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STRUCTURE IN THE PROTON AND THE NEUTRON

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INTRODUCTION

In this paper we attempt to summarize the electron-scattering work (1954-1958), which has led to an elucidation of the electromagnetic structures of the proton and neutron. Earlier summaries by the present author,[1,2] and a more recent review by the author and his colleagues,[3] have kept the findings up-to-date. In this article the most recent experimental results will be added.

RESULTS AND INTERPRETATION

In investigations of both the proton[4,5] and the neutron,[6,7] large deviations from scattering by point particles were found for incident electrons in the energy range 200-600 Mev. Such deviations could be attributed to the effects of finite size, since the observed scattering was always less than point scattering at large angles. A reduction in scattering at large angles results from interference effects between the various parts of a structure of finite size. Such effects are well known in other domains of physics, such as in x-ray diffraction and low-energy electron diffraction studies of molecules and atoms. However, the extension of size measurement to such small distances as 0.8×10^{-13} cm, the root-mean-square (rms) radius of a proton, leads one to wonder whether the usual concepts of space and time apply to this small realm. We are thus obliged to ask the question: Does quantum electrodynamics and do our usual ideas of space and time apply to these observations? These are very fundamental questions to which we can give no certain answers at the present time. We shall see that the consistency of our theoretical interpretation of the experimental facts certainly favors the idea that quantum electrodynamics, etc. still holds at these small distances. But we must be cautious, and an unexpected behavior, which will be mentioned below, pointing to differences between the proton and neutron <u>charge</u> distributions, may possibly foreshadow an incipient breakdown of the usual laws.

Since considerable progress can be made with the presently known laws, and since, as stated above, a consistent interpretation binds all the present experimental facts together, we have adopted the point of view that the observations yield evidence of finite size. This point of view

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will be maintained in the present paper. A discussion of other evidence leading to the same conclusion will be found in a recent paper by S. D. Drell,[8] and an earlier discussion of the same problem appeared some time ago[9].

The Proton: The experimental results and their theoretical interpretation will first be treated for the proton. In this case a reliable theory of the electron-scattering process has been worked out by M. Rosenbluth.[10] This theory extends the usual Mott scattering formula to the case where both a charge and a magnetic moment are present in the point particle. The magnetic moment may be allowed to have an anomalous part, which we call the Pauli moment. For the proton this amounts to 1.79 nuclear magnetons. The residual, or intrinsic part of the magnetic moment that accompanies a charge with spin one-half, will be called the Dirac moment. The Dirac moment amounts to 1.00 nuclear magneton. When we permit the proton's charge and magnetic moment to be spread out in space, a phenomenological description of these properties can be made in terms of "form factors."[9-11] The form factors can be introduced in a unique way into the scattering equation and give angular distribution curves which are related to the simpler point particle equation of Rosenbluth.

The differential electron-scattering cross section may be shown to be:

$$\frac{d\sigma}{d\Omega} = \left(\frac{e^2}{2E_o}\right)^2 \frac{\cos^2 \theta/2}{\sin^4 \theta/2} \frac{1}{1 + \frac{2E_o}{Mc^2} \sin^2 \theta/2}$$

$$\cdot \left\{ F_1^{\,2} + \frac{\hbar^2 q^2}{4M^2c^2} \left[2(F_1 + KF_2)^2 \tan^2\theta/2 + K^2 F_2^{\,2} \right] \right\} \qquad (1)$$

where

$$q = \frac{\frac{2E_o}{c\hbar} \sin \theta/2}{\left[1 + \frac{2E_o}{Mc^2} \sin^2 \theta/2\right]^{1/2}}$$

where F_1 and F_2 are the Dirac and Pauli form factors, respectively, and each is a function of the magnitude of the four-vector momentum-energy-transfer (q). Other symbols have their usual meanings.[2] When F_1 and F_2 are each allowed to have the value unity, the above formula (1) reduces to the Rosenbluth scattering formula for a point particle. Figure 1 illustrates the Rosenbluth cross section at 400 Mev and several other curves, corresponding to Eq. (1) for several choices of rms radii for an exponential distribution of charge density and magnetic moment density inside a proton. The figure thus shows how the cross section is severely reduced at large angles, while the small angle results are not greatly

affected. A density distribution as smooth and monatonic as an exponential does not show diffraction features.[1]

Now Fig. 2 shows the first data which showed finite size deviations from point scattering in the proton. The experimental points are taken from reference 4. Confirmation of these early results was obtained by Chambers and Hofstadter,[12] who showed that the model proposed in reference 4 would fit all the experiments in the energy range between 200 and 550 Mev. A representative scattering curve[12] taken at 550 Mev is shown in Fig. 3. The deviations between point scattering and the observed scattering are nearly as great as a factor of ten at the large angles. Figure 3 shows a solid line drawn below the point particle curve. This solid curve, which passes through the experimental points quite well, is actually a theoretical curve based on an exponential model of the proton whose rms radius is 0.8×10^{-13} cm. This theoretical curve corresponds to a choice of an exponential model of the density distribution for both the charge and the moment. This means that F_1 and F_2 have been taken to be equal to each other at the same value of the momentum transfer. Such an assumption is by no means a necessary one, but this choice yields a remarkable fit to all the data of Chambers and Hofstadter, as well as to all the earlier data. The above authors could not exclude certain simple models, such as a gaussian or a hollow exponential distribution. However, the other acceptable choices led to rms radii very close to 0.8×10^{-13} cm and in general to $F_1 = F_2$.

The various acceptable models yield nearly the same distributions of charge and magnetic moment density in the middle regions of the proton's space extension. This fact is shown by the crossing-over of three acceptable distributions in Fig. 4. The ordinate of these curves is $4\pi r^2 \rho$ where ρ is the charge density within the proton. The ordinate is therefore proportional to the amount of charge in a shell at radius r. From 0.6 to 2.0×10^{-13} cm, the gaussian, exponential, and hollow exponential models give not very different distributions. On the other hand, the Yukawa model in the figure does not fit the data for any choice of size parameters.

Bumiller and Hofstadter[13] set themselves the problems of determining which among the three acceptable distributions could fit the experimental data at higher energies. They also desired to know how closely $F_1 = F_2$ without reference to a choice of model. Already, in reference 12, evidence was presented that at $q^2 \cong 4 \times 10^{26}$ cm^{-2}, the ratio F_1/F_2 was equal to 1.1 with a possible error as large as $\pm$ 20 per cent. The above authors[13] found that at $q^2 = 9.3 \times 10^{26}$ cm^{-2}, $F_1/F_2 = 1.04 \pm 0.2$. Thus together with the earlier result we are justified in assuming that F_1 and F_2 are at least approximately equal to each other at all q values. With this assumption, attempts were made to fit the new experimental data at energies up to 650 Mev. The results[13] are shown in Fig. 5, where the theoretical (form factor)^2 is also shown as a solid line for the exponential model of a proton. The data are thus consistent with the exponential model used in the earlier interpretations. The gaussian and the hollow exponential models which were suitable up to 550 Mev could now be eliminated by the new data. Furthermore, the ratio of the experimental scattering at 75° to that at 135° was found to lie between 9.0 and 10.0 at energies between 200 and 650 Mev.

Neither a gaussian nor a hollow exponential gives this behavior, while the exponential model does. Thus much evidence now favors the latter model or its equivalent phenomenological form factors.

The above facts are still not sufficient to fix a unique choice of a model. Again it may be suspected that any model which agrees with the exponential model in Fig. 4 will also fit the experimental facts. This is true, and in the same figure we show a model proposed by Clementel and Villi[14] which contains a negative delta function at the origin containing 20 per cent of the protonic charge. The exterior of the proton then contains 120 per cent of a protonic charge, distributed as a Yukawa type function with a 1/r behavior at small distances. This Yukawa distribution is different from the lowermost curve of the figure. The Clementel-Villi distribution is essentially indistinguishable from the exponential distribution between radii 0.5 and 2.0×10^{-13} cm and has the same rms radius as the exponential model. Differences between the C-V model and the exponential model are more noticeable at radii less than 0.5×10^{-13} cm, but the experiments have less sensitivity in this region. It appears likely that a monatonic type of singularity in the proton is not acceptable in any case.

<u>The Neutron</u>: We turn now to a discussion of the neutron. Because free neutrons are not found in sufficient numbers to make an electron-scattering target, it will be anticipated that the measurements will be much more difficult than in the case of the proton. However, suprisingly good accuracy can be obtained by using the neutron in the deuteron. The deuteron is known to be a loosely bound structure in a nuclear physics sense, and the very large momenta transferred to a nucleon in an electron-scattering experiment can easily exceed the internal momenta of the deuteron by a factor of ten. Thus in an approximate way the neutron in the deuteron can be thought of as free. We may call it quasi-free.

The scattering in which a very large momentum is transferred to a nucleon will result in a disintegration of the deuteron and is therefore an example of incoherent scattering from the nucleons. This type of scattering must be distinguished from the coherent scattering from the nucleons which is known otherwise as the elastic scattering from the deuteron. The elastic scattering studies[15] yield information about the ground state of the deuteron and show how the ground state wave functions are affected by the finite extent of the proton and neutron. From the elastic deuteron studies, McIntyre and his colleagues[15] have confirmed the finite proton size. They have also demonstrated that the neutron appears to have little and possibly no spatial extent of its charge cloud. This is a verification of the results of the neutron-electron interaction studies at very low neutron energies.[16]

Returning to the incoherent scattering in the deuteron, it now becomes clear that the finite spatial extent of the neutron's Pauli, or magnetic moment, cloud can be studied at large angles and high energies without being complicated by possible uncertainties in the Dirac cloud. Early experiments were carrried out on the basis of these ideas showed immediately that the neutron's magnetic cloud could not be a

point, as its charge cloud seemed to be. The experiments were rather direct, because experimental comparison with the scattering from free protons could be made under exactly the same scattering conditions. A demonstration of this is afforded by the data of Fig. 6, in which a direct comparison can be made of the incoherent scattering from the deuteron (peak C) and the elastic scattering peak from the free proton(A). The free proton peak is much sharper than the deuteron peak, because the motion of the nucleons in the deuteron produces the same effect as a smeared-out incident energy of the electron beam. In other words, because of the momentum spread of the proton and neutron in the deuteron, the nucleons are sometimes moving with momentum components directed parallel and antiparallel to the direction of the incident electron beam. In the above figure the elastic scattering peak from the deuteron would be much too small to be observed at the large angle of the experiment (135°). A second deuteron peak is shown at D and results from scattering with production of pions. This peak will hot be discussed in this paper, although its study offers another method of comparing the spatial extent of the neutron and proton.

Now the data of Fig. 6 show that the combined scattering of neutron and proton, i.e., the area under the peak C, is not much greater than the area under the peak A. If the neutron were a point particle, as regards its magnetic moment, the area under peak C would be four times as large as the area under peak A. The evaluation of the areas under the two peaks is discussed in considerable detail in references 3 and 7. The theoretical decomposition of the area under peak C into independent additive contributions from the proton and neutron has also been shown to be valid to within approximately 10 per cent by the work of Jankus,[17] Blankenbecler and Drell.[18]

A second example showing the deuteron incoherent peak at a higher energy, but at a smaller angle, is provided in Fig. 7, and shows how the low-energy tail of the incoherent curve approaches small values when experimental electron-pion and other direct pion contributions have been measured and subtracted. Numerous deuteron curves of the type shown in Figs. 6 and 7 have been obtained and compared with corresponding proton curves. Thus absolute values for the differential cross sections for the neutron may be found. Such values then permit a comparison with the cross sections calculated from the Rosenbluth formula, equation 1, when F_1 is placed equal to zero, as explained above for the case of the neutron. Of course, the proper value of the magnetic moment of the neutron (1.91 n.m.) must be inserted for K in this formula. A comparison can also be made with the Rosenbluth formula for the neutron when the neutron's magnetic moment is considered to be a point. The latter comparison then enables the magnetic form factor of the neutron to be computed under a variety of different experimental conditions. This procedure has been followed by Yearian and Hofstadter,[6,7] who find the experimental points shown in Fig. 8. The method by which these points have been obtained is called, for convenience, the "area" method. Using the area method, the figure shows that the neutron's magnetic form factor is indistinguishable from that of the proton within the present experimental limits of error. The rms magnetic radius of the neutron is 0.8×10^{-13} cm for an exponential model. It is possible

that the points of Fig. 8 could be a little lower, because the area method may allow inclusion of small contributions from meson exchange effects. A discussion of these effects is given in references 6 and 7.

Data of the type shown in Figs. 6 and 7 may also be interpreted by means of a theory of Jankus,[17] who has calculated the shape of the incoherent peak of the deuteron. Jankus' theory has been slightly modified in references 6 and 7, so that better agreement with experiment has been realized. Using the modified Jankus theory, Yearian and Hofstadter, following a suggestion of Drell, have compared the peak value of the incoherent deuteron distribution with the elastic cross section of the free proton. This method, which may be called the "peak" method, gives results very similar to the area method. However, the peak method is much less subject to errors, due to meson exchange effects and to interactions in the final state. The method may be slightly sensitive to the choice of the neutron-proton potential, however. In any case, the agreement between the two methods is to be considered highly satisfactory. The peak method gives the results shown in Figs. 8 and 9. The rms radius of the magnetic cloud in the neutron is found to be about 0.8×10^{-13} cm for the 500-Mev data and about 0.9×10^{-13} cm for the 600-Mev data. Both sets of measurements indicate a slightly larger radius than the area method, but the differences are well within the experimental errors.

New data, so far unreported, have recently been obtained for the deuteron by Sobottka and the author[19] at smaller angles. These data also show the ratio of the incoherent scattering to the elastic scattering in the deuteron. A sample of the type of data is shown in Fig. 11. It is hoped that data taken at both the small and large angles will lead to further information about the charge form factor of the neutron. An analysis of the data of Fig. 11 and other new data is now in progress.

CONCLUSIONS

Successful theoretical treatments of the electromagnetic structure of the proton and neutron have not yet been proposed, as far as the present author knows. The experiments reported in this paper have therefore been interpreted in the framework of presently existing, and partially imperfect, theory. Thus it is necessary to wait until further experiments are carried out and further developments are made in theory before it will be possible to know the full meaning of the results reported. The apparent structure effects in the nucleons reported in this paper may someday turn out to be partly due to electrodynamic failures or to other changes in space-time concepts. That the whole effects are of this type appears doubtful to the author. Nevertheless, just on the chance that this is so, the experiments are being continued at still higher energies, and a study of electron-electron scattering has been initiated in the author's Stanford laboratory.

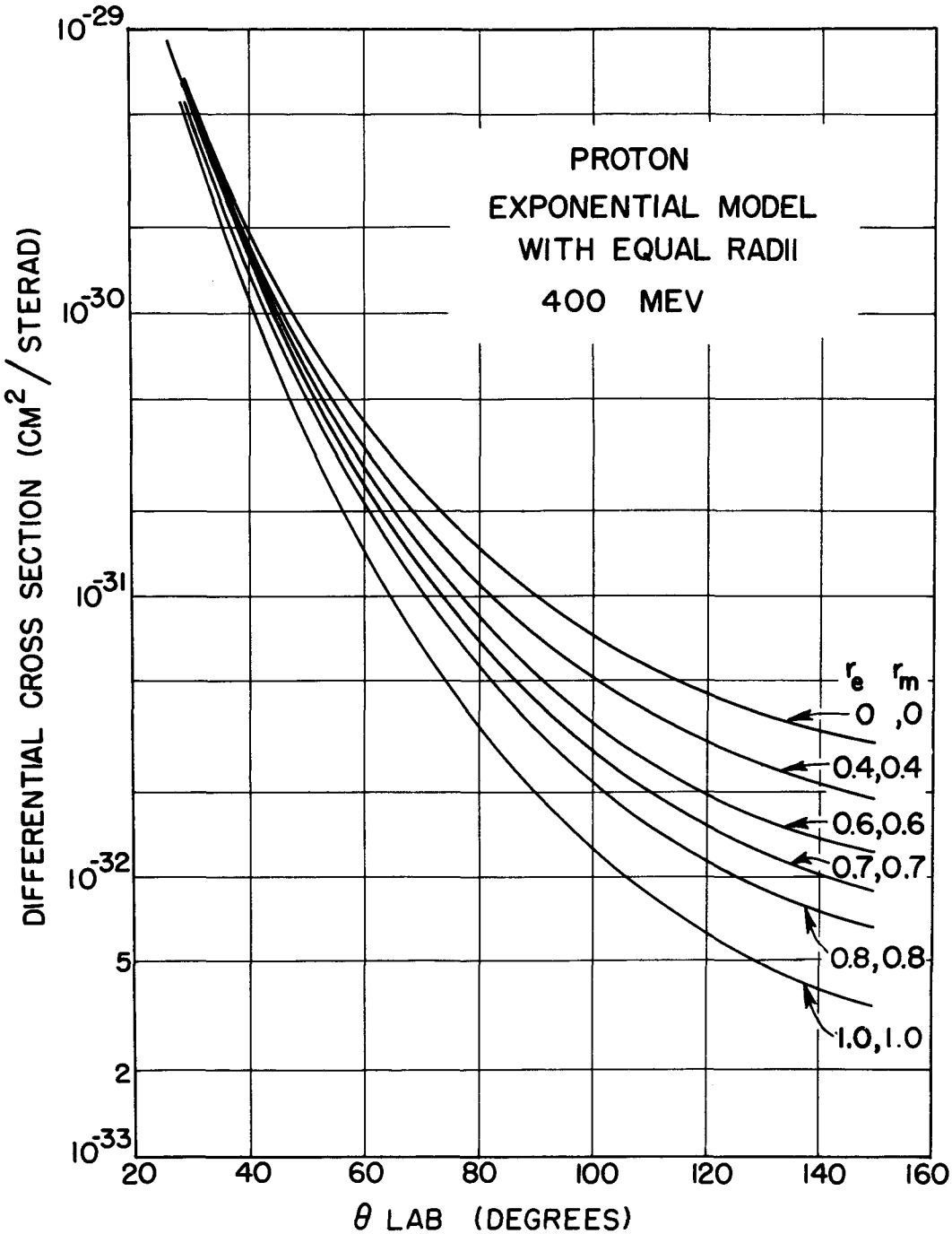

Fig. 1. The Rosenbluth point-proton curve ($r_e = r_m = 0$) is shown in this figure along with angular distributions at 400 Mev for various rms radii of the proton assuming an exponential model of charge density and magnetic moment density.

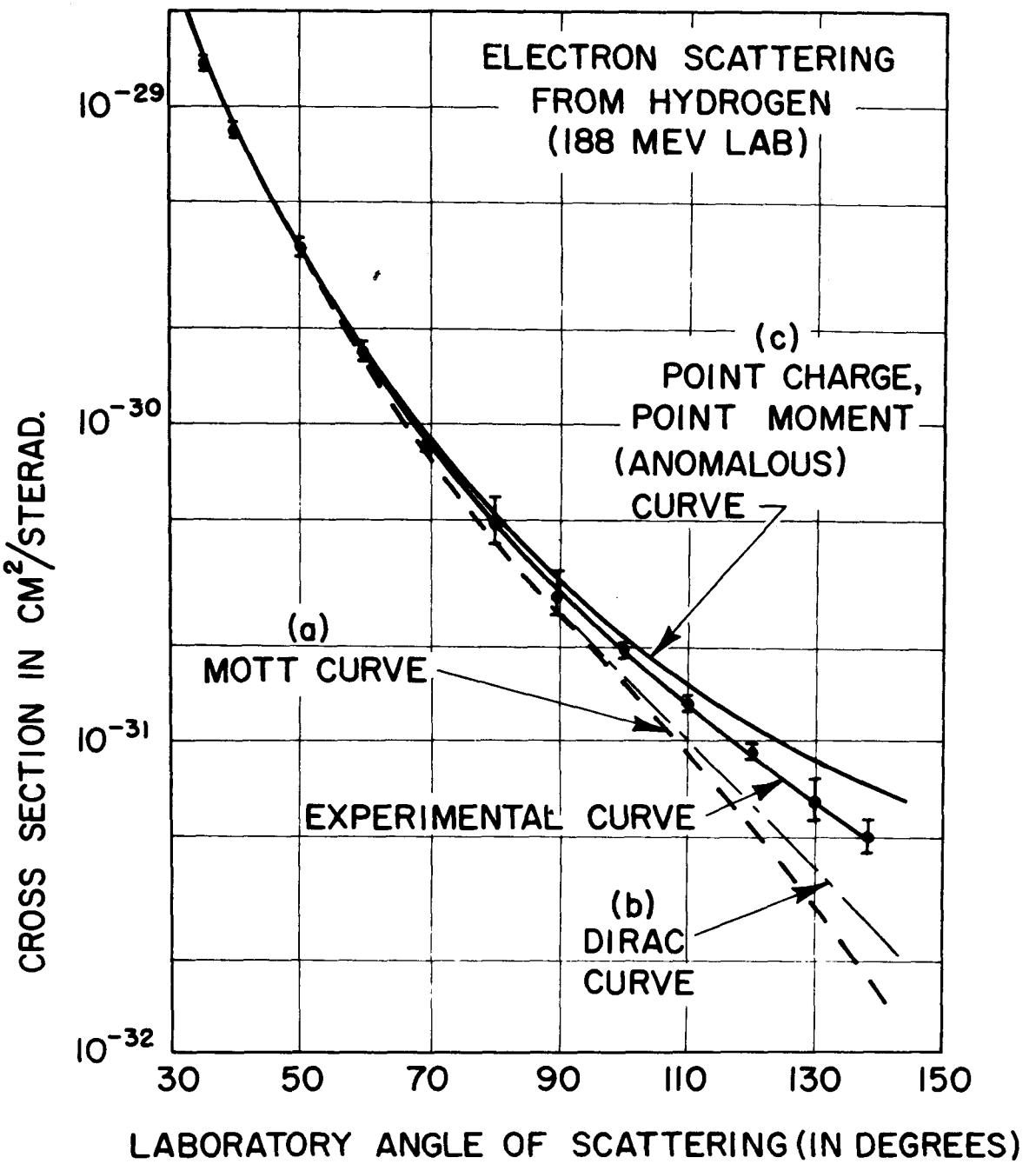

Fig. 2. The first evidence of finite size in the proton obtained with electrons of 188 Mev. The deviations below point-proton, Rosenbluth scattering (curve c) are ascribed to a finite size. Mott scattering (curve a) corresponds to $F_1 = 1$ and the second term of the bracket in Eq. (1) equal to zero. Dirac scattering (curve b) corresponds to $F_1 = 1$, $F_2 = 0$ in Eq. (1). The solid line through the experimental points refers to a theoretical curve with rms radii of 0.7×10^{-13} cm.

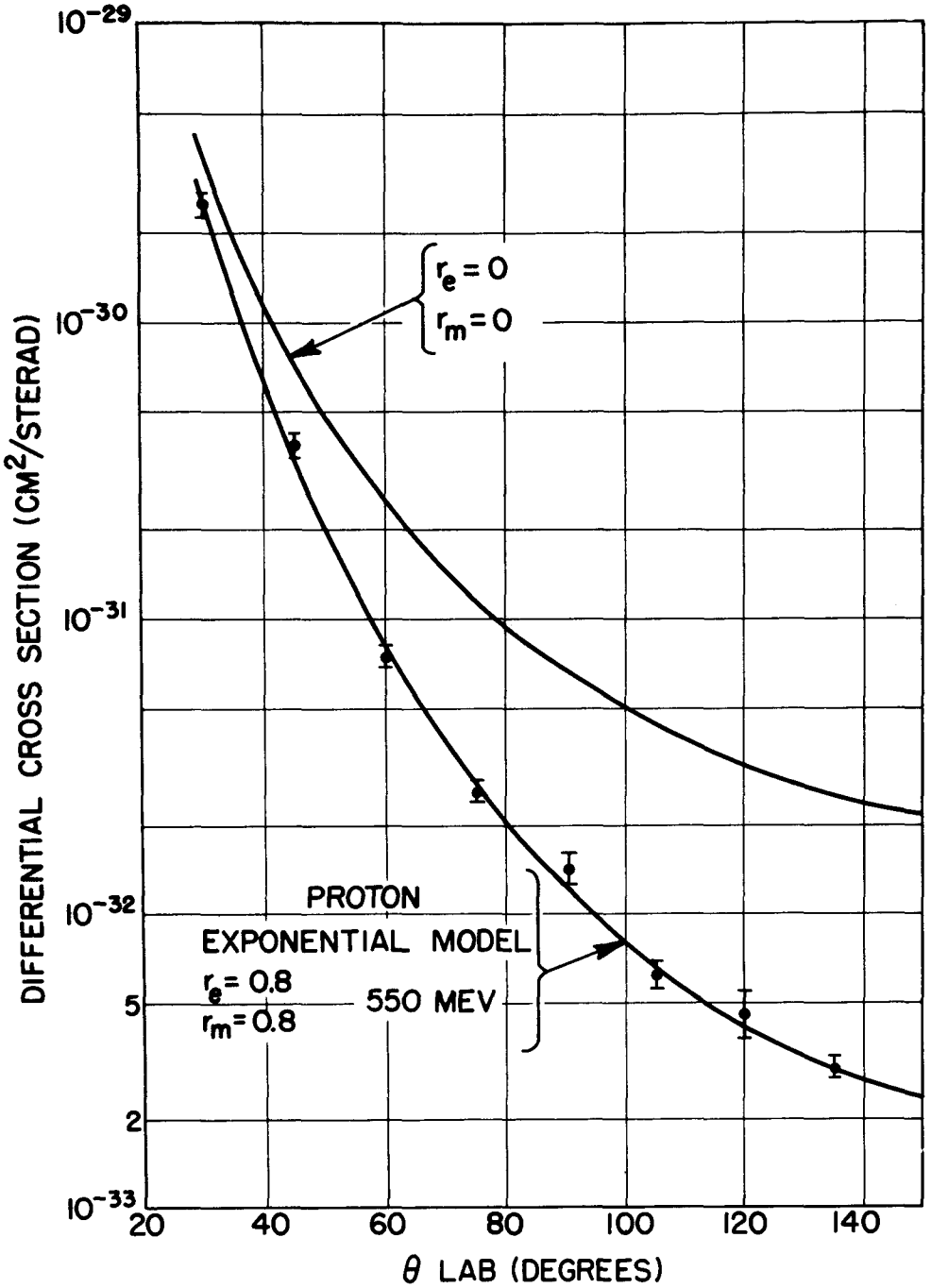

Fig. 3. Confirmation of the results in Fig. 2 at a higher energy (550 Mev).[12] The upper curve is the point-proton curve and the lower one corresponds to an exponential model with rms radii 0.8×10^{-13} cm. The deviations from point scattering approach a factor of ten at large angles.

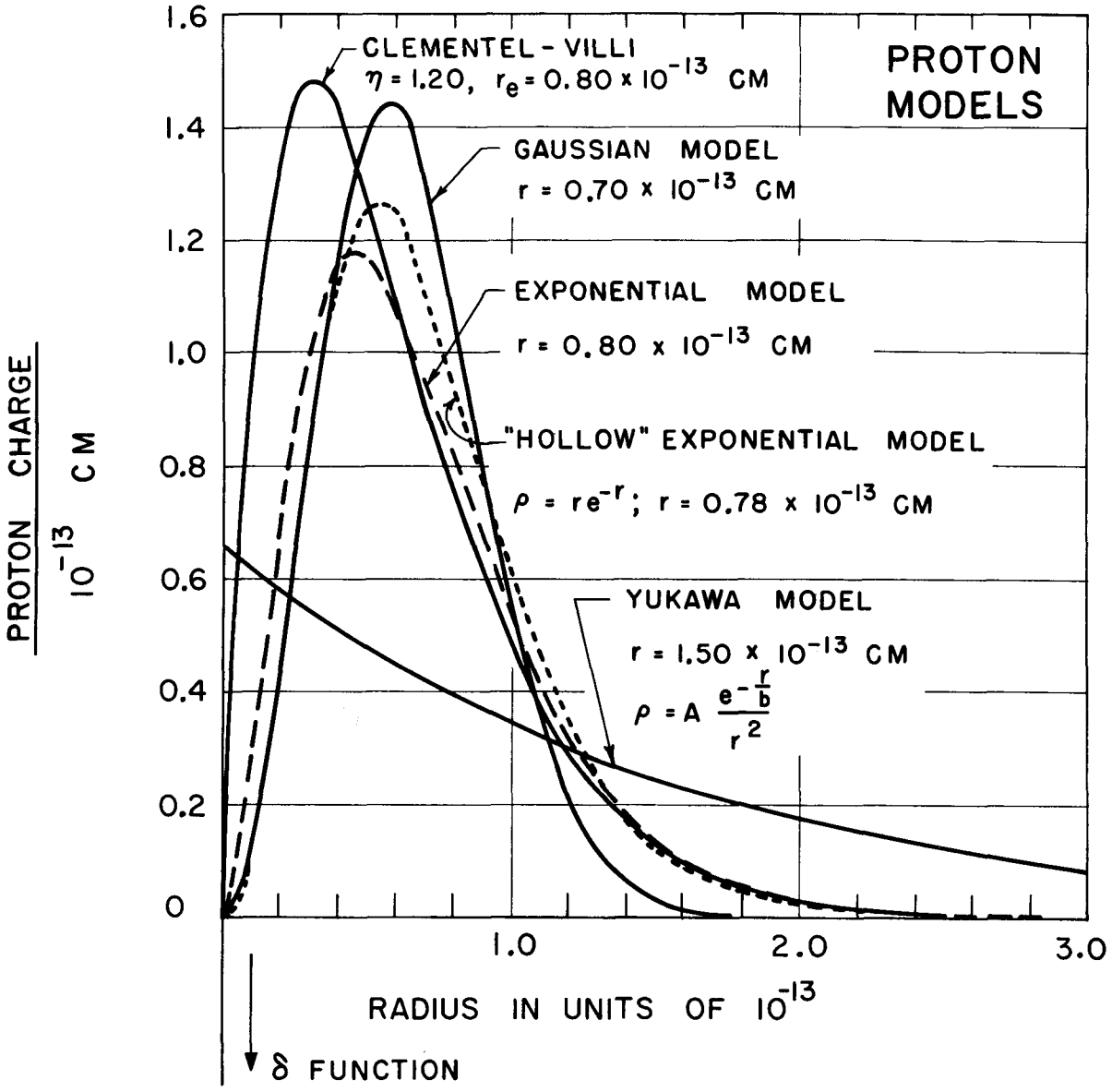

Fig. 4. This figure shows $4\pi r^2 \rho$ for acceptable and unacceptable models of the proton corresponding to data taken up to 550 Mev by Chambers and Hofstadter.[12] Three acceptable distributions (gaussian, exponential, and hollow exponential) cross over in the region of radii about 1.0×10^{-13} cm. The Yukawa model is unacceptable. The Clementel-Villi model is an acceptable model.

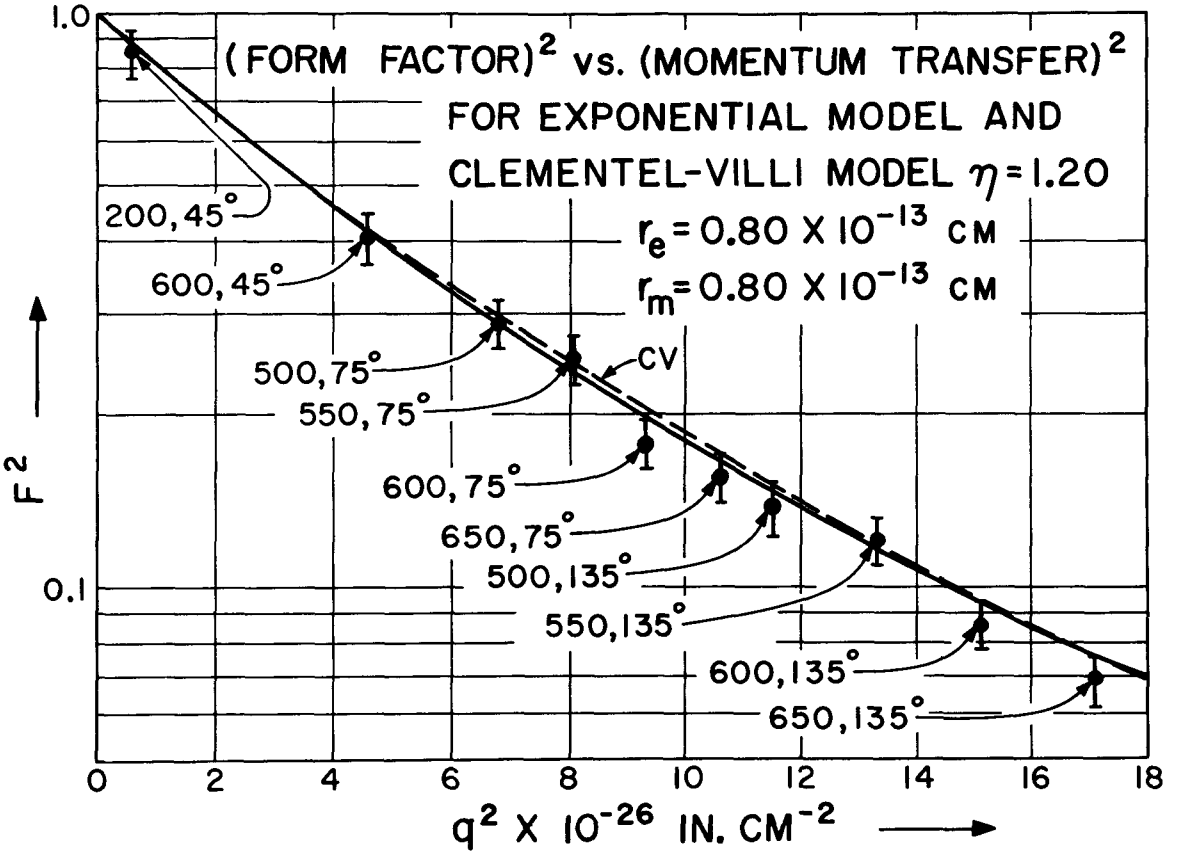

Fig. 5. The experimental (form factor)^2 for the proton at the conditions given in the figure. Solid and dashed lines are theoretical curves as indicated.

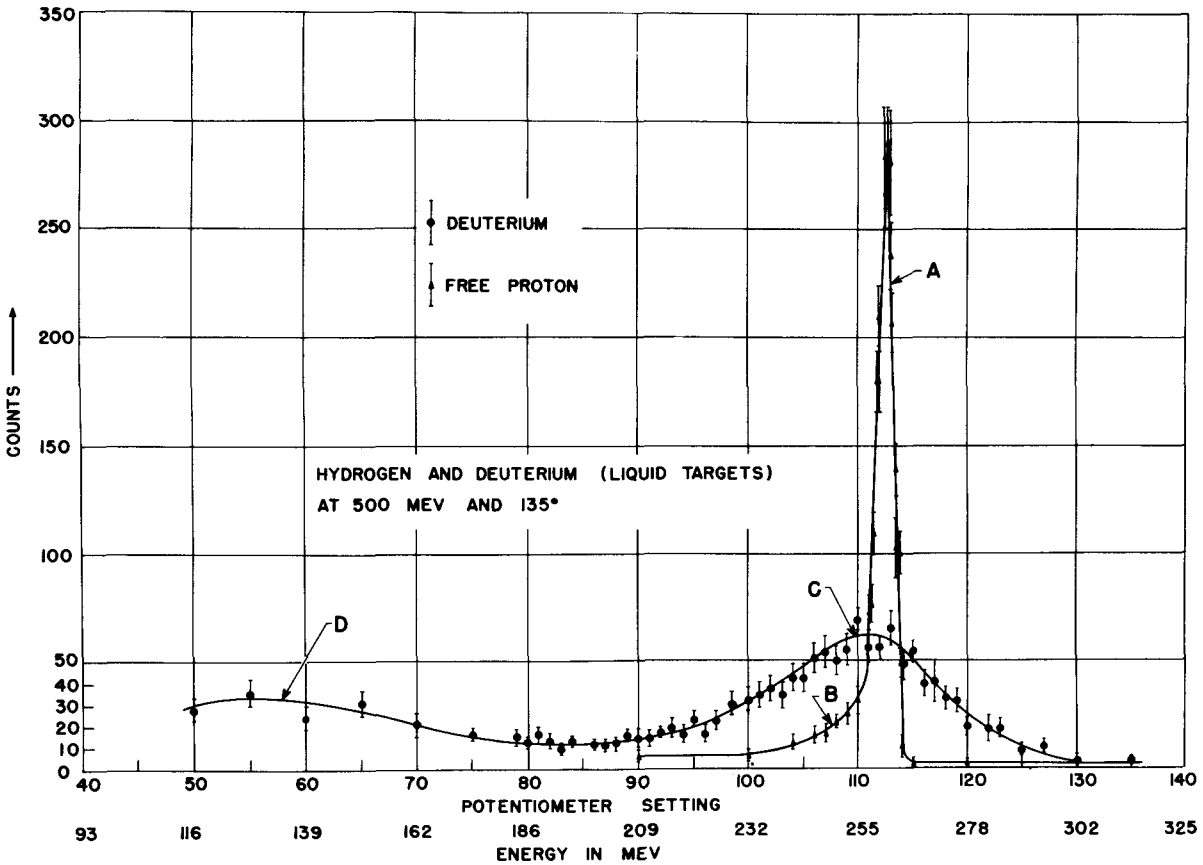

Fig. 6. Direct comparison of the incoherent scattering (500 Mev, 135°) from the deuteron (peak C) and the elastic scattering from the free proton (peak A). The deuteron curve should be multiplied by 0.87 to allow for the relative densities of liquid hydrogen and liquid deuterium under experimental conditions. The comparison of areas shows immediately that the neutron's magnetic moment is a spread-out structure.

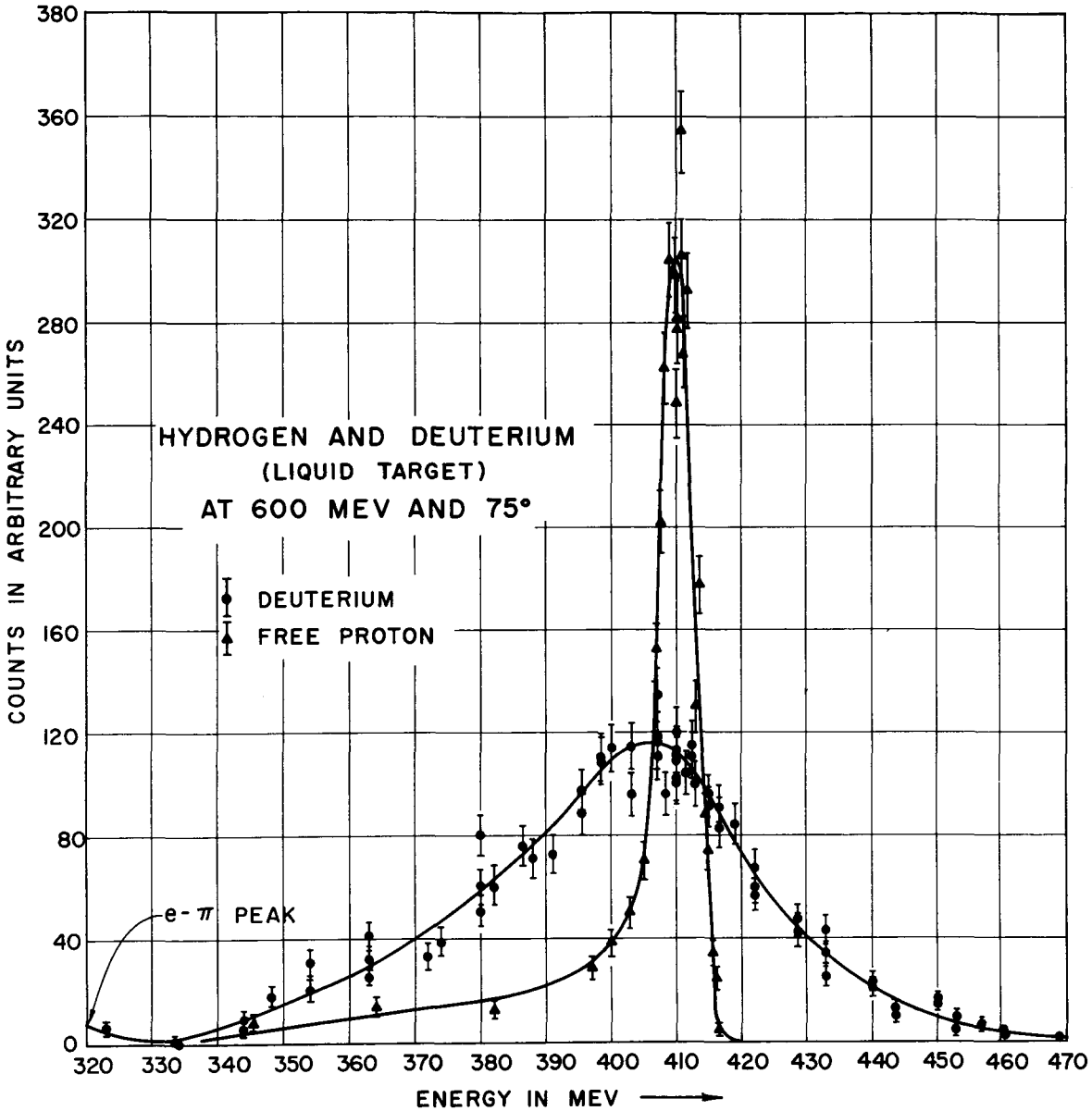

Fig. 7. Data similar to Fig. 6 except that the electron energy is higher (600 Mev) and the scattering angle smaller (75°). The conclusion is similar to that implied by Fig. 6. The deuterium curve should be multiplied by 0.87 for relative evaluation of area.

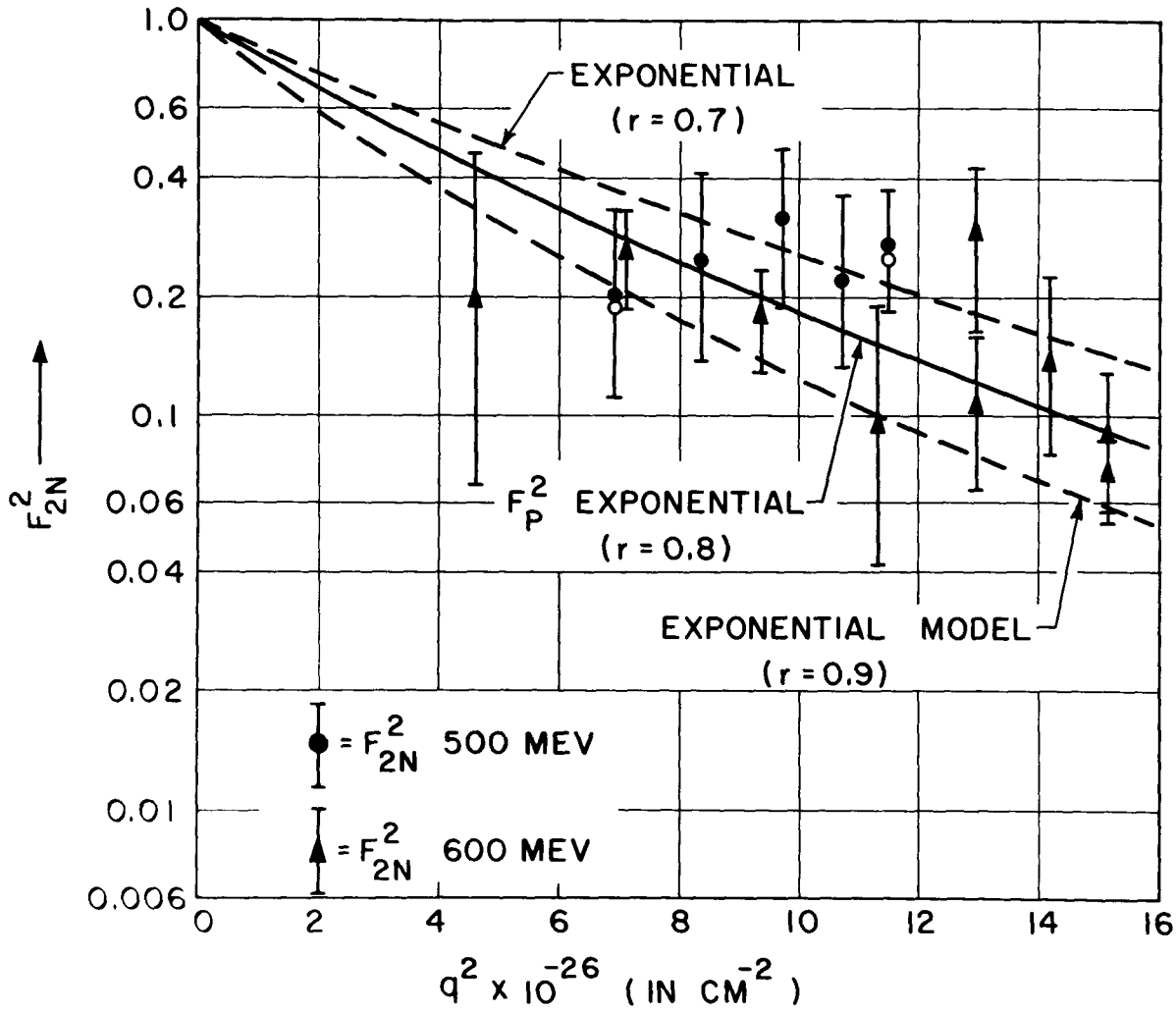

Fig. 8. The (form factors)^2 for the neutron obtained by the area method of Yearian and Hofstadter.[6,7] The rms magnetic radius of the neutron is close to 0.8×10^{-13} cm for an exponential model.

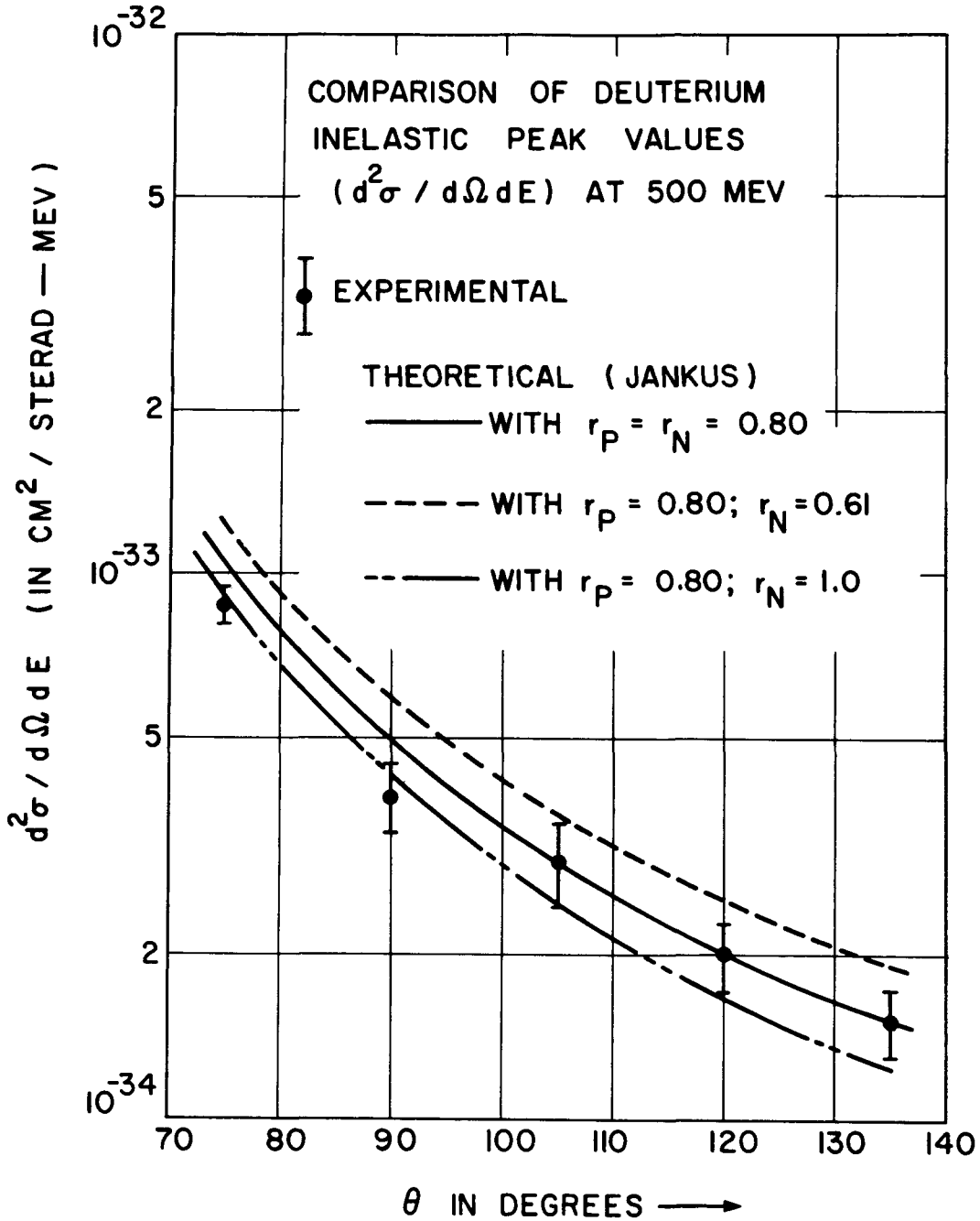

Fig. 9. The differential cross section of the incoherent deuteron scattering at the peak of the inelastic continuum at 500 Mev for various scattering angles. Theoretical curves obtained with the modified Jankus theory and form factors corresponding to an exponential model show that the experimental points[6,7] are fit by an rms radius near 0.8×10^{-13} cm.

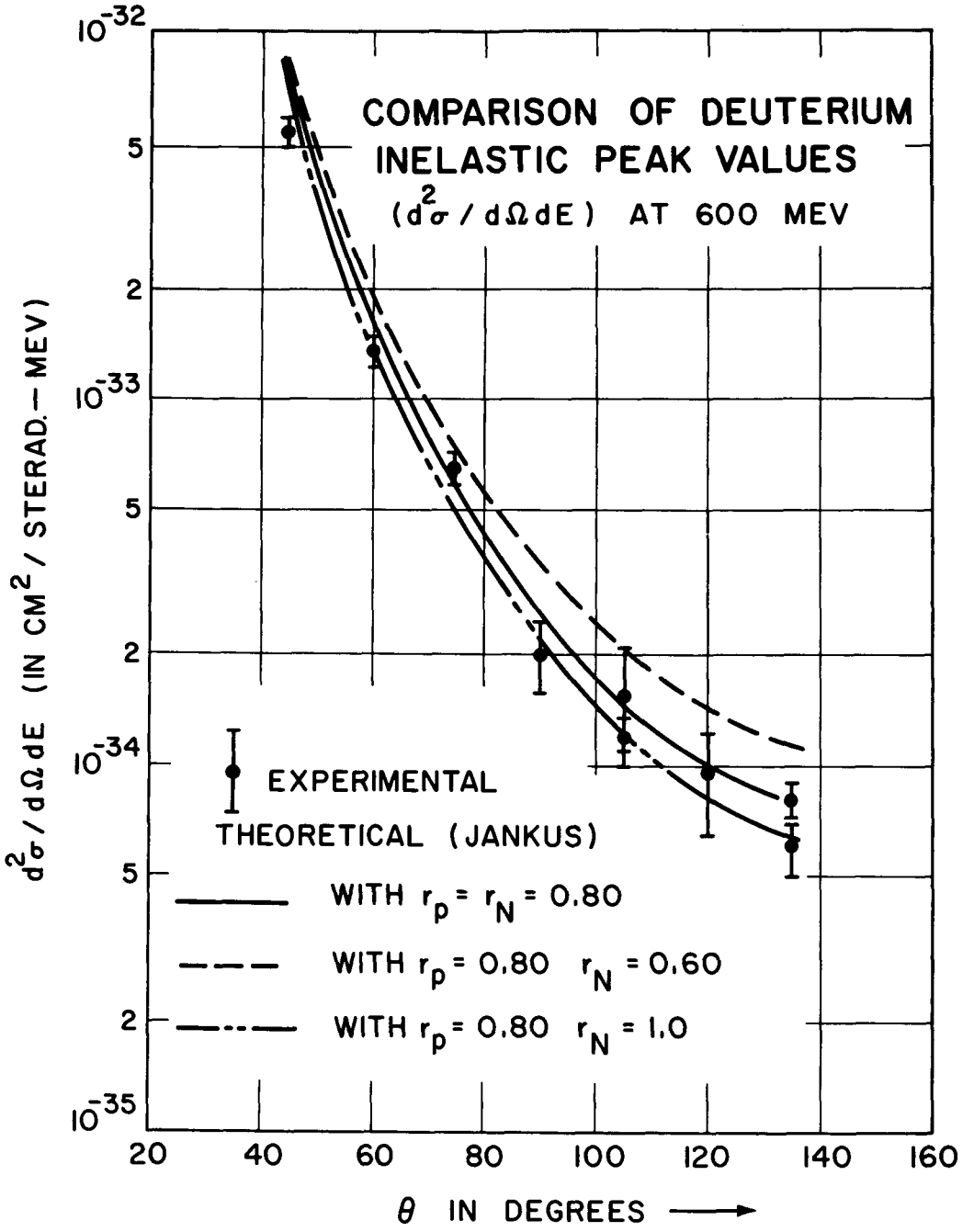

Fig. 10. "Peak" data similar to those of Fig. 9 except that the electron energy is 600 Mev. The experimental points fall between the theoretical curves corresponding to rms radii of 0.8×10^{-13} cm and 1.0×10^{-13} cm.

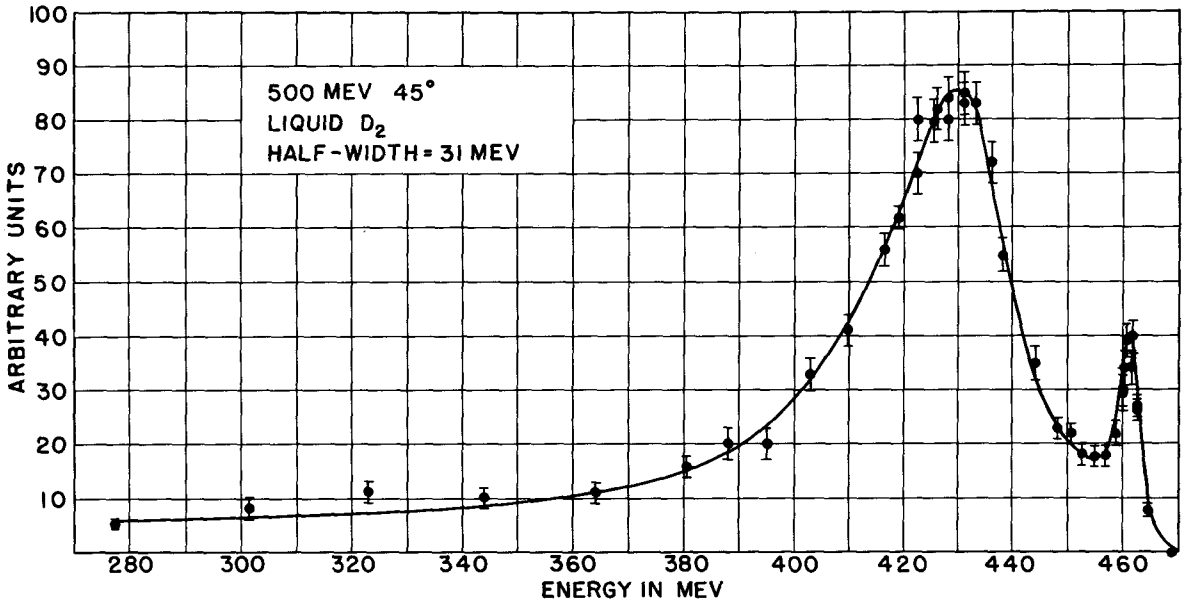

Fig. 11. Recent data of Sobottka and the author showing the elastic and inelastic deuteron peaks at 500 Mev, 45°, obtained with a liquid deuterium target.

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STRUCTURE IN THE PROTON AND THE NEUTRON

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ABSTRACT

This paper will attempt to survey the recent work on the structures of the proton and the neutron carried out by high-energy electron-scattering methods used at Stanford University over the years 1954-1958. Early work[1-3] established finite size effects in the proton and led to information about the charge and magnetic density distributions in the proton. The rms size was established to be close to $(0.77 \pm 0.10) \times 10^{-13}$ cm, and the density distributions of charge and anomalous magnetic moment were shown to be approximately of the same shape. The form factors could be described in terms of several alternative models given, for example, by an exponential, gaussian, hollow exponential, hollow gaussian, etc. distribution of densities. Many other shapes were excluded by the experimental data. Recent work by Bumiller and Hofstadter[4,5] now fixes one among these models that is appropriate to the proton and provides an extremely good fit at all angles between energies of 200 and 650 Mev. The new evidence clearly favors the exponential model with rms radius $(0.80 \pm 0.04) \times 10^{-13}$ cm. Recent studies[4,5] of the proton have attempted to answer the question of how closely similar the charge and magnetic form factors are. This work now shows that the distributions have the same sizes and shapes to within 5 per cent, and each distribution is given very closely by the exponential model described above with radius $(0.80 \pm 0.04) \times 10^{-13}$ cm. Early work on the inelastic continuum in the deuteron[6] established that the neutron's magnetic structure was extended and not a point. It was further shown that the neutron's size was approximately the same as that of the proton. This work has recently been extended by Yearian and Hofstadter,[5,7,8] to a determination of the variation of the neutron's magnetic form factor over the range where the proton's form factor is known. The new results show: (1) the neutron is not a point, (2) the neutron's magnetic structure has a size lying between the limits 0.61×10^{-13} cm and 0.80×10^{-13} cm. The first value $(0.61 \times 10^{-13}$ cm) is determined by examining the total deuteron electrodisintegration cross section at a given angle and incident energy and comparing this cross section with that of the free proton under the same conditions. The second value $(0.80 \times 10^{-13}$ cm) is found by examining the peak of the deuteron electrodisintegration cross section. Because of possible contributions to the total cross section by mesonic exchange effects, the second method is believed to be slightly more accurate. The neutron size is, therefore, approximately $(0.70 \pm 0.10) \times 10^{-13}$ cm and probably the larger size 0.80×10^{-13} cm is correct. Thus the magnetic clouds of the neutron and proton are closely the same. The bearing of these results on the validity of electrodynamics will be discussed. Because of the small radius implied by the neutron-electron experiments, there is an anomaly between the neutron and the proton. This is represented by the small charge radius for the neutron and the much larger radius of the proton. Additional information on the structure of the deuteron and on the

production of pions by electrons is also furnished by the same experiments and will be discussed at the meeting.

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