of these factors will be considered separately in its effect on the nuclear binding energy.

Consider first the nuclear size. From scattering and other experiments with heavy nuclei it is found that nuclear radii are proportional to $A^{1/3}$. In fact,

$$R = 1.48 \times 10^{-13} A^{1/3} \text{cm} \tag{8-2}$$

fairly well fits the known data, although this formula does not mean much if applied to the very lightest nuclei. For the present purpose, the formula implies that the average density of constituent particles is about the same in all nuclei. It is quite likely that the density within a single nucleus does not vary much from one region within the nucleus to another. If a certain binding energy resulting from nuclear forces is to be associated with two nuclear particles within the nucleus a given distance apart, it is clear from the foregoing that this binding energy per unit volume of the nucleus is constant, inasmuch as the average distances between constituent particles are everywhere the same. We conclude, therefore, that the binding energy of nuclei is essentially proportional to their volume or to A. In terms of the energy of the nucleus (the negative of the binding energy), we have then $E_1 \sim -a_1 A$, where a_1 is some positive coefficient which these considerations have not sufficed to determine. This is not completely accurate, since we have failed to consider the fact that the nuclear constituents at the nuclear surface are not bound as strongly as particles inside. The number of such particles is proportional to the surface area so that we must subtract a number proportional to $A^{2/3}$ (or R^2) from our previous estimate E_1. Thus:

$$E_1 = -a_1 A + a_2 A^{2/3} \tag{8-3}$$

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