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SOME ASPECTS OF REACTOR THEORY

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SOME ASPECTS OF REACTOR THEORY

Alvin M. Weinberg

My purpose in these remarks will be primarily to outline what seem to me to be the major issues in reactor theory; second, to stress some results which ought to be classical, but which I find are not as well known as they should be; and finally, to ask some questions both of reactor theory, and of reactor theorists, to which I have not as yet heard convincing answers. I hope my remarks will serve the purpose of placing in better perspective a few of the classical problems in reactor theory which have not received as much attention as they ought.

There is really only one problem in chain reactor theory, just as there is really only one problem in quantum mechanics. In quantum mechanics it is the computation of the wave function, as a function of time, space, energy, and spin variable, given the physical parameters and initial conditions; in reactor theory it is the computation of the neutron flux as a function of time, space, energy, and angular direction, given the neutron cross sections, the source distributions, and the initial conditions. The eigenvalues, in the one case the stationary energies, in the other the critical reactor multiplication constant, are computed more or less as a corollary to the computation of the fluxes or wave functions.

During the war the exposition of reactor theory proceeded independently, and along rather different lines, at Los Alamos on the one hand, and at Chicago and Montreal, on the other. At Los Alamos the chain reactor systems were small and homogeneous. The properties of such systems could be computed in principle directly from the Boltzmann equation in which measured cross sections were used. At Chicago and Montreal, where interest was centered on the large unenriched

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heterogeneous chain reactors, the probem was always divided into two parts: first, the computation of the "intensive" or "microscopic" properties of the chain reactor such as the thermal utilization, the resonance escape probability, etc., and second, the computation of the "extensive" properties such as critical size by using the locally computed properties in a fictitious equivalent uniform chain reactor. This division into "macroscopic" and "microscopic" chain reactor theory was natural because the lattice structure so complicated the local neutron distribution as to make the capture probability in fuel of a thermal neutron, i.e., the thermal utilization, a problem of considerable proportion in itself. However, the resulting division of the theory into macroscopic and microscopic is one of its least satisfactory aspects. To this I shall return later.

Asymptotic Reactor Theory

From the macroscopic (or Los Alamos) viewpoint, the properties of a chain reactor are to be determined from solutions of an appropriate Boltzmann equation. Near a boundary - say the edge of the reactor - the solution of the Boltzmann equation is always much more complicated than far from the boundary, and so it is of interest to consider the asymptotic flux distribution - that is, the distribution far from boundaries. This asymptotic distribution is an intrinsic characterization of the reactor medium; consideration of the properties of the asymptotic distribution is therefore centrally important in the theory.

Now an asymptotic distribution will exist only if the properties of the chain reactor do not change from point to point. We therefore concern ourselves with a uniform reactor; since we consider distributions which are far from boundaries, we can imagine the uniform reactor filling all space.

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We denote the total number of fission neutrons created per second per c.c. by $F(\underline{r})$. Further, the probability that a single fission at $\underline{r}'$ will give rise to one primary fission neutron at $\underline{r}$ during the course of one generation we call $G(|\underline{r} - \underline{r}'|)$. The kernel G is a displacement operator because one part of the reactor is just like every other part. Then, in the steady state,

$$F(\underline{r}) = k \int F(\underline{r}') P(|\underline{r}' - \underline{r}|) d\underline{r}' \tag{1}$$

where $k = \int G(\underline{r})d\underline{r}$ is the total number of primary fission neutrons created by a single fission neutron, and $P(\underline{r}) = \dfrac{G(\underline{r})}{\int G(\underline{r})d\underline{r}}$.

An equation such as (1) in which the total fission neutron creation rate (independent of energy) is the variable was first suggested by Placzek and Volkoff.[1] Because the reactor is isotropic and homogeneous, P is a displacement operator, and the equation (1) is invariant under the Euclidean group. Thus, as can be verified directly, (1) possesses a solution which satisfies

$$\Delta F(\underline{r}) + B^2 F(\underline{r}) = 0 \tag{1a}$$

provided the buckling B^2 satisfies

$$k \bar{P}(B^2) = 1 \tag{1b}$$

where $\bar{P}(B^2)$ is the three-dimensional Fourier transform of P. The result which I have quoted here is the essential part of what is called the fundamental theorem of pile theory,[2] or again, it is the basic idea of the so-called extrapolated end point method. For the buckling B^2 is determined not only by the

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characteristic equation (1b) but also, in the actual finite system which is to be related to the fictitious infinite system, by the geometric relation

$$B^2 = \frac{\pi^2}{H^2}$$

in the case of a slab of extrapolated height H. The power distribution in that case is

$$F(\underline{r}) = F_o \cos \frac{\pi x}{H} .$$

It has always struck me as being entirely remarkable that perhaps the most important fact about a chain reactor - that its power distribution is sinusoidal with wave number B - follows essentially from what are the group-theoretic properties of the basic transport equations, and is independent of any details such as slowing down properties, cross sections, etc. Of course this is not to say that the <u>value</u> of B is independent of these; rather, for a critical B, the asymptotic power distribution in a uniform reactor is a consequence of the essential identity of each point of the reactor.

A corollary of the preceding remarks is that the asymptotic solution of the reactor equations must be separable in space and in energy; i.e., far from boundaries the neutron flux of <u>every</u> energy is sinusoidal with the same wave number B. Instead of the very general, but uninformative, (1) let us write the reactor equation in age theory approximation for the neutron flux[3] Φ,

$$\frac{D(u)}{\sigma_s(u)} \Delta \sigma_s \Phi - \sigma_a \Phi + f(u) \left[\int_0^{u_o} k(u') \sigma_a \Phi (\underline{r}, u') du' + k(u_o) \sigma_a(u_o) \Phi_s \right] =$$

$$\frac{\partial}{\partial u} \xi \sigma_s \Phi \qquad (2)$$

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where σ_s, σ_a are the macroscopic scattering and absorption cross-sections; D is the diffusion coefficient; u is the lethargy measured from the top of the fission spectrum; u_o is thermal lethargy; f(u) is the fission neutron spectrum; $k = \left(\frac{\nu\sigma_f}{\sigma}\right)$ is the multiplication constant at lethargy u; ξ is the logarithmic energy decrement.

The solution of (2) is

$$\Phi(\underline{r}, u) = \psi(\underline{r})X(u)$$

provided

$$\Delta\psi + B^2\psi = 0$$

$$X(u) = \left[\exp - \int_0^u \frac{\sigma_a(y) + B^2D(y)}{\xi\,\sigma_s(y)}\,dy\right]\left[\int_0^u f(z)\exp\int_0^z \frac{B^2D(x) + \sigma_a(x)}{\xi\,\sigma_s(x)}\,dx\right]dz$$

$$= \int_0^u f(z)\,p(z,u)\exp - B^2\tau(z,u)dz\,. \qquad (3)$$

Herein B^2 is determined as the solution of

$$k_f\,e^{-\tau_f B^2} + \frac{k_s\,e^{-\tau_s B^2}}{1 + L^2B^2} = 1 \qquad (4)$$

where the "fast" and "slow" multiplication constants, k_f and k_s are

$$k_f = \int_0^{u_o}\int_0^w \frac{\nu\,\sigma_f(w)}{\xi\,\sigma_s}\,p(z,w)\,F(z)dzdw; \quad k_s = \frac{\nu\,\sigma_f(u_o)}{\sigma_a(u_o)}\int_0^{u_o} p(z,u_o)F(z)dz\,. \qquad (5)$$

The "slow" and "fast" ages τ_s and τ_f are then defined by

$$k_f \exp - \tau_f B^2 = \int_0^{u_o} \int_0^{w} k(w) \frac{\sigma_a(w)}{\xi \sigma_s} p(z,w) e^{-B^2 \tau(z,w)} F(z)dzdw;$$

$$k_s \exp - \tau_s B^2 = k(u_o) \int_0^{u_o} p(z,u_o) e^{-B^2 \tau(z,u_o)} F(z)dz$$

and $p(z,u)$ is the probability that a neutron will escape resonance capture in slowing from lethargy z to lethargy u, while $\tau(z,w)$ is the age from z to w.

In the usual slow neutron reactors, it is assumed that all fast fission occurs at the single lethargy, o. Thus $\tau_f = \tau_s = \tau$, and (4) reduces to the customary

$$\frac{\eta \epsilon p f e^{-\tau B^2}}{1 + L^2 B^2} = 1$$

because, by definition, the fast effect is $\epsilon = \frac{1}{1 - k_f}$.

Equation (4) is the most general characteristic equation based on age theory for a reactor in which fission takes place at arbitrary neutron energy. It first appeared in the literature in 1944 in a paper by Wigner on breeders. In more recent years, it has been ignored to a rather surprising degree - perhaps because most resonance reactors are strongly reflected, and therefore the asymptotic solution is not very important practically.

It is natural to inquire whether there is an asymptotic angular distribution which, like the spatial distribution, is energy independent; i.e., whether the complete asymptotic neutron flux distribution can be written as a product

$$\Phi(\underline{r}, \underline{\Omega}, E) = \psi(\underline{r}) X(E) w(\underline{\Omega}).$$

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To appreciate what is involved in deciding this, we write the general one-dimensional Boltzmann equation in an infinite, multiplying medium in which moderation is by elastic collisions (cf. "Theory of Neutron Chain Reactions," ORNL CF-51-5-98, p. III-13)

$$\mu \frac{\partial \Phi}{\partial x}(x,\mu,E) + \sigma \Phi = \frac{(1+M)^2}{4M} \int_{E}^{E/\alpha^2} \int_{-1}^{1} \sigma_s(E')\Phi(x,E',\mu')\, \delta\left[\mu_o - g(E,E')\right] \frac{dE'}{E'} d\mu'$$

$$+ \frac{\nu}{2} f(E) \int_{0}^{E_o} \int_{-1}^{1} \sigma_f \Phi(x,E',\mu') dE' d\mu' \qquad (6)$$

where

$$g(E,E') = \frac{1}{2}\left[(M+1)(E/E')^{1/2} - (M-1)(E'/E)^{1/2}\right],$$

$\alpha^2 = \left(\frac{M-1}{M+1}\right)^2$, M = mass of moderator, $\mu_o = \underline{\Omega} \cdot \underline{\Omega}'$ = direction cosine of scattering angle. The δ function implies that the angle of scattering and the energy loss are correlated. Now $\delta\left[\mu_o - g(E,E')\right]$ can be expanded:

$$\delta\left[\mu_o - g(E,E')\right] \cong \sum \frac{2\ell+1}{2} P_\ell(\mu) P_\ell(\mu') P_\ell\left[g(E,E')\right]$$

+ azimuthal terms. Thus, the correlation between energy loss and angle means in a sense that the scattering law in the first place is anisotropic, and in the second place that the character of the anisotropy is energy dependent. Now it has been shown by Wigner that on group theoretic grounds the angular distribution of the flux in the velocity independent case involves the same number of spherical harmonics as the scattering law for the individual events. It would therefore be reasonable to expect that in the energy dependent case, where

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because of the correlation, an infinite number of energy dependent spherical harmonics are needed to describe the scattering, that the flux too will involve an infinite number of harmonics, the coefficient of each harmonic itself being energy dependent. In other words, there does not exist an asymptotic solution of (6) which is separable in space and in angle.

To see how this comes about, it is worth recasting the Boltzmann equation into an infinite set of linear integral equations.

We put

$$\Phi(x,\mu,E) = \frac{e^{iBx}}{\sigma + iB\mu} \psi(\mu,E),$$

and expand the function $\psi(\mu, E)$ in a Legendre series:

$$\psi(\mu,E) = \sum \frac{2\ell + 1}{2} \psi_\ell(E) P_\ell(\mu) .$$

There results an infinite set of integral equations which can be written

$$\psi_n(E) = \frac{(1+M)^2}{4M} \sum_\ell \int_E^{E/\alpha^2} P_n\left[g(E,E')\right] \frac{\sigma_s}{\sigma} Q_{\ell n}(B,E')\psi_\ell(E') \frac{dE'}{E'}$$

$$+ \frac{\nu}{2} f(E) \sum_\ell \int_0^{E_0} \frac{\sigma_f}{\sigma} Q_{\ell n}(B,E') \delta_{on} \psi_\ell(E') dE'$$

where

$$\frac{2}{2\ell+1} Q_{\ell n}(B,E) = \sigma \int_{-1}^{1} \frac{P_\ell(\mu) P_n(\mu)}{\sigma + iB\mu} d\mu .$$

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In terms of the angular components of the flux, $\Phi_\ell(E)$, where the Φ are defined by

$$\Phi(x,\mu,E) = e^{iBx} \sum_\ell \frac{2\ell+1}{2} \Phi_\ell(E) P_\ell(\mu)$$

$$\Phi_n = \sum_\ell \frac{1}{\sigma} Q_{\ell n} \psi_\ell,$$

the integral equations are

$$\sigma \Phi_\ell(E) = \frac{(1+M)^2}{4M} \sum_n Q_{\ell n} \int_E^{E/\alpha^2} P_n\left[g(E,E')\right] \sigma_s \Phi_n(E') \frac{dE'}{E'} \qquad (7)$$

$$+ \delta_{o\ell} \frac{\nu}{2} f(E) Q_{\ell o} \int_o^{E_o} \sigma_f \Phi_o(E')dE'.$$

These integral equations have the following interesting significance. The quantity $\sigma_s \Phi_n(E')$ is a sort of collision density of the n-th angular mode of the neutron flux; these neutrons after a collision are distributed in energy with a probability function $\frac{(1+M)^2}{2ME} P_n\left[g(E,E')\right]$ - i.e., the collisions are non-spherically symmetric. The energy distribution of all n-th mode neutrons contributing to the energy E is given by the first integral in Eq. (7). A certain fraction of these, $Q_{\ell n}(B,E)$, make a collision in the l-th mode without escaping from the reactor.

The matrix $Q_{\ell n}$, which Welton calls the non-leakage probability matrix, is therefore seen to be a generalization of the usual non-leakage probability. Each component $Q_{\ell n}$ can be thought of as giving the probability that a collision of an n-th mode neutron will be followed by an l-th mode collision before the neutron escapes from the reactor.

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The lowest $Q_{\ell n}$, Q_{oo}, which relates the Φ_o before collision to the Φ_o after collision is the familiar

$$Q_{oo} = \frac{\sigma}{B} \tan^{-1} \frac{B}{\sigma} \approx 1 - \frac{1}{3} \frac{B^2}{\sigma^2} \text{ for } \frac{B}{\sigma} \ll 1, \tag{8}$$

i.e., it is the Fourier transform of the monoenergetic transport kernel.

To solve (7) for Φ, and the eigenvalue B in general, seems out of the question. However, the infinite set of integral equations does reduce to a set of linear equations if all cross sections are energy independent, as shown by B. Davidson[4]; or the set reduces to a single integral equation in Φ_o if the correlation between energy loss and scattering angle implied in the δ function kernel of (6) is ignored. In this latter very simple case the equation (7) may be written

$$\frac{\sigma \Phi_o}{Q_{oo}} = \frac{(1 + M)^2}{4 M} \int_E^{E/\alpha^2} \sigma_s \Phi_o \frac{dE'}{E'} \tag{9}$$

where, for simplicity, we have assumed that the fission distribution is a delta function at energy E_o. Now (9) is just in the form of the equation for moderation in an infinite medium with total cross section $\frac{\sigma}{Q_{oo}}$.

An approximate asymptotic solution is

$$\sigma_s \Phi_o E \xi = \text{const} \times \exp - \int_E^{E/\alpha^2} \frac{\sigma/Q_{oo} - \sigma_s}{\sigma_s \xi} \frac{dE'}{E'}.$$

and from the extrapolated condition at E_o, viz.,

$$\sigma_{s_o} \Phi_o E_o \xi = \nu \int_o^{E_o} \sigma_f \Phi (E') dE',$$

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there results the characteristic equation

$$1 = \frac{Q_{oo}\,\sigma_{s_o}}{\sigma_o} \int_o^{E_o} k(E') \frac{dp}{dE'}(E_o, E')\; e^{-\int_{E'}^{E_o}\left[1 - Q_{oo}(B^2, x')\right] dx'/x'}\, dE'$$

which is a slight generalization of (4) for a resonance reactor. The main difference is that the non-leakage probability here is

$$e^{-\int_{E'}^{E_o}\left[1 - Q_{oo}(B^2, x')\right] dx'/x'}$$

which reduces, by virtue of (8), to the familiar

$$e^{-B^2\tau(E', E_o)}$$

if B is much less than σ, i.e., if the reactor is large compared to the mean free path.

"Practical" Multigroup Methods

The asymptotic theory has beauty and elegance. Its practical usefulness is limited of course because in small reflected reactors the non-asymptotic solutions tend to overwhelm the asymptotic solution. Largely on this account numerical methods have been devised - mainly at KAPL, but also at ORNL, at Rand and at GE-ANP, for handling the reactor equation in the age-diffusion approximation. In the group method as originally expounded at the Metallurgical Laboratory in Chicago, only the hardiest calculator - such as F. L. Friedman - would allow the number of groups to exceed 3. The demonstration by KAPL, and subsequently ORNL, that with IBM equipment it is perfectly feasible to handle as many as 32 groups is indeed a striking and, at least to me, unexpected development.

All things considered, the success of the multigroup methods has been extraordinary. I have listed here a table, prepared by R. A. Charpie, of some 8 resonance reactor critical masses observed both at KAPL and at ORNL, together with multigroup computed values. The cross section uncertainties apparently are not so important but that good agreement is obtained in almost every case.

The multigroup equations, being based on the age theory approximation, must be in error if the resonance cross sections are strongly self-absorbed. In addition, transport corrections are not within its competence. On this account it has often been suggested that an approximation directly to the Boltzmann equation ought to be developed, and Safonov[5] has proposed such a "Boltzmann" group method. It is my impression that just as IBM equipment made the step from two-groups to thirty-two groups possible, so it may be that the new digital computers, such as the ORACLE, might make numerical methods based on the Boltzmann equation practical.

Other Post-war Advances

The flowering of the multigroup method into a practical and effective method for computing reactors is, to my mind, the most important post-war development in the theory. There are, of course, several other advances which I have not time to discuss in detail, but which I should like to mention. These include Goertzel's investigation of the minimum critical mass problem; the thorough-going investigations of the non-linear kinetic properties of high powered chain reactors by almost all of the theoretical reactor groups; and, of course, the deeply penetrating and sophisticated analyses of the Boltzmann equation by our English competitors, notably B. Davidson.

Unsolved Problems

I should like now to turn to a few of the theoretical reactor problems on which progress has been less marked. The list includes both problems which are of practical importance and others of rather more theoretical interest.

The most interesting result that has emerged from the operation of the MTR is that the success of the whole operation depends on the careful programming of the shim rod positions. This arises because, before the reactor burns out, the bulk of the heat must be extracted from less than the full fuel area, and this leads to gross peaks in the heat load. As the reactor burns and the shim rods are withdrawn, the available effective heat transfer area increases; the hot spot problem becomes correspondingly less severe as the reactor burns out. Such flux peaks have of course been noticed in the Hanford operation; however, in an enriched, high burn-up reactor where as much as 10% of the fuel is destroyed, the effects are extremely pronounced. This is seen in the accompanying figure in which the longitudinal power distribution at the MTR before and after a 370 MWD run is plotted.

In a reactor like MTR where the purpose is to achieve a certain maximum flux, the uneveness of the distribution is not so important. If 2×10^{14} anywhere in the reactor is acceptable, then the power of the machine can always be tailored to match the effective heat transfer area. At the beginning of the run, 10 megawatts will be removed from 1 kg.; at the end, 30 MW from 3 kg. However, in a reactor where a certain total power output must be maintained, the design must realistically take account of the condition at the worst time during the run. If all the shim rods come in from one side, this will occur at the beginning of the run.

One possible way to overcome this burn-out peaking, which has been proposed several times, is to distribute a heavy poison such as boron throughout the reactor. The burn-out of the fuel is balanced by the burn-out of the poison; in this way the k excursion, and therefore the change in shim rod configuration, can be drastically reduced.

The realization that the shim rod programming would be crucial in operating the MTR - or any high burn-out enriched reactor - while understood in principle, was not appreciated by those of us concerned with the MTR until after the gadget was put into full powered operation. Looking back at the calculations I realize now that the estimate of the absolute maximum to average ratio was rather naive; it did not take into account adequately that this ratio can change drastically during the operation.

To the reactor theorist this presents a real problem - how to compute the spatial details of the power distribution in a system in which irregular shim rod positions cause serious local peaks. What is involved is the numerical solution of coupled wave equations - one equation for each neutron group. Evidently relaxation methods are extremely unwieldy in three-dimensional systems - and yet, the incentive to compute the flux peak with an accuracy of say 20% is strong enough to warrant an attack on this numerical problem with the new high speed computing machinery which is available to the AEC.

A second problem in nuclear reactor theory that I should touch upon is really a problem in hydrodynamics; it is the estimation of turbulent fluctuations in a high powered boiling or homogeneous reactor. The experiments described by W. M. Breazeale on boiling the LITR and the SUPO show that the power output of a boiling system is subject to large fluctuations. In this case the fluctuations arise primarily because the boiling is not even, and the average bubble density, and therefore the average density of the reactor fluid, vary irregularly in time. Such density fluctuations imply quick reactivity surges, and these of course are reflected in the observed power fluctuations.

There are, however, rather more subtle reactivity fluctuations which one can think of in a turbulent system. Consider a quiet, just critical chain reactor. The fission neutron creation rate $F(\underline{r})$, in the steady state satisfies

$$\frac{1}{k} F(\underline{r}) = \frac{1}{1+\beta} \int F(\underline{r}') P(\underline{r}',\underline{r}) dr' + \frac{\beta}{1+\beta} \int F(\underline{r}') H(\underline{r}',\underline{r}) dr'$$

where β is the delayed neutron fraction.

The term $\frac{k}{1+\beta} \int F(\underline{r}') P(\underline{r}',\underline{r}) d\underline{r}'$ represents the contribution to the total fission neutron creation rate of the prompt neutrons; the term $\frac{k\beta}{1+\beta} \int F(\underline{r}') H(\underline{r}',\underline{r}) d\underline{r}$ is the contribution of the delayed neutrons. The kernel $H(\underline{r}',\underline{r})$ is the probability that a fission at $\underline{r}'$ will result in a delayed neutron somewhere in the system which will ultimately lead to a new fission at $\underline{r}$. Thus if $Q(\underline{r}',\underline{r}'')$ is the probability that a fission at $\underline{r}'$ will lead to a delayed neutron emitted at $\underline{r}''$, then

$$H(\underline{r}',\underline{r}) = \int Q(\underline{r}',\underline{r}'') P(\underline{r}'', r) d\underline{r}'' .$$

In a completely quiescent reactor, $Q(\underline{r}',\underline{r}'') = \delta(\underline{r}' - \underline{r}'')$ since the delayed neutrons are created at the same place as the prompt neutrons, and $H(\underline{r}', \underline{r}) = P(\underline{r}',\underline{r})$; in a circulating fuel system, $H(\underline{r}',\underline{r})$ will have a different form, $H_1(\underline{r}',\underline{r})$ which depends on the hydrodynamic details of the flow.

Now suppose the originally quiet reactor starts to circulate. Then to maintain criticality the multiplication constant will have to change to k_1, and the new distribution, $F_1(\underline{r})$, will satisfy

$$\frac{1}{k_1} F_1(\underline{r}) = \frac{1}{1+\beta} \int F_1(\underline{r}') P(\underline{r}',\underline{r}) d\underline{r}' + \frac{\beta}{1+\beta} \int F_1(\underline{r}'') H_1(\underline{r}'',\underline{r}) d\underline{r}'' .$$

By the usual technique of perturbation theory it can be shown that

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$$\frac{1}{k} - \frac{1}{k_1} = \frac{\beta}{1+\beta} \frac{\iint F_1(\underline{r}'') \left\{ H(\underline{r}'',\underline{r}) - H_1(\underline{r}'',\underline{r}) \right\} F^*(\underline{r})\, d\underline{r}''d\underline{r}}{\int F_1(\underline{r})\, F^*(\underline{r})\, d\underline{r}}$$

where $F^*(\underline{r})$ is the flux adjoint to $F(\underline{r})$. In terms of $F_d(\underline{r})$, the number of delayed neutrons produced at $\underline{r}$ per c.c. per second, the difference $\Delta k = k_1 - k$ is

$$\frac{\Delta k}{k} = k_1 \frac{\iint \left[F_d(\underline{r}) - F_{d_1}(\underline{r}) \right] P(\underline{r},\underline{r}')\, F^*(\underline{r}')\, d\underline{r}\, d\underline{r}'}{\int F^*(\underline{r})\, F_1(\underline{r})\, d\underline{r}}. \qquad (10)$$

For a completely uniformized delayed neutron distribution in a bare reactor, (10) leads to a reactivity loss of about $\frac{2}{5}\beta$. The turbulent fluctuations arise because the delayed neutron emission pattern fluctuates in time, and this, according to (10) will make Δk change randomly.

The turbulent fluctuations due to the delayed neutrons are evidently of second order importance since the whole delayed neutron contribution, β, is only 0.8% in U^{235}. However, in a very high powered chain reactor in which there is a very strong temperature gradient and a large temperature coefficient, a certain temperature profile corresponds to criticality; random fluctuations from this temperature profile will in general imply fluctuations in reactivity and in power. An estimate of these fluctuations by F. H. Murray is that for a temperature coefficient of $10^{-3}/^{\circ}C$ and a power density of 20 kw/ℓ the root mean square turbulent fluctuation would be 20%. The size of these fluctuations increases with the average temperature gradient - i.e., the power output. Thus the turbulent fluctuations could place an absolute upper limit to the possible power which might be derived from a circulating fuel chain reactor.

In the case of an unstable slurry reactor, the turbulent fluctuation could in principle be even greater. Here the fission density itself is subject to

local variations, since the concentration of fuel is likely to change from place to place as the swirling fluid creates its eddies. Such fluctuations in fission density again represent deviations from the critical fission distribution, but this time the fluctuation is in the whole fission pattern, not just the pattern of delayed neutron precursors. As compared to the delayed neutron induced fluctuations, the direct density fluctuations could be $\frac{1 - \beta}{\beta} \sim 100$ times as great. Critical experiments with slurries which are planned at ORNL should give us some feeling for how important these fluctuations actually are. While the impression is that the intrinsic fluctuations due to turbulence will not prove troublesome, a complete theoretical analysis is still awaited on this point.

The last problem which I should like to remark on is the old one of the relation between microscopic and macroscopic pile theory in a heterogeneous reactor, i.e., the computation of the diffusion area, L^2, in a multiplying lattice.

This problem is almost the oldest one in reactor theory. It was considered by Teller in a paper[6] on the exponential experiment, and was then treated by Plass for a cubic lattice. Plass's results, expressed as numerical comparison between the usual relation $L^2 = (1 - f) L_1^2$ and exact L^2 are listed in the accompanying table. However, Plass's results, being numerical, could never be put in a very neat form, and therefore the problem has never been considered entirely settled.

To restate the question, we consider an all-thermal neutron, heterogeneous reactor. From the usual microscopic, or lattice, theory, we compute the thermal utilization, f, and the diffusion length, $L^2 = (1 - f)L_1^2$, L_1 being the moderation diffusion length. Both L and f are intensive quantities and are therefore computed for an infinite system of structure similar to that of the finite system.

Now to compute the critical mass - which is an extensive quantity - it is necessary to relate f, the number of neutrons absorbed in U per neutron created in the infinite system, to f_{eff}, the number of neutrons absorbed in U per neutron created in the finite system. The usual relation between f_{eff} and f - which implies the relation between macroscopic and microscopic reactor theory - is

$$f_{eff} = \frac{f}{1 + L^2B^2} = \frac{f}{1 + (1 - f)\, L_1{}^2B^2} \tag{11}$$

i.e.,

$$k_{eff} = \eta\, f_{eff} = \frac{k}{1 + (1 - f)\, L_1{}^2B^2} .$$

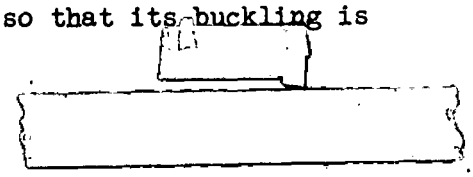
It is not entirely convincing to assume a relation like (11) and to make an estimate of L^2 in a non-multiplying heterogeneous system of similar absorbing properties. Rather it is necessary to compute f_{eff}, ab initio, and to see whether the resulting equation for f_{eff} can be written in the form

$$f_{eff} \approx \frac{f}{1 + \bar{L}^2B^2} , \tag{12}$$

and <u>then</u> express

$$\bar{L}^2 = (1 - f)\, L_1{}^2 + \text{corrections.} \tag{13}$$

There is one not quite trivial case in which the problem can be solved exactly, and which I would like to mention since it has never been published. Consider an infinitely long all-thermal slab lattice reactor whose transverse dimensions are H_1 and H_2, so that its buckling is



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$$B^2 = \frac{\pi^2}{H_1^2} + \frac{\pi^2}{H_2^2} .$$

We seek to calculate f_{eff}, under the assumption that the slow neutrons are produced in the moderator like

$$q = Q \cos \frac{\pi y}{H_1} \cos \frac{\pi z}{H_2} .$$

From its definition,

$$f_{eff} = \frac{\int \sigma_{a_u} n(\underline{r})\, d\underline{r}}{\int_{mod} q(\underline{r})\, d\underline{r}} ,$$

where $n(\underline{r})$ is the neutron density at $\underline{r}$, σ_{a_o} is the absorption cross-section in uranium, and the integrals are over the uranium and moderator regions respectively. The neutron density is computed, as usual, from the pair of equations

$$\frac{\sigma_{a_1}}{\kappa_1^2} \Delta n_1 - \sigma_{a_1} n_1 + q = 0 \quad \text{in moderator}$$

$$\frac{\sigma_{a_o}}{\kappa_o^2} \Delta n_o - \sigma_{a_o} n_o = 0 \quad \text{in uranium}$$

together with the usual boundary conditions of continuity of flux and current.

The result of this computation is

$$\frac{1}{f_{eff}} = \left(1 + \frac{B^2}{\kappa_1^2}\right) \frac{\sigma_{a_1} x_1}{\sigma_{a_o} x_o} \sqrt{(\kappa_o^2 + B^2)}\; x_o \coth \sqrt{\kappa_o^2 + B^2}\; x_o$$

$$+ \left(1 + \frac{B^2}{\kappa_o^2}\right) \sqrt{\kappa_1^2 + B_1^2}\; x_1 \coth \sqrt{\kappa_1^2 + B_1^2}\; x_1$$

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x_1 and x_o being the half thickness of the moderator and uranium slabs. The infinite reciprocal thermal utilization is, as usual,

$$\frac{1}{f} = \frac{\sigma_{a_1} x_1}{\sigma_{a_o} x_o} \kappa_o x_o \coth \kappa_o x_o + \kappa_1 x_1 \coth \kappa_1 x_1 .$$

From (12), the correct value for the heterogeneous diffusion length is

$$\bar{L}^2 = \frac{1}{B^2} (1 - f/f_{eff})$$

which becomes, if the lattice parameters are small compared to the dimensions H_1 and H_2

$$\bar{L}^2 \approx (1 - f) L_1^2 + f L_o^2 - f \frac{\kappa_1^2 x_1^2}{45} (L_1^2 B^2 + 2) x_1^2 .$$

The first term, $(1 - f) L_1^2$, is always the most important, and therefore in this very simple case, the usual formula is justified. It would, it seems to me, still be worthwhile to reinvestigate the problem with a view to obtaining second order corrections in more complicated cases.

Is Reactor Theory Worthwhile?

My remaining remarks are really non-technical. They concern the question of worthwhileness of computing critical masses - and by implication, the worthwhileness of reactor theory itself.

It is sometimes said that accurate calculation of a critical mass is really not so important, since one can always do a critical experiment. While such talk has been and still is surprisingly prevalent, I don't think it makes much sense. Granted that the exact value of a critical mass may not be crucially

important - except, say, in the design of chemical plants - what is extremely important is a knowledge of the maximum uncertainty in a computed mass. One cannot escape the feeling that this tendency to deprecate the importance of being able to compute masses reliably is mainly a rationalization of our lack of confidence in our ability to compute masses reliably.

Whether we really should feel so defensive in this matter is a little obscure. The examples which I have given of the many successes of multi-group theory, or of the rather good agreement between calculated and computed MTR critical masses should increase our confidence. Nevertheless, I sometimes am depressed at the very large number of experimental critical masses which are now known but for which there has been insufficient theoretical analysis, and which therefore add little to the estimate of how reliable our calculations are. The old compilation of Masket[8] list some 52 separate critical experiments or exponential experiments. Since this compilation there must have been several hundred more assemblies. Unfortunately too many of these experimental data have been amassed without thorough-going theoretical analysis: the result is that we are still tentative where we should be definite. What is badly needed is a thorough review of <u>all</u> critical masses. Our motto should be no measured critical mass without a calculation alongside it.

To some extent our lack of confidence in our computations is, I fear, a reflection of a sort of odd defensiveness which has grown up in this country in respect to the general subject of reactor theory. Reactor theory in many ways is a demanding discipline which requires physical insight of high order. Yet, I suppose because it has such strongly important practical overtones on the one hand, and because its development is outside of the university community on the other, papers on the theory almost never appear in the regular physical literature.

In this respect we are less sensible than our foreign competitors. Several papers on some aspect of theory have appeared each year in almost all of the standard British, Canadian, French, Swiss, and Italian (though not Russian) journals. No paper on reactor theory has appeared in The Physical Review in the last 3 years, in spite of the fact that all reactor theory has been declassifiable during this period.

That there are problems of considerable physical validity in the theory I think is evident to any who care to study the situation. That we who are professionally engaged in the enterprise have a strong responsibility to establish a firm scholarly tradition in the field is also self-evident. Insofar as we have allowed reactor engineering to engulf all our effort, we reactor physicists have been guilty of short-sightedness for which, in the long run, we will not be thanked by those future reactor engineers who find the theory insufficient. It is on this plea then that I close: that we insist on maintaining both a deeply scientific and a deeply critical attitude toward the development of reactor physics; that we actively seek and find enough of interest in it to maintain the discipline of and by itself, as a valid scientific activity; and that we rest content that such scholarly enterprise will, in the long run, prove doubly worthwhile for being the foundation of what we all hope will be the eminently useful technology of nuclear power.

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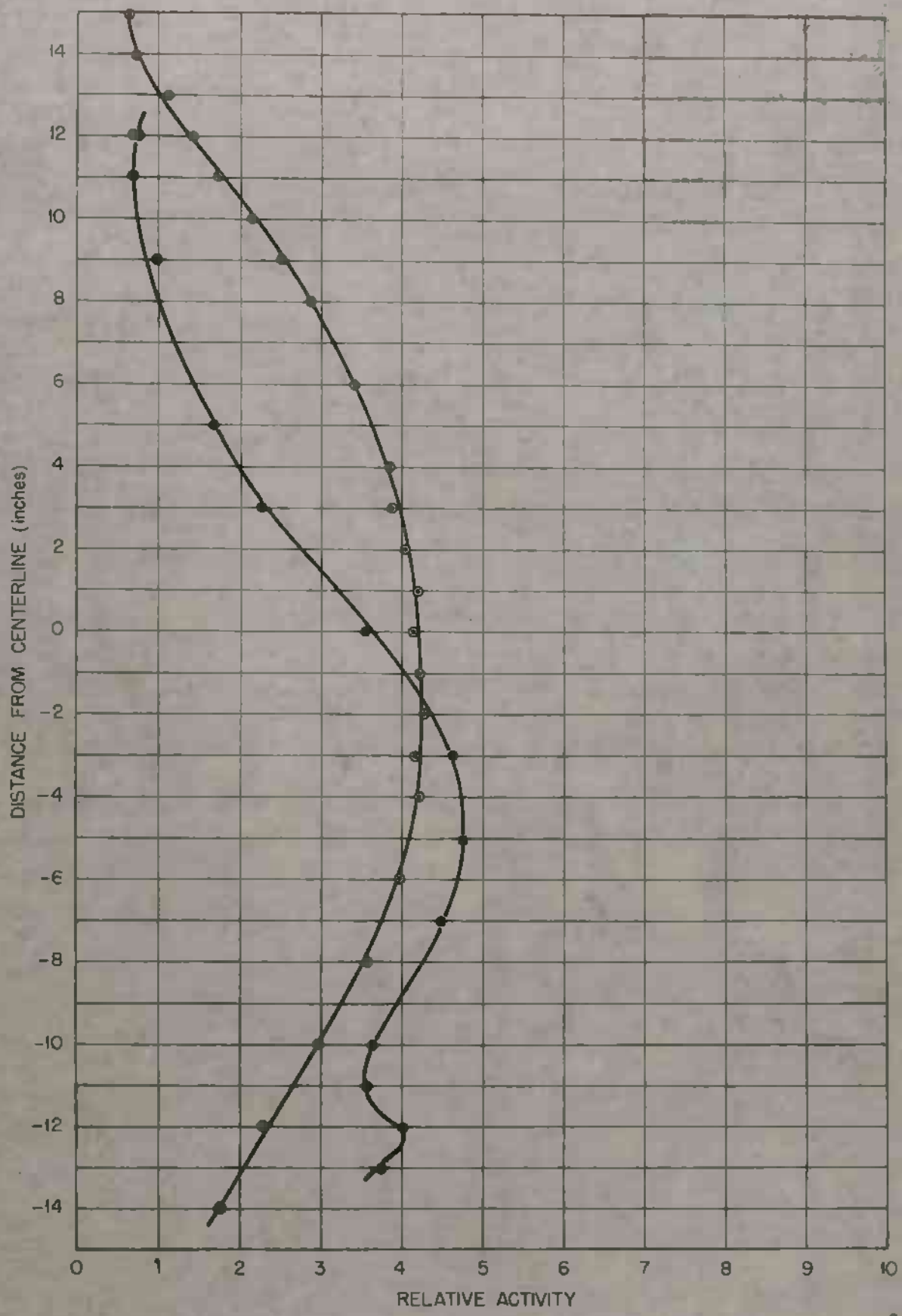

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Table A

PLASS'S RESULTS

r_o = 3 cm., graphite lattice

f	$(1 - f)L_1^2$ (cm^2)	L^2 exact (cm^2)
.9366	146.5 cm^2	139.1 cm^2
.8989	233.7	234.9
.8522	341.6	344.4
.7983	466.2	455.9
.7390	603.3	558.3

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