$$E - U>0 \quad u = A\sin\left[\sqrt{2m(E-U)/\hbar^2}\; r\right] + B\cos\left[\sqrt{2m(E-U)/\hbar^2}\; r\right]$$

$$E - U=0 \quad u = A'r + B' \qquad (5\text{-}8)$$

$$E - U<0 \quad u = A''\exp\left[-\sqrt{2m(U-E)/\hbar^2}\; r\right] + B''\exp\left[+\sqrt{2m(U-E)/\hbar^2}\; r\right]$$

Boundary conditions determine the values of A, B, A', B', etc. These conditions are that u/r ($=\psi$) is finite everywhere and vanishes at infinity. In addition, the first derivative must be continuous.

With these conditions and the solutions of (5-8) in mind, Figure 20 can be constructed. In all instances, u must be zero at $r = 0$, since $u/r = \psi$ must remain finite. The variation of u with r is fairly straightforward for the cases $E>0$ and $E = 0$. The situation when $E<0$ needs explanation. Near the origin, the usual oscillation is observed with increasing period as the function $E - U$ decreases. At $r = r_0$ (see Figure 20), the oscillation stops, and at greater values of r, the exponential solution, last equation, (5-8), must reduce to a single *negative* exponential, since the positive exponential would not satisfy the condition that u/r is finite at $r = \infty$. That this reduction to a single negative exponential is not possible for all values of E is shown in Figure 20 ($E<0$) where for $E = E_1$, the coefficient of the positive exponential is negative and for $E = E_2$, the coefficient is positive. Between E_1 and E_2, there must be some value of E for which the coefficient vanishes. There may be a number of values of E for which u/r is finite at $r = \infty$. These are the allowed values of E for $E<0$ corresponding to the discrete spectrum or the bound states of the system.

The case $E>0$ corresponds to the case of an incident particle (positive kinetic energy). As shown in Figure 20, the wave function outside the nuclear radius is a sine function $A\sin(\sqrt{2mE/\hbar^2}\; r+\delta)$ where δ is a phase shift dependent on the wave function within the nucleus to which the sine must be joined (at $r = R$). The sine function does not, when extrapolated, seem to come from the origin (dotted line figure) but appears to have its origin at a distance "a" from $r = 0$. This distance is related to δ by the equation $a/\lambda = \delta/2\pi$ with $\lambda = 2\pi\hbar/\sqrt{2mE}$ (the de Broglie wavelength of the incident particle).

Now it can be proved* that the scattering cross section is directly dependent on this phase shift δ in such a way that when δ is small (or an integral multiple of π), the scattering cross section is small, and when δ is $\pi/2$ (or an integral multiple of $\pi/2$), the cross section is a maximum. The relation between δ and σ_s is:

$$\sigma_s = (4\pi\hbar^2/m^2v^2)\sin^2\delta \text{ (s scattering only)} \qquad (5\text{-}9)$$

The limitation to s scattering means that the incident particle has zero angular momentum. On a classical basis, a particle with velocity v at large distances from the nucleus moving in such a direction that it would pass the nucleus (if unaffected by nuclear forces) at a distance b, Figure 21, has an angular momentum mvb. According to quantum mechanical principles, this must be quantized, or $mvb = l\hbar$ ($l = 0, 1, 2\ldots$). Thus $b = l\hbar/mv$, or $b = l\lambda\!\!\!^{-}$ ($\lambda\!\!\!^{-}$ = de Broglie wavelength x 2π). The region between $l = 0$ and $l = 1$ or $b = 0$ to $b = \lambda\!\!\!^{-}$ is the region of s scattering. Between $b = \lambda$ and 2λ is the p scattering region. Now if the nuclear size is less than λ, that is, $R<\lambda = \hbar/mv$, then it is obvious that no p scattering is possible. Particles passing at "p distances" from the nucleus will not be aware of the nucleus, due to the short range character of nuclear forces. Recalling the discussion in section 2.2, it is apparent then that *there is s scattering only if the neutrons are slow.*

For very low velocities, the formula (5-9) can be simplified. This is due to the fact that the wave function inside the nucleus will change very little with changes in E when E is small. For this reason, "a" does not vary. However, λ increases as $E^{\frac{1}{2}}$, with the net result that a/λ becomes

*Rasetti, F. "Elements of Nuclear Physics," 1936. Prentice-Hall, p. 204ff; Mott, N.F. and H.S.W. Massey, "Theory of Atomic Collisions," Oxford, 1933.

a-93-43