BNL 16828
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BROOKHAVEN NATIONAL LABORATORY
Associated Universities, Inc.
Upton, New York

ACCELERATOR DEPARTMENT
Informal Report

MASTER

HIGH ENERGY ELECTROMAGNETIC AND WEAK INTERACTION PROCESSES

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January 11, 1972

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INTERSECTING STORAGE ACCELERATOR NOTES

HIGH ENERGY ELECTROMAGNETIC AND WEAK INTERACTION PROCESSES

T. D. Lee
Columbia University, New York, N. Y.

January 11, 1972

This is based on a lecture given at Brookhaven National Laboratory, on August 5, 1971.

This research was supported in part by the U. S. Atomic Energy Commission.

In this talk, I shall first review some known features of the high energy electromagnetic and weak interaction processes, and then try to speculate on some particular aspects of their future possibilities. At present, both quantum electrodynamics and the usual (current X current) theory of the weak interaction have been remarkably successful. For a purely leptonic system, there is now nearly perfect agreement between theory and experiment, which extends over all presently known phenomena, such as $(g-2)$ of e and μ, Lamb shift, ρ-value in μ-decay, etc. For a system involving hadrons, the agreement between theory and experiment is helped, on the one hand, by theorists' inability to do exact calculations, which makes any serious disagreement difficult, and on the other hand, by the recent important discovery of the "scaling property".

1. Scaling Hypothesis

The scaling property is the consequence of the scaling hypothesis which was first suggested[1] by Bjorken and others. Here, we wish to state the scaling hypothesis in a form somewhat different from its original formulation, one that is perhaps more directly related to experimental results, and appears to be symmetric with respect to leptons and hadrons. For definiteness, we consider a purely leptonic or semi-leptonic reaction which can be either a second order electromagnetic process or a first order weak interaction process, e.g.,

$$e^+ + e^- \rightarrow \mu^+ + \mu^-$$

$$e^+ + e^- \rightarrow \text{hadrons}$$

$$\nu_\mu + n \rightarrow \mu^- + \text{hadrons, etc.}$$

Furthermore, for the semi-leptonic reaction we shall always sum over all final hadronic channels. Let $d\sigma$ be the appropriate differential cross-section, which can be, in general, written as

$$d\sigma = f(s, q^2, m_\ell, m_N) \times \begin{cases} \alpha^2 \\ G^2 \end{cases} , \qquad (1)$$

where the factors α^2 and G^2 are, respectively, the squares of the fine structure constant and of the Fermi constant, depending on whether the process is electromagnetic or weak,

$$s = (\text{center-of-mass energy})^2 ,$$

q^2 represents the various relevant (4-momentum transfer)^2, m_ℓ denotes the various lepton masses (m_e or m_μ), and m_N denotes the various hadron masses (which can be either the nucleon mass m_N itself, or the ρ and pion masses, etc.).

The scaling hypothesis states that (i) if s and $|q^2|$ are much larger than m_ℓ^2 then it is a good approximation to set $m_\ell = 0$ in the expression for $d\sigma$ and (ii) if s and $|q^2|$ are much larger than m_N^2, then it is a good approximation to set $m_N = 0$ in the expression for $d\sigma$, provided that all final hadronic channels are summed over. Accordingly, for s and $|q^2|$ larger than a few $(\text{GeV})^2$, one may set, as a good approximation, $m_\ell = m_N = 0$; therefore, (1) becomes simply

$$d\sigma = f(s, q^2) \times \begin{cases} \alpha^2 \\ G^2 \end{cases} . \qquad (2)$$

Apart from the coupling constant α^2 or G^2, the differential cross section $d\sigma$ now depends only on s and the various q^2. These quantities represent (in the natural units $\hbar = c = 1$) the only physical observables with the dimension $(\text{length})^{-2}$. All the consequences of the scaling hypothesis can then be easily derived by a pure and simple dimensional analysis[2]. The scaling hypothesis means simply the absence of any basic physical energy scale, such as m_ℓ and m_N. As we shall see, this enables us to connect various cross sections at a relatively low energy range to those at a much higher energy range.

2. Applications

(i) To illustrate the use of the scaling hypothesis, we shall first consider the following two electromagnetic processes, one purely leptonic and the other semi-leptonic:

$$e^+ + e^- \rightarrow \mu^+ + \mu^- \tag{3}$$

and

$$e^+ + e^- \rightarrow \text{hadrons}\,. \tag{4}$$

It follows from the scaling hypothesis that for

$$s = (\text{center-of-mass energy})^2 \gg m_\mu^2$$

one may set $m_e = m_\mu = 0$. The total cross section for the purely leptonic reaction (3) depends then only on α^2 and s. From simple dimensional considerations, one sees that

$$\sigma(e^+e^- \rightarrow \mu^+\mu^-) = \text{constant} \cdot \alpha^2/s\,.$$

The constant can be evaluated by using quantum electrodynamics, which is consistent with the scaling hypothesis provided that radiative corrections are neglected; one finds then

$$\sigma(e^+e^- \to \mu^+\mu^-) = \frac{4\pi\alpha^2}{3s} .$$

Similarly, according to the scaling hypothesis, if one sums over all final hadronic channels in the semi-leptonic reaction (4), for $s >$ a few $(\mathrm{GeV})^2$ one may set $m_N = m_\ell = 0$. A simple dimensional analysis leads to

$$\sigma(e^+e^- \to \text{hadrons}) = \text{constant} \cdot \alpha^2/s$$

where the constant may be determined by a relatively low energy experiment, which then enables one to predict the cross section in a much higher energy region. The present colliding beam results from Frascati[3] are in agreement with the predicted s^{-1} dependence.

One notes that for a fixed hadron channel, say $e^+e^- \to \rho^o$, the scaling hypothesis is clearly non-applicable, since in such a case the physical dimension related to the mass and the width of ρ^o can never be neglected.

(ii) Next, we consider the following weak processes:

$$\nu_e + e^\pm \to \nu_e + e^\pm \tag{5}$$

and

$$\nu_\mu + N \to \mu^- + \text{hadrons} . \tag{6}$$

Let q^2 denote the (4-momentum transfer)^2 between the incident neutrino and the

target, and s be the (center-of-mass energy)^2, as before. For the purely leptonic reaction (5), if s and q^2 are $\gg m_e^2$, on account of the scaling hypothesis, one may set $m_e = 0$ in the expression for $d\sigma$. Similarly, for the semi-leptonic reaction (6), if s and q^2 are greater than a few $(\mathrm{GeV})^2$, one may set $m_\mu = m_N = 0$, provided all hadron channels are summed over. In either case, the differential cross section is proportional to G^2 and the proportionality factor depends only on q^2 and s. Recalling that the dimension of G is $(\mathrm{length})^2$, one finds from simple dimensional considerations that the differential cross sections of both reactions must be of the form

$$\frac{d\sigma}{dq^2} = G^2 \cdot f\left(\frac{q^2}{s}\right) \tag{7}$$

where f is a dimensionless function depending only on the ratio (q^2/s), which varies from 0 to 1 since $s = q^2_{max}$, the maximum value of q^2. The corresponding total cross sections are of the form

$$\sigma = \text{constant} \cdot G^2 s \ . \tag{8}$$

According to the usual (Current X Current) theory of the weak interaction, one can readily show that

$$\frac{d\sigma}{dq^2}(\nu_e e^- \to \nu_e e^-) = \frac{G^2}{\pi}$$

and

$$\frac{d\sigma}{dq}(\nu_e e^+ \to \nu_e e^+) = \frac{G^2}{\pi}\left(1 - \frac{q^2}{s}\right)^2$$

which agree with (7). In Figure 1, we reproduce the results from the CERN neutrino experiment[4], which gives, after averaging over $N = p$ and n,

$$\sigma(\nu_\mu + N \rightarrow \mu^- + \text{hadrons}) \cong 0.6 \times 10^{-38}\ (\text{cm}^2/\text{nucleon}) \times \left[(E_\nu)_{lab} \text{ in GeV}\right]$$

in good agreement with (8).

(iii) As a further example, one may consider the following two electromagnetic processes:

$$e^\pm + \mu^\pm \rightarrow e^\pm + \mu^\pm \tag{9}$$

and

$$e^\pm + p \rightarrow e^\pm + \text{hadrons} \tag{10}$$

in which one sums over all final hadron channels, as is done in the "deep" inelastic experiments of SLAC[5]. It is customary to introduce

$$W^2 \equiv (\text{invariant mass of the final hadron system})^2$$

and the scaling variable

$$\omega \equiv 1 + q^{-2}(W^2 - m_N^2) \ .$$

According to the scaling hypothesis, for s and q^2 greater than a few $(\text{GeV})^2$, one may set $m_\ell = m_N = 0$; therefore, ω becomes simply

$$\omega = 1 + (W^2/q^2)$$

and the differential cross section depends only on q^2, $s = q^2_{max}$ and W^2. From simple dimensional considerations, one deduces that for the deep inelastic ep scattering

$$\frac{d^2\sigma}{dq^2\, d\omega} = \frac{\alpha^2}{(q^2)^2}\ F\left(\frac{q^2}{s}, \omega\right) \ . \tag{11}$$

Identical results can be derived for the purely leptonic reaction (9). Because $e\,\mu$ scattering is an elastic process, the scaling variable ω equals 1 and the corresponding function F is, therefore, proportional to $\delta(\omega - 1)$. One may write, instead of (11),

$$\frac{d\sigma}{dq^2}(e^-\mu^+ \to e^-\mu^+) = \frac{\alpha^2}{(q^2)^2} f\left(\frac{q^2}{s}\right) . \tag{12}$$

The $\frac{q^2}{s}$ dependence in both (11) and (12) can be explicitly evaluated by using quantum electrodynamics, since it involves only lepton variables. One finds (for $m_\ell = m_N = 0$)

$$\frac{d\sigma}{dq^2}(e^-\mu^+ \to e^-\mu^+) = \frac{4\pi\alpha^2}{(q^2)^2}\left[1 - \frac{q^2}{s} + \frac{1}{2}\left(\frac{q^2}{s}\right)^2\right]$$

and

$$\frac{d^2\sigma}{dq^2\,d\omega}(e^-p \to e^- + \text{hadrons}) = \frac{4\pi\alpha^2}{(q^2)^2}\left[\left(\frac{1}{\omega} - \frac{q^2}{s}\right)(\nu W_2) + \left(\frac{q^2}{s}\right)^2 W_1\right]$$

where W_1 and νW_2 are called structure functions; both are dimensionless and depend on ω only. As shown in Figure 2, the validity of the scaling hypothesis has been verified by the recent SLAC data[5].

The scaling hypothesis can be readily applied to other reactions such as $p + p \to \ell^+ + \ell^- + \text{hadrons}$, $p + p \to \ell^\pm + \nu + \text{hadrons}$, etc. These are discussed in the Appendix.

3. Breakdown of the Scaling Property

If the scaling hypothesis were exact, then a large part of high energy physics would be simply to verify dimensional analysis, and that, after a little while, could become quite dull. Fortunately, as we shall see, there are good reasons to believe that the scaling hypothesis is not an exact law of nature in the extremely high energy limit, but is only approximately valid in the intermediate energy range which includes the one presently accessible to experimental study. There are two independent reasons why one expects a breakdown of the scaling property:

(i) It is well-known that the mass of a charged particle cannot be zero. Theoretically, this impossibility is connected with the so-called "mass singularity" in quantum electrodynamics[6]; experimentally this is reflected by the large radiative corrections known to be present in any high energy electron collision processes. In quantum electrodynamics, only in the Born approximation, is the approximation $m_\ell = 0$ a valid one in the high energy limit. Higher order diagrams invariably lead to terms proportional to powers of $\alpha \ln (s/m_\ell^2)$, which dominate in the limit $s \to \infty$. Thus, even for leptons, there is no reason to believe that the scaling hypothesis is an exact law of nature in the infinite energy limit. Nevertheless, it does serve as a good approximation over a wide, though limited, energy range. Similarly, just on account of electromagnetic radiative corrections, one expects that the scaling hypothesis for the hadrons also should not be an exact description of the infinite energy limit. The presence of radiative corrections due to meson fields most likely would, in addition, introduce further deviations; this is reasonable when one recalls the generality in the concept underlying the "mass singularity", which should by no means be restricted only to quantum electrodynamics.

It might be argued that if one could sum up the series of all higher powers of $\alpha \ln s/m_\ell^2$, perhaps the entire sum might approach zero as $s \to \infty$. However, it is important to note that in any case this possibility has very little bearing on the existing experimental verification of the scaling property. At the present SLAC energy range, there is no doubt that (at least) the electromagnetic radiative correction is dominated by the lowest order term which gives a rising deviation from the scaling property $\sim 0\left[\alpha \ln s\right]$ with increasing energy. Whatever is responsible for the scaling property[7] appears to have no connection with the sum of these higher order radiative corrections, leaving aside whether such a sum might or might not converge at infinite energy.

Indeed, in the usual experimental determination of, say, structure functions in deep inelastic ep scattering, the leptonic contribution to electromagnetic radiative corrections has already been taken into account, since it is calculable and also happens to be the dominant one. On the other hand, the hadronic contribution, though smaller in magnitude, is inextricably contained in the experimental data, which should, therefore, consist of some non-scaling terms $\sim 0\left[\epsilon \ln (s/m_N^2)\right]$ where $\epsilon \sim \alpha$ just on the basis of quantum electrodynamics, and might be much larger if there are additional deviations from the scaling property due to strong interactions. The present experimental data is consistent with $\epsilon \lesssim 10^{-1}$.

(ii) It has been known for quite some time that the conventional theories of quantum electrodynamics and the weak interaction, despite their spectacular successes at the presently available energy range, must undergo major changes in some higher energy region. As the following discussions will show, there should then exist a new fundamental energy scale much greater than 1 GeV. The existence of a new energy scale

would imply further large departures, other than those simple $\epsilon \ln s$ terms, from the predictions of the scaling hypothesis. To understand the reasons for expecting a new energy scale, we shall first review some of the arguments that demonstrate the inadequacy of the present theories of quantum electrodynamics and weak interactions[8].

4. Limitation of the Fermi Theory

Except for CP violations, all the known weak reactions can be represented by an effective Lagrangian density

$$\mathcal{L}_{eff} = \frac{G}{\sqrt{2}} \left(\mathcal{J}_\lambda^{wk} \right)^\dagger \mathcal{J}_\lambda^{wk} \tag{13}$$

where G is the Fermi constant $\cong 10^{-5}/m_N^2$,

$$\mathcal{J}_\lambda^{wk} = J_\lambda^{wk} + j_\lambda^{wk}$$

in which j_λ^{wk} is the usual lepton current, related to the charged lepton field ψ_ℓ and the corresponding neutrino field ψ_{ν_ℓ} by

$$j_\lambda^{wk} = i \sum_{\ell = e, \mu} \psi_\ell^\dagger \gamma_4 \gamma_\lambda (1 + \gamma_5) \psi_{\nu_\ell} ,$$

and J_λ^{wk} denotes the hadron current. The use of an effective Lagrangian for weak interactions was first introduced by Fermi[9]. The rule of an effective Lagrangian is that its lowest order matrix element determines directly the corresponding element of the S-matrix; the higher order matrix elements are divergent and must be discarded, in analogy to the pseudo-potential method used in studying nuclear forces, also introduced by Fermi.

That such an effective Lagrangian must become inadequate at high energy may be demonstrated by considering the reaction

$$\nu_\mu + e^- \rightarrow \nu_e + \mu^- \quad .$$

According to the effective Lagrangian (13), its cross section should be (for an unpolarized e^- and neglecting the lepton masses)

$$\sigma = \frac{4}{\pi} G^2 p_\nu^2$$

where p_ν is the ν_μ-momentum in the center-of-mass system; furthermore, it consists of only s-wave scattering. From unitarity, there exists an upper bound

$$\sigma < \tfrac{1}{2} \pi \lambda\!\!\!^{-2}$$

where $\pi \lambda\!\!\!^{-2}$ is the usual upper-bound for an inelastic s-wave scattering cross-section, and the additional $\frac{1}{2}$ factor is to take into account that only half of the unpolarized e^- can interact. Thus, the prediction of the Fermi theory must break down at[10]

$$p_\nu < \left[\frac{\pi^2}{8G^2} \right]^{\frac{1}{4}} \sim 300 \text{ (GeV/c)} \quad . \tag{14}$$

Independently of the underlying equations of the weak interaction, the particular effective Lagrangian density $\mathcal{L}_{eff}$, given by (13), must be modified at high energy; i.e., at small distances. For example, (13) can be replaced by a non-local expression

$$\mathcal{L}_{eff} = \frac{G}{\sqrt{2}} \int \left[\mathcal{J}_\mu^{wk}(x) \right]^\dagger D_{\mu\lambda}(x - x') \, \mathcal{J}_\lambda^{wk}(x') \, d^4x' \quad . \tag{15}$$

To generate such apparent nonlocality, some agents are needed to transmit the action of the current at a point x to a different point x'. The simplest possibility is to assume[11] the existence of a charged spin 1 intermediate boson $W^{\pm}$. On account of the unitarity limit, one expects the mass m_W to have an upper bound of about 300 GeV. Of course, the actual structure may turn out to be much more complicated than a single $W^{\pm}$, but it can hardly be simpler.

At present, from the neutrino experiments[4] there exists only a rather poor lower bound

$$m_W > 2.5 \text{ GeV} \quad . \tag{16}$$

5. Divergence Difficulties

Both the conventional quantum electrodynamics and the conventional weak interaction theory (either the Fermi theory, or the intermediate boson theory) have serious divergence difficulties, which we shall review briefly:

(i) Electromagnetic mass differences

Quantum electrodynamics is a renormalizable theory. The observed mass of any particle can be written as

$$m_{obs} = m_o + \Delta m$$

where m_o is the unrenormalized mass and Δm is the electromagnetic mass shift.

In the conventional theory, both m_o and Δm are divergent. Consider now, for example, the mass difference between $\pi^{\pm}$ and π^{o}. It is reasonable to assume that this mass difference is due purely to the electromagnetic interaction; therefore

$$m_o(\pi^+) = m_o(\pi^o)$$

and

$$m_{obs}(\pi^+) - m_{obs}(\pi^o) = \Delta m(\pi^+) - \Delta m(\pi^o)$$

which, according to the usual theory, is infinite in clear violation of the observed fact. The same divergence difficulty exists for all observed (therefore finite) mass differences between hadrons of the same isospin multiplet.

(ii) Radiative corrections

The ratio of the observed Fermi constant in β-decay and that in μ-decay may be written as

$$\left(\frac{G_\beta}{G_\mu}\right)_{obs} = \left(\frac{G_\beta}{G_\mu}\right)_o + \text{radiative correction} \quad .$$

Since the left-hand side is, by definition, determined by the observed decay rates, it is therefore finite. Furthermore, the ratio $(G_\beta/G_\mu)_o$ is, according to the universality assumption of the weak interaction, related to the Cabibbo angle θ by [12]

$$\left(\frac{G_\beta}{G_\mu}\right)_o = \cos\theta \; ;$$

consequently, the radiative correction should be finite, and yet it is infinite in the

conventional theory. The same difficulty extends to electromagnetic radiative corrections to other weak decays.

(iii) Higher order weak processes

Both Fermi theory and the conventional intermediate boson theory are unrenormalizable. It is not possible to use such theories to evaluate higher order weak processes; these calculations result only in vast amounts of meaningless infinities.

From an observational point of view, the most striking feature about these divergence difficulties is their total absence in nature. The fractional electromagnetic mass differences between hadrons of the same isospin multiplet are not just finite but are all of the right order of magnitude; i.e., of the order of the fine structure constant α. The same is true for all radiative corrections to weak decays as determined from the observed decay rates and the Cabibbo angle. This strongly indicates that all these physical observables are indeed due to second order electromagnetic processes. Similarly, from order of magnitude estimations, one expects the rates of all higher order weak processes, at least at the low energy region, to be extremely small. At present, except for the very small mass difference between K_L^o and K_S^o, none of the higher order weak processes has been observed. This lack of positive results shows conclusively that these higher order weak processes are indeed of extremely small rates, and certainly not infinite.

Under some rather general theoretical assumptions, one can show that such infinities, being closely connected with the equal-time commutator between the current operator and its derivatives[13], cannot be eliminated through strong interactions. Thus, it appears that in order to remove these divergence difficulties there must be some fundamental changes in our basic formulation of quantum electrodynamics and the weak interaction.

6. Indefinite Metric (A Theoretical Possibility)

The simplest way to remove all these divergence difficulties is to assume the existence of fields of negative metric [14, 15]

(i) Heavy photon

We assume that in the electromagnetic interaction Lagrangian density $\mathcal{L}_\gamma$, the usual zero mass positive-metric photon field A_μ is replaced by a complex field $A_\mu + i B_\mu$, so that [15]

$$\mathcal{L}_\gamma = e \mathcal{J}_\mu^\gamma (A_\mu + i B_\mu) \tag{17}$$

where $\mathcal{J}_\mu^\gamma$ is the usual electromagnetic current operator and B_μ is the negative-metric heavy photon field of mass m_B. [In the free Lagrangian, A_μ and B_μ are separate.] The usual $\frac{1}{q^2}$ propagator of the photon is now replaced by the propagator of $(A_\mu + i B_\mu)$

$$\frac{1}{q^2} - \frac{1}{q^2 + m_B^2} = \frac{m_B^2}{q^2 (q^2 + m_B^2)}$$

which behaves like q^{-4} as $q^2 \to \infty$, and therefore removes all previous logarithmic divergences in electromagnetic mass differences and radiative corrections to weak decays.

Because of its heavy mass, the heavy photon B^o is unstable. It decays into usual positive-metric particles, such as e^+e^-, $\mu^+\mu^-$, $\pi^+\pi^-$ and other hadron modes. Thus, in a typical reaction, say

$$p + p \to p + p + B^o \hookrightarrow \begin{cases} e^+ + e^- \\ \mu^+ + \mu^- \\ \text{hadrons} \end{cases} ,$$

only positive-metric particles appear in both the initial and final states, and thereby insure the unitarity of the S-matrix. The heavy photon B^o appears as a resonant state, which corresponds to a pole on the first sheet (i.e., the physical sheet) above the real axis, unlike the usual resonances which are poles on the second sheet below the real axis. The partial width of B^o decaying to the lepton modes can be readily calculated. One finds (neglecting lepton masses)

$$\gamma(B^o \to e^+e^-) = \gamma(B^o \to \mu^+\mu^-) = \tfrac{1}{3}\alpha m_B \quad . \tag{18}$$

At present, one knows that

$$m_B > 5 \text{ GeV}$$

from the $(g-2)$ measurement of the muon[16] and from the lepton pair production cross section in the p + uranium experiment[17]. This lower bound can be improved by using the deep inelastic ep scattering data through the following indirect argument: If one assumes the usual scaling properties of structure functions due to the strong interaction, the presence of the heavy photon would introduce a deviation in the scaling properties of the observed W_1 and νW_2 functions in the form

$$[W_1]_N = \left(\frac{m_B^2}{q^2 + m_B^2}\right)^2 [F_1(\omega)]_N \tag{19}$$

and

$$[\nu W_2]_N = \left(\frac{m_B^2}{q^2 + m_B^2}\right)^2 [F_2(\omega)]_N \quad .$$

where ω is the scaling variable, defined earlier. From the absence of such a correction

factor $\left[m_B^2/(q^2 + m_B^2)\right]^2$ in the present SLAC data[5], one may deduce

$$m_B > 9 \text{ GeV} .$$

(ii) Applications to the weak interaction

The use of negative metric can be easily extended to eliminate divergence difficulties in the weak interaction. For definiteness, let us assume the existence of the intermediate boson field W_μ. The weak interaction Lagrangian density is given by

$$\mathcal{L}_{wk} = g\, \mathcal{J}_\mu^{wk} W_\mu + \text{adjoint.} \tag{20}$$

All presently observed weak transitions are second order in g^2, transmitted by the covariant W-propagator

$$D_{\mu\nu}(k) = \delta_{\mu\nu} \int \frac{\sigma_1\, dM}{k^2 + M^2} + k_\mu k_\nu \int \frac{(\sigma_1 + \sigma_0)\, M^{-2}\, dM}{k^2 + M^2} .$$

These second order effects can be formally represented by the "effective" Lagrangian given previously by (15).

In order that such a theory may correspond to a renormalizable one, we require[18]

$$\int (\sigma_1 + \sigma_0)\, M^{-2}\, dM = 0 . \tag{21}$$

If one makes the further requirement that such a theory should be a finite one, then in addition to (21), one must have

$$\int \sigma_1\, dM = 0 \qquad \text{and} \qquad \int \sigma_0\, dM = 0 . \tag{22}$$

These conditions can be easily satisfied if one uses indefinite metric. For example, in order to have a renormalizable theory (but not a finite theory before renormalization), the simplest solution of (21) is

$$\sigma_1 = \pm\, \delta(M - m_1)$$

and

$$\sigma_0 = \mp\, (m_0/m_1)^2\ \delta(M - m_0) \tag{23}$$

which corresponds to a charged spin 1 boson $W_1^{\pm}$ of mass m_1 and a charged spin 0 boson $W_0^{\pm}$ of mass m_0. These two bosons are of opposite metric. The upper signs in (23) imply that $W_1^{\pm}$ is of positive metric and $W_0^{\pm}$ of negative metric, and the lower signs imply the opposite. The masses m_0 and m_1 are free parameters, and the previously stated lower bound (16) applies only to m_1. Additional boson states would be required, if one assumes the theory to be also a finite one. In order for both (21) and (22) to be valid, the simplest solution is to assume the existence of two spin 1 bosons of opposite metric and two spin 0 bosons also of opposite metric.

7. New Energy Scale

It must be emphasized that the possibility of using indefinite metric is merely a theoretical speculation. Nevertheless, there is no doubt that our present theory of quantum electrodynamics and weak interactions is not adequate. At high energy, there must be deviations from the present theoretical predictions. The fundamental question is: What is the new energy scale that determines this high energy region? For the Fermi theory, a natural possibility is to regard the basic scale to be given by the Fermi constant itself,

$$G^{-\frac{1}{2}} \sim 300 \text{ GeV} .$$

On the other hand, if one assumes the weak interaction is governed by the same dimensionless constant α as that in the electromagnetic interaction, then the relevant scale can be much lower,

$$(\alpha/G)^{\frac{1}{2}} \sim 30 \text{ GeV} .$$

[A more careful consideration may lead[19, 20] to 37.29 GeV, or higher values[21].] If this new basic scale emerges in the form of spin 1 or spin 0 bosons, as discussed in the previous section, then one may have a possible spectrum as illustrated in Figure 3. Of course, before experimental verification all theoretical speculations are subject to, at least, the same uncertainties as, say, the time table of the Long Island Railroad which one uses when coming to Brookhaven; nevertheless, without such a time table one would never know how late the train is.

Independently of these theoretical speculations, we recall that throughout the

study of microscopic physics, new frontiers are opened whenever a new basic energy scale is reached, as in the case of atomic and molecular physics with the electron-volt energy scale, nuclear physics with the MeV energy scale, and the present strong interaction physics with the GeV energy scale. In each case, at the energy scale of interest, one encounters a vastly rich structure of multiple energy levels and detailed dynamics; yet, when viewed with a much larger energy scale, this superstructure dissolves simply into the continuum. The recently discovered scaling property strongly indicates that we are now in a transition region: the familiar GeV scale is no longer significant, but the still higher new high energy scale is, as yet, unreached. By definition, the scaling invariance means the absence of any basic physical energy scale; its violation therefore automatically implies the presence of some basic energy scales. As we have shown, in the high energy region, there are irrefutable reasons to expect major changes in the present Fermi theory of the weak interaction, and there are strong reasons to expect important modifications in the present form of quantum electrodynamics. The search for the new basic energy scale that underlies this high energy region is clearly one of the main purposes for studying high energy physics. I am therefore delighted to learn of the design of a future ultra-high energy colliding beam at Brookhaven, which together with the CERN ISR machine and the Batavia NAL machine should lead us to the discovery of the new basic energy scale in physics.

Appendix

(i) As further examples of the application of the scaling hypothesis, let us consider

$$p + p \rightarrow \ell^+ + \ell^- + \text{hadrons} \tag{A.1}$$

and

$$p + p \rightarrow \ell^\pm + \nu + \text{hadrons} \ . \tag{A.2}$$

It is convenient to define

$$M^2 = (\text{ invariant mass of di-leptons })^2 \ .$$

According to the scaling hypothesis, for s and M^2 larger than a few $(\text{GeV})^2$ one may set $m_\ell = m_N = 0$. The differential cross section $d\sigma/dM$ for either (A.1) or (A.2), after summing over all hadron channels, depends then only on s and M^2. From purely dimensional analysis, one finds[22]

$$\frac{d\sigma}{dM}(pp \rightarrow \ell^+\ell^- \text{ hadrons}) = \alpha^2 M^{-3} \cdot f(M^2/s) \tag{A.3}$$

and

$$\frac{d\sigma}{dM}(pp \rightarrow \ell\nu \text{ hadrons}) = G^2 M \cdot g(M^2/s) \ . \tag{A.4}$$

where f and g are dimensionless functions that depend only on the ratio (M^2/s).

By using the Fermi theory and quantum electrodynamics, these two cross sections can be readily related:

$$\frac{\frac{d\sigma}{dM}(pp \rightarrow \ell^\pm\nu \text{ hadrons})}{\frac{d\sigma}{dM}(pp \rightarrow \ell^+\ell^- \text{ hadrons})} = \frac{\xi M^4}{\alpha^2}\left(\frac{G}{4\pi}\right)^2 ;$$

i.e., $g = (4\pi)^{-2}\,\xi f$, and therefore

$$\frac{d\sigma}{dM}(pp \to \ell\nu \text{ hadrons}) = \xi M \left(\frac{G}{4\pi}\right)^2 \cdot f\left(\frac{M^2}{s}\right) \tag{A.5}$$

where ξ is related to the weak interaction currents V_μ, A_μ and the electromagnetic current J_μ by

$$\xi \equiv \left[\frac{\int \langle pp \mid V_\mu^\dagger(x)\, V_\nu(0) + A_\mu^\dagger(x)\, A_\nu(0) \mid pp\rangle\; e^{iq\cdot x}\, d^4x}{\int \langle pp \mid J_\mu^\dagger(x)\, J_\nu(0) \mid pp\rangle\; e^{iq\cdot x}\, d^4x}\right]_{\text{average}} . \tag{A.6}$$

The precise value of ξ depends on the model, but it should be $\sim 0(1)$.

At present, only reaction (A. 1) has been observed by Lederman and his collaborators[17] for proton on uranium at several different values of M^2, but in a rather limited incident proton laboratory momentum range varying from 22 to 29.5 GeV/c. Thus, unlike the previous examples of semi-leptonic reactions discussed in section 2, there is as yet no solid experimental evidence for the validity of the scaling hypothesis. However, if one assumes that the scaling formula (A.3) is indeed applicable to the existing experiment, then the dimensionless function $f(M^2/s)$ is already determined, and is given approximately by[17, 23]

$$f(M^2/s) \equiv \text{Lederman factor} \cong \exp(-10\, M^2/s) . \tag{A.7}$$

Especially in this case, further experiments are clearly needed in order to verify the validity of the scaling properties (A.3), (A.5) and the Lederman factor (A.7).

(ii) Assuming the existence of the heavy photon B^o and/or the spin 1 intermediate boson $W^{\pm}$, one expects to see large deviations from the scaling property discussed above and in section 2 when the relevant $|q^2|^{\frac{1}{2}}$ is comparable to the masses of these heavy bosons. For a space-like q^2, the modification in the cross section consists of simply a multiplicative factor

$$\left(\frac{m^2}{q^2 + m^2}\right)^2$$

where $m = m_B$ the heavy photon mass, or m_W the spin-1 intermediate boson mass. The same modification holds for a time-like q^2, provided that it is far away from the resonance region; i.e.,

$$|q^2 + m^2| \gg \tfrac{1}{4}\gamma^2$$

where γ = the width of B^o or $W^{\pm}$. For q^2 in the resonance region, one has the production of B^o and $W^{\pm}$. As an example, one may consider any lepton-pair production process

$$a \rightarrow b + \ell^+ + \ell^-$$

where $\ell = e$ or μ, and a and b denote some hadron-lepton complexes. In the resonance region, the B^o can be produced via the reaction

$$a \rightarrow b + B^o \hookrightarrow \begin{cases} \ell^+ + \ell^- \\ \text{hadrons} \end{cases}$$

Let $\sigma(a \to b + B^o)$ be the cross section of B^o production integrated over its width, and $\frac{d\sigma}{dM}(a \to b + \ell^+ + \ell^-)$ be the corresponding differential cross section of charged lepton pair production at a di-lepton mass M, calculated in the conventional quantum electrodynamics without assuming B^o. It can be readily verified that these two cross sections are related by[23], at $M = m_B$,

$$\sigma(a \to b + B^o) = \frac{3\pi M}{2\alpha}\left[\frac{d\sigma}{dM}(a \to b + \ell^+ + \ell^-)\right] . \qquad (A.8)$$

For example, by using (A.3), one finds

$$\sigma(pp \to B^o \text{ hadrons}) = \frac{3\pi\alpha}{2m_B^2} f\left(\frac{m_B^2}{s}\right) \qquad (A.9)$$

$$\cong (1.3 \times 10^{-29}\ \text{cm}^2)\,(m_B \text{ in GeV})^{-2}\ f(m_B^2/s) .$$

Similarly, the cross section $\sigma(a \to b + W^\pm)$ of $W^\pm$-production, integrated over the $W^\pm$-width, is related to the corresponding differential cross section $\frac{d\sigma}{dM}(a \to b + \ell^\pm + \nu)$ at the di-lepton mass $M = m_W$ by

$$\sigma(a \to b + W) = \frac{6\pi^2}{\sqrt{2}\,G}\,\frac{1}{M}\left[\frac{d\sigma}{dM}(a \to b + \ell + \nu)\right] . \qquad (A.10)$$

$\frac{d\sigma}{dM}(a \to b + \ell + \nu)$ is calculated according to the Fermi theory, without assuming $W^\pm$. Thus, for example from (A.5), one finds

$$\sigma(pp \to W \text{ hadrons}) = \frac{3}{8\sqrt{2}}\,\xi\, G\, f(m_W^2/s) \qquad (A.11)$$

$$\cong \xi(1.2 \times 10^{-33}\ \text{cm}^2)\ f(m_W^2/s) .$$

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nected to the theoretical possibility that the physical proton should be regarded as a bound-state composite rather than an elementary particle. For further discussions, see the recent paper by S. D. Drell and T. D. Lee (Columbia University-SLAC preprint).

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Figure Captions

Figure 1. Experimental data from CERN on total neutrino cross sections as a function of laboratory neutrino energy (Ref. 4).

Figure 2. Experimental data from SLAC on structure functions in deep inelastic ep scattering (Ref. 5).

Figure 3. A speculation on the spectrum of various integer-spin bosons which are the carriers of the weak and electromagnetic interactions. The mass of spin 0 intermediate bosons could be equal to or lower than that of spin 1 intermediate bosons. Because of the conservation law of the electromagnetic current, in a simple theory the neutral spin 0 intermediate boson, if it exists, remains a free field.

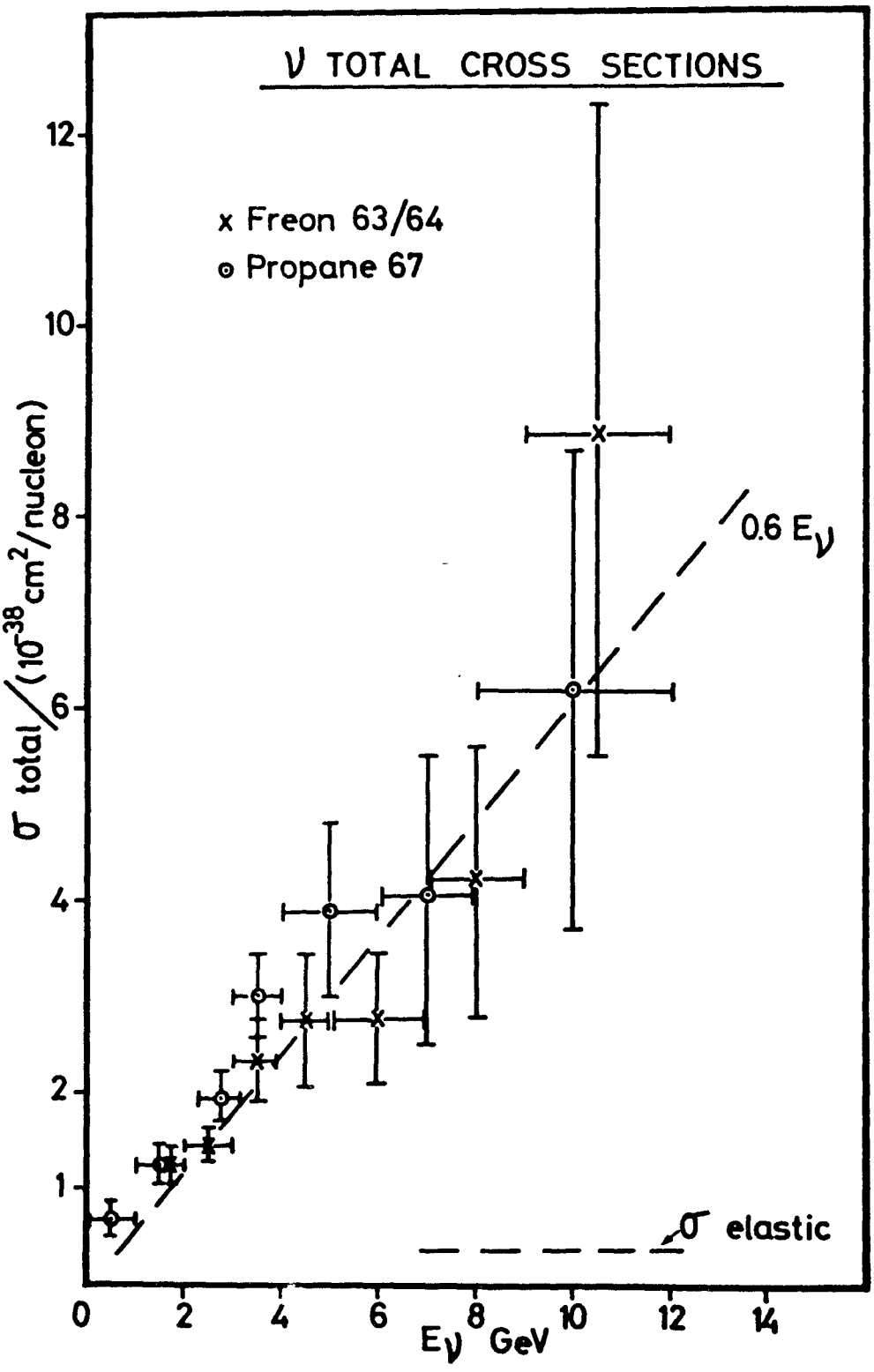
ν TOTAL CROSS SECTIONS
x Freon 63/64
⊙ Propane 67
0.6 Eν
σ elastic
σ total/(10⁻³⁸cm²/nucleon)
Eν GeV
0
2
4
6
8
10
12
14
1
2
4
6
8
10
12

FIGURE 1

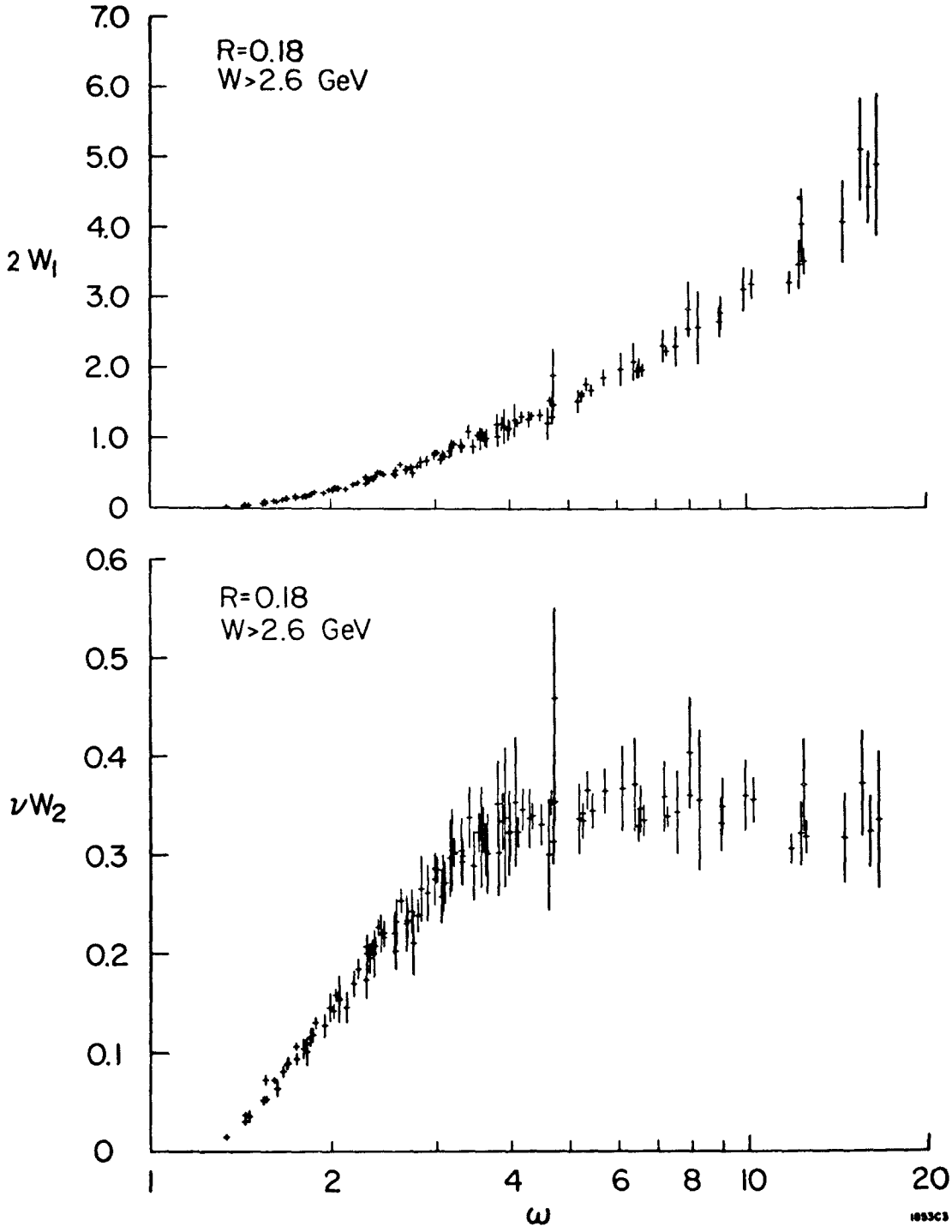
R=0.18
W>2.6 GeV
2 W1
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0
R=0.18
W>2.6 GeV
νW2
0.6
0.5
0.4
0.3
0.2
0.1
0
1
2
4
6
8
10
20
ω

FIGURE 2

			Spin	Mass
W_0^+	W_0^0	W_0^-	0	?
W_1^+	$W_1^0 = B^0$	W_1^-	1	37.29 Gev ?
	γ		1	0

FIGURE 3