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CONSERVATION LAWS IN WEAK INTERACTIONS

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CONSERVATION LAWS IN WEAK INTERACTIONS*+

T. D. Lee

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+ Notes on four Lectures given at Harvard University in March 1957.

CONSERVATION LAWS IN WEAK INTERACTIONS

LECTURE 1

Part I: Review of the Status of Elementary Particle Physics

Types of Interactions.

The interactions among elementary particles can be divided into three groups:

1. Strong interactions--production and scattering of pi mesons, nucleons, hyperons and K-particles. These are characterized by a dimensionless coupling constant, $f^2 / \hbar c$, of the order of 1.

2. Electromagnetic interactions--characterized by a coupling constant $e^2 / \hbar c = 1 / 137$.

3. Weak interactions--decay of the neutron, pi meson, K-particle and hyperons. These are all characterized by coupling constants which are of the order $G^2 / \hbar c \sim 10^{-13}$.

(4. Gravitational interactions--characterized by Newton's constant which has the dimensions of an inverse squared mass $Gm_p^2 / \hbar c = 2 \times 10^{-39}$ where m_p is the proton mass.)

Symmetry Properties and Exact Conservation Laws.

Symmetry properties possessed by all three of these types of interactions, and which therefore lead to exact conservation laws are as follows:

1. Conservation of energy and momentum--implied by the homogeneity of space and time; and invariance under space and time

translations.

2. Invariance under proper Lorentz transformations--related to the isotropy of space-time and leading to the conservation of angular momentum, etc.

3. Conservation of charge, Q--related to gauge invariance.

4. Conservation of heavy particle number, N--related to the fact that we are here.

5. (?) Invariance under time reversal, T. This is equivalent to invariance under CP (see below).

The operations C, P and T are defined as follows:

Charge conjugation, C:

Particle → Anti - particle $\quad \vec{r} \rightarrow \vec{r} \quad t \rightarrow t$

Space inversion, P:

Particle → Particle $\quad \vec{r} \rightarrow -\vec{r} \quad t \rightarrow t$

Time reversal, T:

Particle → Particle $\quad \vec{r} \rightarrow \vec{r} \quad t \rightarrow -t$

That these three operations are related is the substance of the Schwinger-Luders-Pauli Theorem, (SLP), which states that in any field theory derived from a Lagrangian involving only derivatives of finite order, invariance under the proper Lorentz transformations implies invariance under each of the six products: CPT, TCP, PCT, etc. Consequently, for such theories, invariance under T is equivalent to invariance under CP.

Approximate Conservation Laws.

In addition to the exact conservation laws, there are a number

of conservation laws obeyed by only some of the interactions. We call these approximate conservation laws:

1. Strong interactions. The observed charge independence of nucleon-nucleon and pion-nucleon interactions, and the existence of mass multiplets among these particles lead to the introduction of $\vec{I}$, the isotopic spin. This consept is extended to hyperons and K-particles. The strong interactions conserve, individually, P, C, $\vec{I}$, and I_z. The conservation of N, Q and I_z implies the conservation of strangeness, S, needed to explain the observed production of strange particles.

2. Electromagnetic interactions. The conservation of $\vec{I}$ breaks down. This is not surprising in view of the fact that the electromagnetic field distinguishes between charges. The quantities P, C and I_z, however, remain conserved.

3, Weak interactions. It has been recently discovered that P and C are not conserved in at least some of the weak interactions. However, the possibility that PC, and therefore T, are conserved still remains. There would seem to be a connection between the weakness of an interaction and the number of conservation laws it violates. One might ask if there is any limit to this tendency.

In discussing the weak interactions, it is convenient to divide them into two distinct groups.

(A) Interactions involving a neutrino:

β - decay

$\pi \rightarrow \mu + \nu$ (π - decay)

$\mu \rightarrow e + 2\nu$. (μ - decay)

These violate P and C separately. Isotopic spin has no meaning in these processes, since the concept has never been usefully defined for the leptons which occur in the final state of the interactions.

(B) Interactions not involving a neutrino:

$$\Lambda \rightarrow p + \pi^-$$

$$K \rightarrow 3\pi$$

$$K \rightarrow 2\pi$$

I_z is not conserved for these processes. In fact $\Delta I_z = \pm 1/2$. There is also a strong suggestion (but no proof yet) that P may not be conserved in the K-particle decays.

It is important to note that the coupling constants associated with these two groups of interactions are of comparable magnitude, even though the lifetimes vary from 10^{-6} to 10^{-11} seconds. A further connection between them is indicated by the fact that P may be violated in (B) as well as in (A).

Part II: The θ-τ Problem

This difficulty stimulated people to investigate the entire problem of parity conservation in weak interactions. We will now review our knowledge of the situation.

The table gives the relative abundance, the mass and the lifetime of the various K-particle decay modes:

$$\tau \rightarrow 2\pi^+ + \pi^-, \qquad \tau' \rightarrow \pi^+ + 2\pi^o, \qquad K_{\mu 2} \rightarrow \mu^+ + \nu,$$

$$K_{\pi 2} \rightarrow \pi^+ + \pi^o, \quad K_{\mu 3} \rightarrow \mu^+ + (\pi^o + \nu?), \quad K_{e3} \rightarrow e^+ + (\pi^o + \nu?) .$$

Type	Abundance	Mass (m_e) Primary	Mass (m_e) Secondary	Lifetime (sec.)
τ	5.56 ± 0.41	966.3 ± 2.1	966.1 ± 0.7	$1.19 \pm 0.05 \times 10^{-8}$
τ'	2.15 ± 0.47	967.7 ± 4		
$K_{\mu 2}$	58.2 ± 3.0	967.2 ± 2.2	965.8 ± 2.4	$1.24 \pm 0.02 \times 10^{-8}$
$K_{\pi 2}$	28.9 ± 2.7	966.7 ± 2.0	962.8 ± 1.8	$1.21 \pm 0.04 \times 10^{-8}$
$K_{\mu 3}$	2.83 ± 0.95	969 ± 5		$0.88 \pm 0.23 \times 10^{-8}$
K_{e3}	3.23 ± 1.30			$1.44 \pm 0.46 \times 10^{-8}$

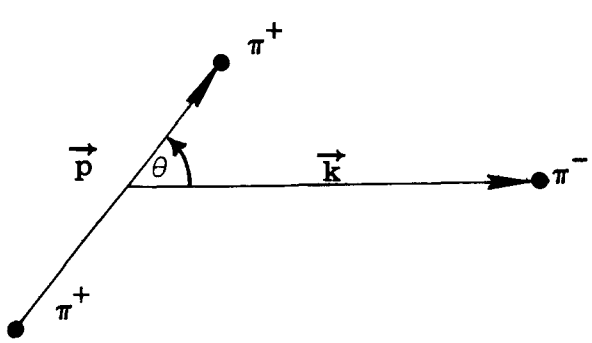
We note that the masses associated with these modes are equal (also that the masses obtained from the primary particle and from its decay products agree, thus verifying the conservation of energy). Furthermore, the lifetimes are equal within 5%. This leads us to suspect that what we have are different decay modes of a single particle.

Spin and Parity Assignments for θ and τ.

To investigate this possibility, we look at those modes where only pi mesons (known to be of spin 0 and odd parity) are emitted. We will compare the $\theta^+ \equiv K^+_{\pi 2}$ and the $\tau^+ \equiv K^+_{\pi 3}$ modes.

(A) $\theta^+ \rightarrow \pi^+ + \pi^o$. Assuming P is conserved, the intrinsic parity of the θ is just the orbital parity of the final state, since the intrinsic parity of the final state is $(-1)^2 = 1$. That is, parity of $\theta = (-1)^J$, where J is the orbital angular momentum of the emitted pi mesons in the center of mass system. By conservation of angular momentum J is also the spin of the θ. Thus, we can give the following spin and parity assignments to the θ: (0,+), (1,-), (2,+),... .

(B) $\tau^+ \rightarrow \pi^+ + \pi^+ + \pi^-$. The three pi mesons in the final state are characterized by the momenta $\vec{p}$ and $\vec{k}$.



$\vec{p}$ = relative momentum of the two π^+

$\vec{k}$ = momentum of the π^- relative to the center of mass of the two π^+.

Let us assume that spin of $\tau = 0$. Then the orbital angular momentum, ℓ_p, associated with $\vec{p}$, must be equal to the orbital angular momentum, ℓ_k, associated with $\vec{k}$, since the two must add to a resultant $J = 0$. Then the parity of the final state is $(-1)^3(-1)^{\ell K}(-1)^{\ell p} = -(-1)^{2\ell p} = -1$, and hence, if we assume conservation of parity, the assignment must be (0,-). On the other hand, if the spin of τ is ≥ 1, we cannot make any general statement about the parity, since there then will be alternate choices of ℓ_p and ℓ_k which could combine to form the same total J and which would correspond to different orbital parity. So we have the following possible spin and parity assignments for the τ:

$$(0,-), \quad (1,+) \quad (1,-), \quad (2,+), \quad (2,-), \ \ldots$$

We can now make three statements concerning θ and τ.

(i) If the spin of τ is 0 and if parity is conserved, then $\theta \not\equiv \tau$. This follows directly from the above possible assignments.

(ii) If in τ decay, there are observed π^+ or π^- mesons of zero energy, and if parity is conserved, then $\theta \not\equiv \tau$. For, let J be the relative orbital angular momentum of two of the mesons, the third having momentum zero and hence orbital angular momentum zero. Then the parity of the final state is $(-1)^3(-1)^J$ where J is the spin of τ. Hence by conservation of parity, the parity of τ is $-(-1)^J$, whereas we have seen above that in general the parity of θ is $(-1)^J$.

(iii) If in τ decay we find a π^- of zero energy, then the spin of τ is even, even if parity is not conserved. This follows from the fact that the relative orbital angular momentum of the two π^+ must be even because of Bose statistics. Since the π^- is emitted with 0 angular momentum, the total angular momentum of the final

state is even and hence the spin of τ is even.

Now since a number of low energy (about 1/3 Mev.) π^- mesons have been observed, the most likely assignments for τ are (0,-), (2,-). Irrespective of conservation of parity, statement (iii) would indicate the likelihood of even spin for τ.

The Statistical Analysis of Dalitz.

To determine the extent to which the detection of low energy pions in τ decay affects the probability that $\tau \equiv \theta$, we consider in detail the statistical analysis of Dalitz.

By energy and momentum conservation, two independent parameters are sufficient to describe the final state. They are chosen to be $\mathcal{E} = k^2 / k^2_{max}$ and θ = angle between $\vec{p}$ and $\vec{k}$. The distribution function is given by

$$|\psi(k,\theta)|^2 \quad d \text{ (phase space)}$$

where ψ (k,θ) is the three pion wave function. Non-relativistically one has

$$d(\text{phase space}) = \text{const} \sqrt{\mathcal{E}(1-\mathcal{E})} \quad d\ (\cos\theta)\ .$$

If $\tau \equiv \theta$ and if parity is conserved, it is easy to prove

1. As $\mathcal{E} \to 0$, $|\psi|^2 \to \mathcal{E}^n$ $\quad n \geq 1$
2. As $\mathcal{E} \to 1$, $|\psi|^2 \to (1 - \mathcal{E})^n$ $\quad n \geq 2$
3. As $\theta \to 0$. $|\psi|^2 \to \sin^2\theta$.

The proof of these assertions is done here for the case

$\tau = (1,-)$, that is, a polar vector. The wave function ψ of the final state must then be an axial vector because of the odd intrinsic parity of the three pi mesons. It also must be an even function of $\vec{p}$ in order to produce a wave function symmetrical between the two π^+. The most general form for such a wave function is $(\vec{k} \times \vec{p})(\vec{k} \cdot \vec{p})f(k^2, p^2, \vec{k} \cdot \vec{p})$ which apart from a constant, gives rise to

$$|\psi|^2 = \mathcal{E}^2(1-\mathcal{E})^2 \sin^2 \theta \cos^2\theta |f|^2.$$

For other spins and parities of τ one must use the appropriate vector and tensor combinations of $\vec{p}$ and $\vec{k}$.

If we are dealing with a local theory with only derivatives of finite order in the interaction Hamiltonian, f is regular and so we may expand in powers of k^2, p^2 and $\vec{p}.\vec{k}$: $f = 1 + 0\ (k^2R^2)$ where R is taken conveniently as the pi meson Compton wavelength. This completes the proof of 1, 2, 3.

However, the observed distribution of pi mesons from τ decay is in contradiction with the statements 1, 2, 3. From the analysis of 1000 cases, Dalitz concludes that $|\psi|^2 = 1$ for all k and θ (uniform distribution). Were we to use f = 1 we would find on the basis of the above distribution function that if $\theta \equiv \tau$ and if parity is conserved, the probability of obtaining the observed distribution of 1000 events would be $< 10^{-15}$.

We notice that if k represents the average pion energy of 26 Mev., $(kR)^2 \sim 1/3$ and it becomes incorrect to take f = 1. However, only the regions of small k^2, p^2 or $\vec{k}\cdot\vec{p}$ are important for the

validity of our argument, and in these regions f is certainly very close to being a constant.

The argument holds if one also takes into consideration both relativistic effects and Coulomb enhancement at low energies.

We may then conclude that if parity is conserved, $\theta \not\equiv \tau$, and we are forced to admit the existence of at least two kinds of K-particles.

We can adjust ourselves to the equality of the masses by postulating a new kind of symmetry operation, parity conjugation, which relates particles of different parity in the same way that charge conjugation operates in the case of electrical charge. We might also say that the lifetimes are only apparently the same, due to some cascade process which converts one kind of K-particle into the other kind extremely rapidly. There is, however, no experimental evidence for this conclusion.

Since it has been observed that parity is not conserved in other weak interactions, the explanation of the θ-τ puzzle by parity non-conservation seems quite plausible. However, whether or not this is actually the case and whether or not there are two types of K-particles are still open questions.

LECTURE 2

Part I: Present Limits on the Validity of Parity Conservation.

If there exists a weak interaction which violates parity conservation, then all atomic and nuclear wave functions no longer will be eigenstates of P but will be instead mixtures of states of

opposite parity. That is,

$$\psi = \psi_P + F\psi_{-P}$$

where **P** is the usual parity assigned to the state and F is the probability amplitude that a component ψ_{-P} of opposite parity is admixed in ψ. F is then given by $F \sim G^2 / \hbar c \sim 10^{-13}$, characteristic of the weak interactions.

What can we learn about limits on F^2 from our present experimental knowledge of atomic and nuclear states?

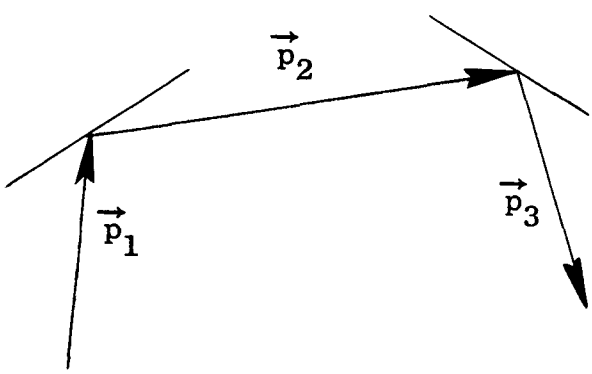
(i) Atomic spectroscopy:

Because of the reflection properties of the various electric multipole operators, the relative parity of the atomic states involved in a transition is alternately +1 and -1 for consecutive multipole orders. Also the intensity of one multipole transition relative to the preceding one, is roughly $(r/\lambda)^2$ where r is the dimension of the system, and λ is the wavelength of the radiation. If parity is not conserved, a transition that was previously forbidden can now occur with an intensity F^2 relative to the usual intensity for a transition of that order. Thus the experimentally observed parity selection rules imply that $F^2 \leq (r/\lambda)^2$. If we take $r = 10^{-8}$cm and $\lambda = 10^{-5}$cm, we get $F^2 \leq 10^{-6}$. By observing radiation of longer wavelength, this restriction can be brought down to $F^2 \leq 10^{-10}$, and possibly to a limit $F^2 \leq 10^{-14}$. These experiments show us that the ordinary electromagnetic interactions conserve parity.

(ii) Nuclear spectroscopy:

The observed selection rules which state, for example, that the β transition $\Delta J = \pm 1$ (yes) is first forbidden, imply that $F^2 < (r/\lambda)^2 \sim 10^{-4}$ (if we take $r \sim 10^{-13}$ cm, $\lambda = 10^{-11}$ cm). Thus for nuclear interactions we have a less stringent condition, which nevertheless is sufficient to guarantee that the strong meson-nucleon interaction conserved parity. Parity violation in strong and electromagnetic interactions for a nuclear system would be incompatible with nuclear spectroscopy.

(iii) Double scattering of protons (Segre, Chamberlain et al.).



In this experiment the symmetry of the distribution of $\vec{p}_3$ with respect to a reflection in the $\vec{p}_1$, $\vec{p}_2$ plane was investigated. The observed absence of any up-down asymmetry in the doubly scattered proton distribution gives $F < 10^{-2}$. An interaction not conserving parity would lead to a pseudoscalar term $\vec{p}_1 \times \vec{p}_2 \cdot \vec{p}_3$ in the scattering amplitude. This experiment is further evidence that nuclear interactions conserve parity.

(iv) The static electric dipole moment of the neutron (Ramsey, Purcell and Smith).

The electric dipole moment of the neutron was shown to be less

than 5×10^{-20} e cm. We might think that if P is not conserved, the neutron would possess a dipole moment $\sim e$ F x (dimensions of system). The results of this experiment imply the condition $F^2 < 10^{-13}$. This would be the most stringent condition on F, though it is still not sufficient to prove conservation of P in weak interactions. However, we will now show that the electric dipole moment of a particle is zero irrespective of whether or not P is conserved, provided only that all interactions are invariant under T. Thus if T is invariant the Ramsey, Prucell, Smith experiment actually gives us no information concerning conservation of parity.

Properties of Time Reversal.

As a preliminary to the proof of this assertion we shall review some facts concerning time reversal. In classical theory, under T: $t \to -t$, $\vec{r} \to \vec{r}$, $\vec{p} \to -\vec{p}$, $\vec{J} \to -\vec{J}$. In quantum theory, both in nonrelativistic quantum mechanics and in relativistic field theory, T, unlike C and P, cannot be represented by a unitary operator. Rather $T = U_T$ x complex conjugation, where U_T is a unitary operator. As an example we consider a non-relativistic, spin 1/2 particle. In this case $U_T = i\,\sigma_2$ where $\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$. We work in a representation in which $\vec{r}$ is diagonal. Then $\vec{r}^* = \vec{r}$ $\vec{p}^* = -\vec{p}$. Using the property that $U_T U_T^\dagger = 1$, we have

$$\text{(a)} \quad T\,\vec{r}\,T^{-1} = U_T\,\vec{r}^*U_T^\dagger = \vec{r}$$

$$\text{(b)} \quad T\,\vec{p}\,T^{-1} = U_T\,\vec{p}^*U_T^\dagger = -\vec{p}$$

$$\text{(c)} \quad T\,\vec{\sigma}\,T^{-1} = U_T\,\vec{\sigma}^*U_T^\dagger = -\vec{\sigma}$$

(c) follows from the fact that although σ_1 and σ_3 are real, σ_2 anticommutes with σ_2 and σ_3. On the other hand σ_2 is pure imaginary, and, or course, commutes with itself. Furthermore, since $\vec{J} = \vec{r} \times \vec{p} + 1/2\vec{\sigma}$, it follows from (a), (b) and (c) that $T^{-1}\vec{J}T = -\vec{J}$. Thus we see that T generates the same transformation as expected classically. These considerations are easily generalized to include particles of spin > 1/2.

Now we will show $\langle\psi \mid \vec{r} \mid \psi\rangle = 0$ if our theory is invariant under T. Suppose that the particle has spin J. Because the matrix elements of any vector operator between states with the same total J are uniquely determined up to a constant factor independent of J_z', we may write,

$$\langle\psi \mid \vec{r} \mid \psi\rangle_{J_z'} = K \langle\psi \mid \vec{J} \mid \psi\rangle_{J_z'} \tag{1}$$

where K is a constant. Take the complex conjugate of (1):

$$\langle\psi^* \mid \vec{r}^* \mid \psi^*\rangle_{J_z'} = K^* \langle\psi^* \mid \vec{J}^* \mid \psi^*\rangle_{J_z'}$$

Both matrix elements are real since they are diagonal matrix elements of a hermitian operator. Thus $K^* = K$ and

$$\langle\psi^* \mid U_T^\dagger U_T \vec{r}^* U_T^\dagger U_T \mid \psi^*\rangle_{J_z'} = K \langle\psi^* \mid U_T^\dagger U_T \vec{J}^* U_T^\dagger U_T \mid \psi^*\rangle_{J_z'}$$

or

$$\langle T\psi \mid \vec{r} \mid T\psi\rangle_{J_z'} = K \langle T\psi \mid -\vec{J} \mid T\psi\rangle_{J_z'}$$

if invariance under T is valid. That is, if $THT^{-1} = H$ then

$T \mid \psi \rangle_{J_z'}$, is an eigenstate of H with the same eigenvalue; indeed it is just $\mid \psi \rangle_{-J_z'}$ since

$$J_z \; T \mid \psi \rangle_{J_z'} = -T \; J_z' \mid \psi \rangle_{J_z'} = -J_z' \; T \mid \psi \rangle_{J_z'}$$

Hence

$$\langle \psi \mid \vec{r} \mid \psi \rangle_{-J_z'} = -K \langle \psi \mid \vec{J} \mid \psi \rangle_{-J_z'} ,$$

while on the other hand

$$\langle \psi \mid \vec{r} \mid \psi \rangle_{-J_z'} = +K \langle \psi \mid \vec{J} \mid \psi \rangle_{-J_z'}$$

Thus

$$\langle \psi \mid \vec{r} \mid \psi \rangle_{-J_z'} = 0$$

Another way of proving this is to note that it can be shown that if our theory is invariant under T, the ψ_p and ψ_{-p} are 90° out of phase. Then $\langle\psi| \vec{r} \mid \psi\rangle$ is pure imaginary and hence zero, since $\vec{r}$ is a hermitian operator.

Therefore, we see that if the theory is invariant under T, then the electric dipole moment is equal to zero, irrespective of whether or not P is conserved. That is, the most stringent condition of F^2 is that obtained from atomic or nuclear spectroscopy.

We have concluded that neither the electromagnetic, nor the strong interactions violate parity. There remain only the weak interactions, for which $F^2 \lesssim 10^{-26}$, if they violate parity. This

is consistent with all the experiments mentioned above, so that we must look for experiments which involve the weak interactions directly.

The ''Old'' Experiments.

β-decay experiments can be expected to provide evidence about parity conservation in weak interactions. We will see, however, that in all the ''old'' β-experiments the question of parity conservation was not even touched.

(v) β-decay:

The interaction Hamiltonian for β-decay which allows violation of parity can be written

$$H = \sum_i \{C_i(\psi_p^\dagger O_i \psi_n)(\psi_e^\dagger O_i \psi_\nu) + C_i'(\psi_p^\dagger O_i \psi_n)(\psi_e^\dagger O_i \gamma_5 \psi_\nu)\}$$

$$+ \text{ Hermitian conjugate}$$

where O_i are the five possible covariants that can be formed from Dirac matrices. That is i runs over S(scalar), V (vector), T (tensor) A (axial vector), P (pseudoscalar) where

$$O_S = \gamma_4, \qquad O_V = \gamma_4 \gamma_\mu,$$

$$O_T = -\frac{1}{2\sqrt{2}} i \gamma_4 (\gamma_\mu \gamma_\nu - \gamma_\nu \gamma_\mu),$$

$$O_A = -i \gamma_4 \gamma_\mu \gamma_5, \qquad O_P = \gamma_4 \gamma_5 .$$

This is the most general interaction which can be formed from a direct coupling of fields, and is invariant under proper Lorentz

transformations. Had we placed the γ_5 between the heavy particle operators in the C_i' terms, the roles of A and V, and the roles of P and S would be interchanged, while the contribution from T would remain unchanged; the new Hamiltonian then would be equivalent to the above one.

An observable physical quantity F, is obtained as a sum over spins and other variables of the square of a matrix element M, where M is linear in C_i and C_i'. Hence F is of the form:

$$F = \sum_{i\,j} \{f_{ij}\, C_i^* C_j + f_{ij}'\, C_i'^* C_j' + g_{ij}\, C_i^* C_j' \}$$

$$+ \text{ complex conjugate.}$$

In the old days, (before parity was questioned), only f_{ij} or only f_{ij}' terms were present (just which ones, dependent on the assumed parity of the neutrino with respect to the electron), and all the old experiments agreed with either of these assignments. Why was this so? Note first, that if we assume that the ν has zero mass, then if ψ_ν satisfies the Dirac equation, so does $\gamma_5\, \psi_\nu$, since

$$\gamma_\mu \frac{\delta}{\delta x_\mu} \psi_\nu = 0$$

implies

$$\gamma_\mu \frac{\delta}{\delta x_\mu} \gamma_5\, \psi_\nu = 0 \,.$$

Under the transformation $\gamma_5\, \psi_\nu = \psi_\nu'$ the terms in H involving C_i are interchanged with those involving C_i'. Because of this invariance, a theory with C_i alone or with C_i' alone must predict the

same result for any experiment that does not measure the spin of the ν. Hence $f_{ij} = f'_{ij}$ and we can write F as

$$F = \sum_{ij} \{f_{ij}(C_i^* C_j + C_i'^* C_j') + g_{ij} C_i^* C_j'\}$$

$$+ \text{ complex conjugate.}$$

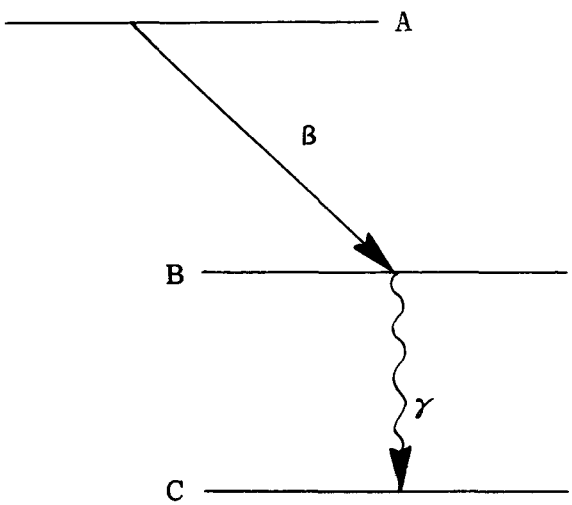
Next we note that the transformation $\vec{r} \to -\vec{r}$, $C_i \to C_i$, $C_i' \to -C_i'$ leaves H invariant, and so must also leave F invariant. This implies that g_{ij} must change sign under this transformation, and thus must be a pseudoscalar. We shall see below that in all the old experiments there never was a possibility of forming a pseudoscalar. Thus g_{ij} was automatically zero, and the predictions of the new theory for those experiments were the same as the predictions of the old theory, with the correspondence

$$(C_i^* C_j + C_i'^* C_j')_{new} \longleftrightarrow (C_i^* C_j)_{old} .$$

This explains why either theory can explain the old data, even though P is not conserved.

Let us now review the old experiments and note that actually no pseudoscalar quantity was measured. The measurement of the electron spectrum gives only $|\vec{p}|$. The electron-neutrino correlation experiments determine only $\vec{p}_e$ and $\vec{p}_\nu$, from which we cannot form a pseudoscalar.

To measure a pseudoscalar quantity, we must observe at least 3 momenta $(\vec{p}_1, \vec{p}_2, \vec{p}_3)$ or a spin and a momentum $(\vec{S}, \vec{p})$. Consider



the following double decay of polarized nuclei A: We measure the distribution of the γ-ray as a function of the angle between $\vec{p}_\gamma$, the direction of the emitted γ-ray, and $\vec{S}_A$ the spin of the polarized nuclei. If the angular distribution contained a term proportional to $\vec{S}_A \cdot \vec{p}_\gamma$, then this would tell us that parity was violated. However, since the γ-rays carry definite parity, the observed probability function must be an even function of $\vec{p}_\gamma$. Thus terms proportional to $\vec{p}_\gamma \cdot \vec{S}_A$ cannot appear, even if parity is violated in β-decay. It can also be shown that the invariance of electromagnetic interactions with respect to time reversal and parity makes it impossible to detect non-conservation of parity by ($\beta - \gamma - \gamma'$) -coincidence experiments. Thus none of the old experiments could provide information as to whether or not parity is conserved.

Part II: Various Tests for Invariance under P, C and T.

We now see the need to look for a new group of experiments which will be suitable to test parity conservation in weak interactions. At the same time we should examine the possibility of non-invariance under C and T, keeping SLP in mind.

Measurement of Pseudoscalar Quantities.

Wu, Ambler, *et al.* measured the β-ray angular distribution from

oriented Cobalt 60 nuclei. They observed a pronounced asymmetry, indicating the presence of a term, $(\vec{S}_A \cdot \vec{P}_e)$. This proves the violation of parity in ß-decay.

Then Frauenfelder at Illinois showed the existence of a correlation between the spin and the momentum of an individual electron emitted in ß-decay. That is, they observed a term $(\vec{S}_e \cdot \vec{P}_e)$, which is even more striking evidence of parity non-conservation. (It has also been discovered that parity is violated in the π - and in the μ-decay. This will be discussed later.)

If we still cling to the requirement of invariance under proper Lorentz transformations in the face of having proved the violation of parity in weak interactions, then SLP implies that invariance under C or T is violated. So we look for experimental information that would place an upper limit on the violation of invariance under C or T.

Equality of Masses and Lifetimes for Particles and Antiparticles.

The equality of the masses and lifetimes of a particle and its antiparticle (for example, π^+ and π^-), might appear to provide evidence for invariance under C. The equality of masses, however, gives very little information, because the mass is brought about aminly by the strong interactions. On the other hand, one might think that the equality of lifetimes to within 1% guarantees invariance under C in weak interactions to 1% since only the weak interactions are involved in the decay. However, we will show that such a conclusion is not justified and indeed that the equality of the masses and lifetimes of particles and antiparticles follows

just from the invariance of the weak interactions under proper Lorentz transformations.

Let us write $H = H_{strong} + H_{weak}$ where H_{strong} includes electromagnetic interactions. We assume that H, H_{strong} and H_{weak} are all invariant under proper Lorentz transformations and that H_{strong} is also invariant under P, C and T, separately. Then SLP implies

$$H = P\ C\ T\ H\ T^{-1}\ C^{-1}\ P^{-1} = P\ C\ U_T\ H^*\ U_T^\dagger\ C^\dagger\ P^\dagger$$

and if we define $0 \equiv PCU_T$ then we have $OH^*O^+ = H$.

We prove two theorems:

<u>Theorem 1</u>: Let A be a stable particle (for example, proton or electron), $\bar{A}$ its antiparticle, m, $\bar{m}$, their respective masses. Then, if we have invariance under proper Lorentz transformations, $m = \bar{m}$, to all orders in the weak coupling.

<u>Proof</u>: From the definition of a stable particle there exist eigenstates, $|A\rangle$, of the total Hamiltonian, H, which satisfy

$$H\ |A\rangle = m\ |A\rangle \tag{2}$$

We may further choose $|A\rangle$ to be an eigenstate, $|A\rangle_{J'_z}$, of J_z.

$$J_z\ |A\rangle_{J'_z} = J'_z\ |A\rangle_{J'_z} \tag{3}$$

Taking the complex conjugate of (2) and then multiplying on the left by 0 we get

$$0\ H^*\ |\ A^*\rangle_{J'_z} = m\ 0\ |\ A^*\rangle_{J'_z}$$

since m is real. Now

$$0\, H^* \mid A^* \rangle_{J'_z} = 0\, H^*\, 0^+\, 0 \mid A^* \rangle_{J'_z} = H\, 0 \mid A^* \rangle_{J'_z} .$$

That is,

$$H\, 0 \mid A^* \rangle_{J'_z} = m\, 0 \mid A^* \rangle_{J'_z}$$

Thus $0 \mid A \rangle_{J'_z} \equiv \phi$ is an eigenfunction of H with the eigenvalue m. Let us examine its properties.

Among the three factors C, P and T of the operation ''0 x complex conjugation'', only C interchanges particles and antiparticles. Hence, ϕ is certainly a state involving $\bar{A}$. Furthermore, $\vec{J}$ is invariant under P and C, and changes sign under T, whence

$$0\, J^*_z\, 0^+ = -\, J_z \quad .$$

From (3) we may then infer

$$J_z \phi = -0 J^*_z 0^\dagger \mid 0\, A^* \rangle_{J'_z} = -0 J^*_z \mid A^* \rangle_{J'_z} = -J'_z\, \phi$$

Thus we see

$$\phi \equiv 0 \mid A^* \rangle_{J'_z} = \mid \bar{A} \rangle_{-J'_z}$$

and

$$H \mid \bar{A} \rangle = m \mid \bar{A} \rangle = \bar{m} \mid \bar{A} \rangle$$

whence $m = \bar{m}$.

This proof applies equally well to arbitrary states of the total Hamiltonian (if it is Hermitian); that is, the complete energy

spectrum is the same for both stable particles and their antiparticles. This is not to say that particles and antiparticles exhibit exactly the same behavior. For this, one would need invariance under charge conjugation, C, which has not been assumed in the foregoing proof.

Theorem 2: Given the following decay processes, governed by the weak interaction, H_{weak}:

$$A \rightarrow B$$

$$\bar{A} \rightarrow \bar{B} \ ;$$

and the condition $B \not\equiv \bar{B}$, where A, B are particles, $\bar{A}$, $\bar{B}$ their antiparticles; then the lifetimes τ, $\bar{\tau}$ of the two processes are the same to lowest order in H_{weak}.

Proof: To lowest order in H_{weak} we may take the particle states, $|\bar{A}\rangle$, $|B\rangle$ to be eigenfunctions of the strong Hamiltonian, H_{strong}. The charge-conjugate states are, of course

$$|\bar{A}\rangle = C \ |A\rangle \ , \ |\bar{B}\rangle = C \ |B\rangle$$

Let us define the two quantities

$$H_{\pm} = 1/2 \ (H_{weak} \pm P \ H_{weak} \ P^{+})$$

Then

$$P^{+} \ H_{\pm} \ P = \pm \ H_{\pm}$$

$$H_{weak} = H_{+} + H_{-}$$

so that H_{+} and H_{-} are, respectively, the parity conserving and

parity non-conserving parts of H_{weak}. Since H_+ and H_- are individually invariant under proper Lorentz transformations, SLP says

$$H_{\pm} = 0\ H_{\pm}^*\ 0^+ = P\ C\ U_T\ H_{\pm}^* U_T^+\ C^+\ P^+$$

or

$$U_T\ H_{\pm}^*\ U_T^+ = C^+\ P^+\ H_{\pm}\ P\ C\ .$$

Therefore, we may write

$$\langle A|H_{\pm}|B\rangle^*_{J'_z} = \langle A^*|U_T^+\ U_T H_{\pm}^* U_T^+\ U_T|B^*\rangle_{J'_z} = \langle A^*|U_T^+\ C^+\ P^+\ H_{\pm} PCU_T|B^*\rangle_{J'_z}$$

Since $|A\rangle$, $|B\rangle$ are eignestates of H_{strong}, which is invariant under T,

$$U_T|B^*\rangle_{J'_z} = |B\rangle_{-J'_z}\ ,\ \langle A^*|_{J'_z}\ U_T^+ = \langle A|_{-J'_z}$$

and

$$\langle A\ |\ H_{\pm}\ |\ B\rangle^*_{J'_z} = \langle A\ |\ C^+\ P^+\ H_{\pm}\ P\ C\ |\ B\rangle_{-J'_z}$$

$$= \langle \bar{A}\ |\ P^+\ H_{\pm}\ P\ |\ \bar{B}\rangle_{-J'_z} = \pm\ \langle \bar{A}\ |\ H_{\pm}\ |\bar{B}\rangle_{-J'_z}\ .$$

Because B and $\bar{B}$ are <u>not</u> identical, the lifetimes of A and $\bar{A}$, to lowest order in H_{weak}, are related to the above matrix elements in a relatively simple way:

$$\tau \propto \Sigma|\langle A\ |\ H_{weak}\ |\ B\rangle|^2 = \Sigma|\langle A|H_+ + H_-\ |\ B\rangle|^2$$

$$\bar{\tau} \propto \Sigma|\langle \bar{A}\ |\ H_{weak}\ |\ \bar{B}\rangle|^2 = \Sigma|\langle \bar{A}|H_+ + H_-\ |\ \bar{B}\rangle|^2$$

But since τ is a scalar quantity, the interference terms between H_+ and H_-, which would be pseudoscalars, must identically vanish. Hence

$$\tau \propto \Sigma\{ |\langle A|H_+ \mid B\rangle|^2 + |\langle A|H_- \mid B\rangle|^2\}$$

$$\bar{\tau} \propto \Sigma\{ |\langle \bar{A}|H_+ \mid \bar{B}\rangle|^2 + |\langle \bar{A}|H_- \mid \bar{B}\rangle|^2\}$$

or $\tau = \bar{\tau}$ to lowest order in H_{weak}.

The above statement can be accurately applied to the physical world since the weak couplings are weak; but we might ask what statement can be made about τ and $\bar{\tau}$ if we include the weak coupling to all orders. In that case the π^+ and π^- produced in the laboratory are not eigenstates of strong interactions and their wave functions may depend on the way they are produced. Furthermore, it is not clear that to all orders of the weak coupling, A and $\bar{A}$ obey a simple exponential law of decay. If we forget about these higher order effects, which is certainly a valid assumption, then we can conclude from theorems 1 and 2, that the equality of masses and lifetimes of a particle and its antiparticle proves only that the theory is invariant under proper Lorentz transformations and that SLP is valid; but we cannot tell whether or not our theory is invariant under C.

LECTURE 3

Possible Tests for Invariance under P, T and C.

We have seen that in order to obtain proof of the violation

of P, it is necessary to measure a pseudoscalar quantity. An example of such a pseudoscalar is $\vec{s} \cdot \vec{p}$. What structure must a quantity have in order that measurement of it give information about time reversal?

We classify all observables 0 into two classes, 0_+ and 0_-, those which are respectively even and odd functions of $\vec{s}$ and $\vec{p}$.

0_+	0_-
even functions of $\vec{s}$, $\vec{p}$	odd functions of $\vec{s}$, $\vec{p}$
$\vec{p}_1 \cdot \vec{p}_2$	$\vec{s} \cdot (\vec{p}_1 \times \vec{p}_2)$
$\vec{s} \cdot \vec{p}$	$\vec{p}_1 \cdot (\vec{p}_2 \times \vec{p}_3)$
	$\vec{s}_1 \cdot (\vec{s}_2 \times \vec{s}_3)$

Under time reversal, we have seen that, classically, $t \to -t$, $\vec{p} \to -\vec{p}$, $\vec{s} \to -\vec{s}$ so that $0_\pm \to \pm 0_\pm$. Quantum-mechanically, as seen in Lecture 1, $T \vec{p} T^{-1} = -\vec{p}$, $T \vec{s} T^{-1} = -\vec{s}$, so that $0_\pm \to T0_\pm T^{-1} = \pm 0_\pm$, in this case as well. Our classification is thus also one of evenness and oddness under T. Naively, we might think that the detection of a quantity 0_- would imply non-invariance under time reversal. However, this is incorrect; indeed, such a quantity has been observed, namely the polarization of protons perpendicular to the scattering plane in p-p scattering. One is here measuring a term of the form $\vec{s} \cdot (\vec{p}_1 \times \vec{p}_2)$ which is in the group 0_-. However, this scattering involves strong interac-

actions only, which are known to be invariant under time reversal.

To determine just what are the rules for detecting invariance under time reversal, we must go into greater detail. Let us decompose the weak Hamiltonian into several terms H_i:

$$H_W = \sum_i g_i H_i \tag{1}$$

where each H_i is a single product (monomial) of single fields. Under time reversal, such a monomial is multiplied by a phase factor. Let us absorb all such phase factors into the g_i so that

$$T\, H_i\, T^{-1} = U_T H_i^{*} U_T^{-1} = H_i \tag{2}$$

and

$$T\, H_W T^{-1} = \Sigma\, g_i^{*} H_i \ .$$

With the conditions (1) and (2) on H_W, therefore, we may say that if H_W is invariant under time reversal, then $g_i = g_i^{*}$, and conversely. Thus, in this representation we need examine only the reality properties of g_i to determine if time invariance is valid.

Dynamics of Decay Processes.

We shall investigate the dynamics of decay processes with the intention of seeing how observable quantities may be expressed in terms of the g_i. We shall relate the properties of the g_i to invariance under P, C and T and thus we shall be able to obtain direct information about these operators from the measurement of observables.

Let us observe the decay $A \rightarrow B$ by measuring the momenta and spins $(\vec{p}_i, \vec{s}_i)$ of the particles involved in the final state B. We let A and B be eigenstates of the strong interaction, H_s. These are not necessarily plane wave states. We then consider two cases.

Case 1. B is a state for which there are no strong interactions (for example, $\pi \rightarrow \mu + \nu$, $\mu \rightarrow e + 2\nu$). Then let $|\vec{p}_i, \vec{s}_i\rangle$, or more simply, $|p,s\rangle$ denote plane wave eigenstates for the particles of B.

From first order, time independent perturbation theory, it follows that, if $|A\rangle$ is non-degenerate, then

$$|B\rangle_{J_z'} = \sum_{ps} \langle p,s|H_W|A\rangle_{J_z'} |p,s\rangle.$$

To first order, $|B\rangle$ is an eigenstate of the Hamiltonian that includes the decay interaction. If H_W is as given in (1), we may write

$$\langle p,s|H_W|A\rangle_{J_z'} = \sum_i g_i M_i(p,s;J_z'),$$

where

$$M_i(p,s;J_z') = \langle p,s|H_i|A\rangle_{J_z'} .$$

We will show that $M_i^*(p,s;J_z') = M_i(-p,-s;-J_z')$. We have

$$M_i^*(p,s;J_z') = \langle p,s^*|H_i^*|A^*\rangle_{J_z'} = \langle ps^*|U_T^+ U_T H_i^* U_T^+ U_T|A^*\rangle_{J_z'}$$

$$= \langle ps|T^{-1}TH_iT^{-1}T|A\rangle_{J_z'} .$$

Since $|p,s\rangle$ and $|A\rangle_{J_z'}$ are both eigenstates of T-invariant, strong Hamiltonians, the operation of T must simply reverse momentum and spin. That is $\langle p,s|T^{-1} = \langle -p,-s|$ and $T|A\rangle_{J_z'} = |A\rangle_{-J_z'}$. Hence

$$M_i^*(ps;J'_z) = \langle -p,-s|H_iA\rangle_{-J'_z} = M_i(-p,-s;-J'_z). \qquad (3)$$

We have $|B\rangle_{J'_z} = \sum_{psi} g_i M_i(p,s;J'_z)|p,s\rangle$.

When we measure an observable 0, we obtain a probability distribution, from which we may infer the expectation value

$$\langle B|0|B\rangle = \sum_{pp'ss'ijJ'_z} g_i g_j^* M_i(p,s;J'_z) M_j^*(p',s';J'_z)\langle p',s'|0|p,s\rangle .$$

If 0 involves only spin and momenta, then in the spin-momentum representation, 0 is diagonal. That is, $p = p'$, $s = s'$. Furthermore, 0 is either even or odd in p,s according to the classification into 0_+ and 0_-. Hence

$$\langle p,s|0_\pm|p,s\rangle = \pm \langle -p,-s|0_\pm| -p,-s\rangle. \qquad (4)$$

Using (3), we have

$$\langle B|0|B\rangle = \sum_{psijJ'_z} g_i g_j^* M_i(p,s;J'_z) M_j(-p,-s;-J'_z)\langle p,s|0|p,s\rangle = \sum_{ij} g_i g_j^* 0_{ij}$$

where $0_{ij} = \sum_{p,s,J'_z} M_i(p,s;J'_z) M_j(-p,-s;-J'_z) \langle p,s|0|p,s\rangle$.

It follows from (4) that $(0_\pm)_{ij} = \pm (0_\pm)_{ji}$. Hence, using the notation $\langle B|0|B\rangle = \langle 0\rangle$, we may re-write this as

$$\langle 0_+\rangle = 1/2 \sum_{i,j} (g_i g_j^* + g_i^* g_j) 0_{ij}$$

$$\langle 0_-\rangle = 1/2 \sum_{i,j} (g_i g_j^* - g_i^* g_j) 0_{ij} .$$

We see that if the theory is invariant under time reversal, then g_i will be real and $\langle 0- \rangle$ will be zero. On the other hand, if we observe a term $\langle 0_- \rangle$, then g_i cannot be real and the theory is not invariant under T. Hence, in the absence of strong interactions in the final state, our naive notions of the relation between $\langle 0_- \rangle$ and invariance under T are valid.

<u>Case 2</u>. The observed final state B is under the influence of strong interactions, including electromagnetic interactions (for example, $n \rightarrow p^+ + e^- + \nu$, $\Lambda^o \rightarrow p^+ + \pi^-$).

In this case, let $|b\rangle$ be the standing wave eigenfunction of the strong Hamiltonian. It can be decomposed into an ingoing part $|b^{in}\rangle$ and an outgoing part $|b^{out}\rangle$:

$$|b^{in}\rangle = |b^{in}\rangle + |b^{out}\rangle$$

(see example below). The wave packets we observe with our apparatus at infinity contain only outgoing waves because the contributions from $|b^{in}\rangle$ interfere destructively at infinity. Hence the state that will give the proper expectation values of the operator 0 is

$$|B\rangle_{J'_z} = \sum_b |b^{out}\rangle \langle b|H|A\rangle_{J'_z}$$

$$= \sum_{bps} \langle p,s|b^{out}\rangle_{J'_z} \langle b|H_W|A\rangle_{J'_z} |p,s\rangle.$$

For example, in the coordinate representation, the S-wave part of $|b\rangle$ at large distances may be written

$$\langle r|b\rangle = \frac{\sin\,(kr + \eta_b)}{r} = \frac{e^{i(kr + \eta_b)} - e^{-i(kr + \eta_b)}}{2ir}$$

where η_b is the S-wave scattering phase shift characterizing the strong interaction. Then

$$\langle r|b^{out}\rangle = \frac{e^{ikr}}{r}\, e^{i\eta_b} .$$

In the momentum representation we would have

$$\langle p,s|b^{out}\rangle = \frac{e^{i\eta_b}}{p^2-k^2} .$$

Apart from the phase factor $e^{i\eta_b}$, we see that there would be no difference in this matrix element for b^{out} and for b^{out*}.

If we write $|b^{out}\rangle = |b^{\overline{out}}\rangle\, e^{i\eta_b}$ we have, for general angular momentum states

$$\langle p,s \mid b^{\overline{out}}\rangle^*_{J'_z} = \langle p,s^*|b^{\overline{out}}\rangle_{J'_z} = \langle p,s^*|U_T^{+}\, U_T|b^{\overline{out}*}\rangle_{J'_z}$$

$$= \langle -p,-s|b^{\overline{out}}\rangle_{-J'_z} .$$

Also $\langle b|H_i|A\rangle^*_{J'_z} = \langle b^*|H_i{}^*|A^*\rangle_{J'_z} = \langle b^*|U_T^{+}U_T H_i{}^* U_T^{+} U_T|A^*\rangle_{J'_z}$

$$= \langle Tb|TH_i T^{-1}|TA\rangle_{J'_z} = \langle b|H_i|A\rangle_{-J'_z} ,$$

since $|A\rangle$ and $\langle b|$ are both eigenstates of the Hamiltonian. Thus if we write

$$|B\rangle_{J'_z} = \sum_{bpsi} g_i e^{i\eta_b}\, M_{ib}(p,s;J'_z)\,|p,s\rangle$$

where $M_{ib}\ (p,s;J_z') = \langle p,s \mid b^{\overline{out}}\rangle_{J'_z} \langle b|H_i|A\rangle_{J_z'}$

then we have just shown $M_{ib}{*}(p,s;J_z') = M_{ib}(-p,-s;-J_z')$.

The expectation value of the observable in the state B is then

$$\langle B|0|B\rangle = \sum_{\substack{psib\\ psjb'\ J_z'}} g_i g_j{*} e^{i\eta_b} e^{-i\eta_{b'}} M_{ib}(p,s;J_z')M^*_{jb'}(p',s';J_z')\langle p',s'|0|p,s$$

As in Case 1, we have $\langle p',s'|0_{\pm}|p,s\rangle = \pm\ \langle -p,-s|0_{\pm}|-p,-s\rangle\ \delta_{pp'}\ \delta_{ss'}$.

Thus

$$\langle 0_{\pm}\rangle = \sum_{jpsibb'J_z'} g_i g_j{*} e^{i(\eta_b-\eta_{b'})} M_{ib}(p,s;J_z')M_{jb'}(-p,-s;-J_z')\langle p,s|0_{\pm}|p,s\rangle$$

$$= \sum_{ijbb'} g_i g_j{*} e^{i(\eta_b-\eta_{b'})} (0_{\pm})_{ijbb'}$$

where $(0_{\pm})_{ijbb'} = \sum_{psJ_z'} M_{ib}(p,s;J_z')M_{jb'}(-p,-s;-J_z')\langle p,s|0_{\pm}|p,s\rangle$.

Now $(0_{\pm})_{ijbb'} = \pm\ (0_{\pm})_{jib'b} = \pm\ (0_{\pm})^*_{ijbb'}$,

so that $\langle 0_{\pm}\rangle = 1/2 \sum_{ijbb'} g_i g_j{*} e^{i(\eta_b-\eta_{b'})} \pm g_i{*} g_j e^{-i(\eta_b-\eta_{b'})}\ 0_{ijbb'}$.

Hence $\langle 0_+\rangle = \sum_{ijbb'} \left\{ \mathrm{Re}\ g_i g_j{*}\cos\ (\eta_b-\eta_{b'}) - \mathrm{Im}\ g_i g_j{*}\sin\ (\eta_b-\eta_{b'})\right\}$

$$\cdot\ 0_{ijbb'}$$

$$\langle 0_-\rangle = i \sum_{ijbb'} \left\{\mathrm{Re}\ g_i g_j{*}\sin(\eta_b-\eta_{b'}) + \mathrm{Im}\ g_i g_j{*}\cos\ (\eta_b-\eta_{b'})\right\} 0_{ijbb'} \tag{5}$$

or $\langle 0_+\rangle = A_+ + B_+$

and $\langle 0_-\rangle = A_- + B_-$

where we have denoted by $A_{\pm}$ the terms in $\langle 0_{\pm} \rangle$ involving Re $g_i g_j{}^*$ and by $B_{\pm}$ the terms in $\langle 0_{\pm} \rangle$ involving Im$g_i g_j{}^*$.

<u>The Rules for Testing Invariance Under T, C and P.</u>

To conclude that time invariance is violated we must observe either B_+ or B_-. The appearance of A_-, due to the presence of strong interactions, shows that we can measure $\langle 0_- \rangle \neq 0$, even when we have invariance under T. This explains the previously mentioned result of measuring $0_- = (\vec{p}_1 \times \vec{p}_2) \cdot \vec{s}$, the polarization of scattered protons. Only if there is no final state interaction, can we say that the observation of $\langle 0_- \rangle$ implies non-invariance under T.

In what follows, let us assume that the ith term in the expansion of H, (that is, $g_i H_i$), is of either even or odd parity. We may then re-write H as $H = \sum_i g_i{}^{(+)} H_i{}^{(+)} + \sum_k g_k{}^{(-)} H_k{}^{(-)}$ + hermitian conjugate, where the $(\pm)$ refer to $\binom{\text{even}}{\text{odd}}$ parity parts.

Before proceeding with the consideration of $\langle 0 \rangle$, we note that invariance under C, together with our earlier assumptions and SLP, would require that $g_i{}^{(+)}$ be pure real while $g_i{}^{(-)}$ be pure imaginary. This follows from the fact that we have, effectively

$$T: \quad (g_i{}^{(+)}, g_k{}^{(-)}) \rightarrow (g_i{}^{(+)*}, g_k{}^{(-)*})$$

$$P: \quad (g_i{}^{(+)}, g_k{}^{(-)}) \rightarrow (g_i{}^{(+)}, -g_k{}^{(-)})$$

while $TP: \quad (g_i{}^{(+)}, g_k{}^{(-)}) \rightarrow (g_i{}^{(+)*}, -g_k{}^{(-)*}) = (g_i{}^{(+)}, g_k{}^{(-)})$

by SLP, if C is invariant. Hence, we may state that the non-vanishing of any of the quantities

$$\mathrm{Im}(g_i^{(+)} g_j^{(+)*}),\ \mathrm{Im}(g_k^{(-)} g_l^{(-)*}),\ \mathrm{Re}(g_i^{(+)} g_k^{(-)*}),$$

would imply non-invariance under C (and also under T, of course, in the first two cases; and under P in the latter).

Let us make a further analysis of $0_{\pm}$ into parts of even and odd parity. It is easy to show that even parity operators, 0, have expectation values, $\langle 0 \rangle$, which involve only the products $g_i^{(+)} g_j^{(+)*}$, $g_k^{(-)} g_l^{(-)*}$; while $\langle 0 \rangle$ for an odd parity operator, 0, involves only $g_i^{(+)} g_k^{(-)*}$.

For even parity 0, therefore, detection of the A terms in (5) tells us nothing direct about P, C, T; but detection of the B terms tells us that C, as well as T invariance is violated. Nothing is learned about P using even parity 0, a fact which we have already seen in a previous lecture. For odd parity 0, on the other hand, detection of the A terms violates both C and P; detection of the B terms violates both T and P.

We summarize these conclusions in a table, in which the notation is self-explanatory:

	A	B
1. Even parity 0	No conclusions	T, C violated
2. Odd parity 0	C, P violated	P, T violated

We can tell if $\langle 0 \rangle$ contains A or B, by noting first whether 0 is of the type 0_+ or 0_- and then by noting whether $\langle 0 \rangle$ is proportional to sin $(\eta_n - \eta_{b'})$ or to cos $(\eta_b - \eta_{b'})$.

Application of the Rules to Specific Experimental Examples.

We illustrate these rules with particular examples.

(1a). $\pi \rightarrow \mu + \nu$. (1b). $\mu \rightarrow e + \nu + \nu$.

Suppose that in (a) we find experimentally, $\langle \vec{s}_\mu \cdot \vec{p}_\mu \rangle$. This violates parity and hence belongs to the second line of the Table. Also, since $\vec{s}_\mu \cdot \vec{p}_\mu$ is 0_+, and since there is no final state interaction, there are only A terms in the expectation value. Hence, both C and P are violated.

A similar argument applies in (b). We note that since there is no final state interaction in these decays, we must measure an operator of the odd type, 0_-, in order to detect violation of T, since only B contains conclusions which demand violation of T. The measurement of $\vec{s}_\nu \cdot (\vec{s}_\mu \times \vec{s}_e)$ in (b), would thus serve as a means for detection of violation of T. It is not likely, however, that such an experiment will be carried out in the foreseeable future.

(2). β-Decay.

Consider first, electron-neutrino angular correlation experiments in which we measure $0_+ = (\vec{p}_e \cdot \vec{p}_\nu)$. Since η is determined by the Coulomb interaction, we can, for small Z, replace cos η by 1, while sin η is proportional to Z. Since $(\vec{p}_e \cdot \vec{p}_\nu)$ is of

even parity, that part of $\langle \vec{p}_e \cdot \vec{p}_\nu \rangle$ which is independent of Z is in class 1A and the part linear in Z is in class 1B.

Next, consider experiments measuring the angular distribution of electrons from β-decay of polarized nuclei in which we measure $(\vec{s}_A \cdot \vec{p}_e)$, an 0_+ odd parity operator. In this case, therefore, the Z independent part of $\langle \vec{s}_A \cdot \vec{p}_e \rangle$ belongs to 2A and the part linear in Z belongs to 2B.

Let us look into the possibility of making these experiments. In Table I are exhibited the spectra, the e - ν angular correlation function and the β-angular distribution for polarized nuclei, expressed as functions of the various coupling constants C_i and of Z. Since the function $F(Z,W) \rightarrow 1$ for small Z, we see specifically the $\langle \vec{p}_e \cdot \vec{p}_\nu \rangle$ and $\langle \vec{s}_A \cdot \vec{p}_e \rangle$ must be linear functions of Z for small Z.

The quantities being determined in the above cases are collected in the following chart:

	A	B
1	Part of $\langle \vec{p}_e \cdot \vec{p}_\nu \rangle$ independent of Z. $\propto$ ap	Part of $\langle \vec{p}_e \cdot \vec{p}_\nu \rangle$ linear in Z $\propto (a' e^2 / \hbar c) Z$ T, C violated
2	Part of $\langle \vec{s}_A \cdot \vec{p}_e \rangle$ independent of Z $\propto \mathrm{Re}(C_T C_T'^* - C_A C_A'^*)$ C, P violated	Part of $\langle \vec{s}_A \cdot \vec{p}_e \rangle$ linear in Z $\propto \frac{e^2 Z}{\hbar c p} \mathrm{Im}\,(C_A C_T'^* + C_A' C_T^*)$ P, T violated

TABLE I

β-decay:

$$H = \sum_i (\psi_p^+ O_i \psi_n)\{C_i(\psi_e^\dagger O_i \psi_\nu) + C_i'(\psi_e^\dagger O_i \gamma_5 \psi_\nu)\}$$

$$N(W,\theta)dW\ d\cos\theta = \frac{\xi}{4\pi^3} F(Z,W)pW(W_o - W)^2[1 + \frac{1}{W}(ap + a'(\frac{e^2}{\hbar c})Z)\cos\theta + \frac{b}{W}]dW\ d\cos\theta$$

where,

$$\xi = (|C_S|^2 + |C_V|^2 + |C_S'|^2 + |C_V|^2)|M_F|^2 + (C_T|^2 + |C_A|^2 + C_T'|^2 + |C_A'|^2)|M_{GT}|^2$$

$$a\xi = \frac{1}{3}(|C_T|^2 - |C_A|^2 + |C_T'|^2 - |C_A'|^2)|M_{GT}|^2 - (|C_S|^2 - |C_V|^2 + |C_S'|^2 - |C_V'|^2)|M_F|^2$$

$$a'\xi = \frac{i}{3}(C_A C_T^* - C_T C_A^* + C_A' C_T'^* - C_T' C_A')|M_{GT}|^2 + i(C_S C_V^* - C_S^* C_V + C_S' C_V'^* - C_S'^* C_V')|M_F|^2$$

$$b\xi = \gamma(C_S^* C_V + C_S C_V^* + C_S'^* C_V' + C_S' C_V'^*)|M_F|^2$$
$$+\gamma(C_T^* C_A + C_A^* C_T + C_T'^* C_A' + C'_A{}^* C_T')|M_{GT}|^2$$

β-angular distribution from polarized nuclei:

$$N(\theta)\ d\cos\theta = (1 + \alpha\cos\theta)\ d\cos\theta$$

For $J \rightarrow J - 1$ (no) $\qquad \alpha = \beta \langle J_z / J \rangle$

For $J \rightarrow J + 1$ (no) $\qquad \alpha = -\beta \langle J_z / J + 1 \rangle$

$$\beta = \mathrm{Re}[C_T C_T'^* - C_A C_A'^* + \frac{ie^2 Z}{\hbar cp}(C_A C_T'^* + C_A' C_T^*)]|M_{GT}|^2 \times \left(\frac{v_e}{c}\right) \frac{2}{\xi + (\xi b / W)}$$

$$\gamma = \sqrt{1 - \left(\frac{e^2 Z}{\hbar c}\right)^2}, \qquad y = \frac{e^2 ZW}{\hbar cp}, \qquad m_e c^2 = 1,$$

$$F(Z,W) = 2(1 + \gamma)(2pR)^{2(\gamma - 1)} \exp\{\pi y \frac{|\Gamma(\gamma + iy)|^2}{|\Gamma(2\gamma + 1)|^2}\}$$

The term a p can be detected, but it serves only to distinguish the relative sizes of C_T and C_A, and of C_S and C_V. It is present in the conventional theory and, in any case, gives no information on violation of C, P or T invariance.

The term, $a' e^2 Z / \hbar c$, could be detected by measuring the e - ν angular correlation as a function of the energy of the emitted electron. Experiments of this kind have been performed but they do not as yet permit us to draw definite conclusions.

The results of the experiments on the β-angular distribution from Co^{60} and Co^{58} nuclei are consistent with the assignment $C_A = C_A' = 0$ and $C_T = -C_T'$. The Z independent part of $\langle \vec{s}_A \cdot \vec{p}_e \rangle$ was detected, thus implying that C and P are not invariant.

The term $(e^2 Z / \hbar c p)(C_A C_T'^* + C_T^* C_A')$ is expected to be very small for a relatively light nucleus such as Co^{60}, since C_A and C_A' are known to be small. To detect it, we should either observe angular distribution of β-radiation from oriented nuclei of higher Z, or measure the velocity dependence of the angular distribution.

From the above experiments we are able to conclude that C and P are violated, and that, at present, the experiments are consistent with invariance under T, although there really have not been any rigid tests for this. We may then ask: What would be necessary and sufficient condition for determining whether or not T is invariant? Since it is difficult to measure the Z dependent terms, upon which the test depends when we use operators of the type O_+; we must look instead for a suitable O_-. If its expectation value is found to have a non-vanishing Z-independent part, then non-invariance under T will be proved.

An example of such a quantity is $0_- = \vec{s}_A \cdot \vec{p}_1 \times \vec{p}_2$, which occurs in recoil experiments from polarized nuclei, as pointed out by Jackson, Treiman and Wyld. Here we find that in the $(J \rightarrow J)$ transitions, with Z small, the coefficient of

$$\frac{\langle J_z \rangle}{J} \frac{\langle \vec{p}_e \times \vec{p}_\nu \rangle}{W_e W_\nu} \text{ is}$$

$$2|M_F| \; |M_{GT}| \sqrt{\frac{J}{J+1}} \; \mathrm{Im}(C_S C_T^* - C_V C_A^* + C_S' C_T'^* - C_V' C_A'^*)$$

$$\cdot \frac{\xi}{(2\pi)^5} p_e W_e (W_o - W_e) \, dW_e d\Omega_e d\Omega_\nu \; .$$

If this experiment should give a positive result we certainly could conclude that T is violated. On the other hand, since C_S, C_T are known to be large relative to C_A, C_V (see below), the absence of such a term would strongly indicate that C_T and C_S are real. This experiment in itself, of course, does not guarantee that T is invariant. We shall see later, however, that this conclusion can be drawn in the case of the two component theory of the neutrino.

Now we might ask: How do all these considerations affect earlier conclusions regarding the values of C_A, C_V, C_T, C_S? The Fierz interference between C_S and C_V terms for Fermi transitions and between C_A and C_T terms for Gamow-Teller selection rules lead to a term b / W in the electron energy spectrum. The absence of such

a term in the observed spectra for Fermi or Gamow-Teller transitions allowed one to conclude, within the framework of the old theory, that $C_A C_T = C_V C_S = 0$. Furthermore, the H_e^6 recoil experiment led to the conclusion $C_A < C_T$, whence $C_A = 0$. A similar experiment involving Fermi selection rules gave $C_V < C_S$, whence $C_V = 0$.

On the other hand, in the new theory, the absence of a b / W term allows one to conclude only

$$\mathrm{Re}(C_T{}^*C_A + C_T'{}^*C_A') = 0 = \mathrm{Re}(C_S^*C_V + C_S'{}^*C_V').$$

The He^6 recoil experiment, which gave a $< 1/3$, now implies only

$$|C_A|^2 + C_A'|^2 \lesssim 1/3\ (|C_T|^2 + |C_T'|^2).$$

We can no longer conclude $C_A = 0$. Similarly we are allowed to conclude only $C_S > C_V$, but not $C_V = 0$.

LECTURE 4

The Two Component Theory of the Neutrino.

This is a theory which allows us to make more definite predictions on β-decay and on the decay of μ-mesons. All of these predictions have been, so far, consistent with experiment.

In 1929, H. Weyl proposed this theory to describe a particle of mass 0 and spin 1/2. It was discussed in Pauli's Handbuch article, and was discarded there because of its non-invariance under space inversion. In view of the recent experimental results, this non-

invariance is, today, an attractive feature of the theory.

Whereas in the conventional Dirac theory we need four anti-commuting matrices, now we require only the three 2 x 2 Pauli matrices, σ_i (i = 1, 2, 3). The wave equation is

$$(\vec{\sigma} \cdot \vec{p})\varphi = i\,\varphi \tag{1}$$

It is easy to show, using arguments similar to those for the Dirac equation, that this theory can be made invariant under proper Lorentz transformations, by a suitable choice of the transformation properties of φ. The operator $\vec{\sigma}$ has the usual interpretation of twice the spin angular momentum.

The energy of our particle is $E = \vec{\sigma} \cdot \vec{p}$. Taking $\vec{p}$ along the z-axis, we obtain

$$E = |p|\sigma_z \,. \tag{2}$$

Consequently, in a one-particle theory the eigenvalues of energy are

$$E = \pm|p| \,.$$

We see from (2) that states of a given sign of energy correspond to a definite orientation of the spin with respect to the direction of motion.

We shall use the following conventions: The state with E + +p will be called a neutrino state; in this state the relationship between spin and direction of motion will be represented by a right-handed screw. The state with E = -p is represented by a left-handed screw. Graphically,

$E = p$: $\equiv \nu$ ν is R

$E = -p$: $\equiv \bar{\nu}$ $\bar{\nu}$ is L

where R and L stand for right-handed and left-handed, respectively.

The hole representing the anti-neutrino will then have opposite momentum and opposite spin to the negative energy state which is vacated. Hence the relation of the spin direction to the momentum of the anti-neutrino is likewise represented by a left-handed screw.

The neutrino that we have defined has no definite parity. For if the operation P, (particle $\rightarrow$ particle, $\vec{r} \rightarrow -\vec{r}$ and $\vec{p} \rightarrow -\vec{p}$), is applied to a neutrino state we would obtain a state with a left-handed neutrino. But no such state exists for a neutrino. Likewise, the operation C, (particle $\rightarrow$ antiparticle, $\vec{r} \rightarrow \vec{r}$ and $\vec{p} \rightarrow \vec{p}$), takes a neutrino state into a state corresponding to a right-handed anti-neutrino, which again does not exist. However, the combined operation CP takes a right-handed neutrino into a left-handed anti-neutrino and hence is a meaningful symmetry operation. The product CP is then conserved for free neutrinos. Whether or not it will be conserved in a theory with interactions depends on the relative phases of the coupling constants. (See Lecture 3).

Irrespective of interactions, the mass of a two-component neutrino must be 0 since it is impossible to include in Equation (1) a term describing a non-zero rest mass and still preserve

Lorentz invariance. For if the neutrino is moving with velocity v in a given frame, ($v < c$, since $m \neq 0$), and if we observe the neutrino from a system moving with velocity $v' > v$, then in this system the momentum is reversed, while the spin remains in the same direction. This again is a non-existent state of the two component theory.

The two component neutrino theory is equivalent to a conventional Dirac theory in which some constraints have been imposed on the wave functions. The Dirac wave equation can be written

$$(\vec{\alpha} \cdot \vec{p})\ \psi = -\ \psi \qquad (3)$$

where $\alpha = \rho,\ \sigma \qquad \gamma_5 = -\rho_1$.
The constraint $\gamma_5\ \psi = -\ \psi$ transforms (3) into (1). Consequently, the β-decay in the two component theory can be obtained from the general form $\sum_i (\psi_p^\dagger\ O_i\ \psi_n)\ \{C_i\ (\psi_e^\dagger\ O_i\ \psi_\nu) + C_i'(\psi_e^\dagger\ O_i\ \gamma_5\ \psi_\nu)\}$ of the decay interaction, by setting $C_i' = -\ C_i$.

<u>Electron Angular Distribution from Polarized Cobalt 60 Nuclei.</u>

The transition $Co^{60} \rightarrow Ni^{60} + e^- + (\nu \text{ or } \bar{\nu})$ is an allowed Gamow-Teller transition: $J = 5 \rightarrow J = 4$ (no). The electron angular distribution in allowed transitions is linear in $\cos\theta$ (see Lecture 3): $D(\theta)d\Omega = 1\ /\ 4\pi\ (1 + \alpha \cos\theta)d\Omega,\ \theta = \measuredangle(\vec{J},\vec{p}_e)$.

Experimentally, Wu, Ambler, Hayward, Hoppes and Hudson determined $\alpha \approx -0.4$. Thus there was pronounced asymmetry favoring emission of the electron in the ''backward'' direction. (By ''forward'' and ''backward'' directions with respect to the spin vector, $\vec{J}$, we shall mean, respectively parallel or anti-parallel to $\vec{J}$ in a

right-handed coordinate system. It might be simpler to think of all angular momenta, including spins, as directed current loops, , rather than as directed vectors . Only the former has an invariant physical significance, independent of the coordinate system.)

Assuming the two-component theory to be valid, we would like to know whether the emitted neutrino is right-handed (= ν, by our convention) or left-handed ($\bar{\nu}$). In order to see how this can be determined, let us first, for the sake of simplicity, adopt an extreme relativistic approximation for the electron: $v_e/c = 1$. The electron mass may then be taken as zero so that γ_5 commutes with the effective Hamiltonian, $\vec{\alpha} \cdot \vec{p}$, in the Dirac equation for the electron field, ψ_e. Hence, ψ_e may be divided into ''right'' and ''left'' parts, ψ_R and ψ_L:

$$\psi_e = \psi_R + \psi_L; \quad \gamma_5 \psi_R = -\psi_R; \quad \gamma_5 \psi_L = +\psi_L,$$

each of which satisfies the equation for ψ_e.

If an anti-neutrino is created in the β-decay, then the matrix element for the creation of $\bar{\nu}$ and e^- is $1/2(\psi_e^+ \, 0_i (1 - \gamma_5) \, \psi_\nu)$

$$= \begin{cases} 1/2(\psi_e^\dagger \, (1 - \gamma_5) 0_i \, \psi_\nu) = (\psi_R^+ \, 0_i \, \psi_\nu) & i = V, A \\ 1/2(\psi_e^\dagger \, (1 + \gamma_5) 0_i \, \psi_\nu) = (\psi_L^\dagger \, 0_i \, \psi_\nu) & i = S, T, P. \end{cases}$$

This is so because $\gamma_5 \, 0_i = 0_i \, \gamma_5$ $\quad i = V, A$

$\gamma_5 \, 0_i = - 0_i \, \gamma_5$ $\quad i = S, T, P.$

For example $\gamma_5 O_v = \gamma_5\gamma_4\gamma_\mu = -\gamma_4\gamma_5\gamma_\mu = O_v\gamma_5$

$$\gamma_5 O_s = \gamma_5\gamma_4 = -\gamma_4\gamma_5 = -O_s\gamma_5 \ .$$

If $(v/c)_e < 1$, $(m_e \neq 0)$, then ψ_L and ψ_R are not eigenstates of the Hamiltonian. Upon projecting the final states of the electron onto eigenstates of $\vec{\sigma} \cdot \vec{p}$, and evaluating the various traces, we would find

polarization = (-v/c) for S, T, P and (+v/c) for V, A.

Since we know the β-interaction to be predominantly S and T, we expect the emitted electrons in β-decay to have a polarization of -v/c, (predominantly left-handed), if the $\bar{\nu}$ is indeed associated with such decay. (This result is completely independent of the nuclear properties). Any deviation from this polarization would show that there must be some V or A mixed in with the S and T. The electron scattering experiments of Frauenfelder found that electrons from Co^{60} are indeed predominantly left-handed, thus giving proof of $Co^{60} \rightarrow Ni^{60} + e^- + \bar{\nu}$, that is, $n \rightarrow p + e^- + \bar{\nu}$.

Now because the $\bar{\nu}$ and e^- are both left-handed, they must both be emitted in the backward direction in an allowed $\Delta J_z \equiv (J_z)_{initial} - (J_z)_{final} = +1$ transition. Consider a special case where $\vec{p}_e$ and $\vec{p}_{\bar{\nu}}$ are along the $\pm$ z axis. Since the light particles carry no orbital angular momentum along the z direction, and since total angular momentum must be conserved, both $\vec{p}_e$ and $\vec{p}_{\bar{\nu}}$ must be parallel to the -z axis. (See Diagram A). Indeed such a backward asymmetry was observed in Co^{60} which is a $J \rightarrow J$ (no) transition, the sam

Similarly for a β^- decay in a $J \rightarrow J$ (no) transition, the same backward asymmetry should be observed if only tensor coupling is important (that is $|M_F| \cong 0$). The magnitude would be reduced because $\Delta J_z = 0, +1$ even for a completely oriented nucleus. For a $J \rightarrow J + 1$ (no) β^- transition the asymmetry is expected to be forward since $\Delta J_z = -1$ for a completely oriented nucleus.

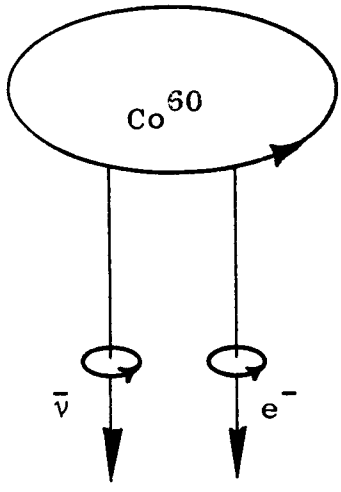


Diagram A

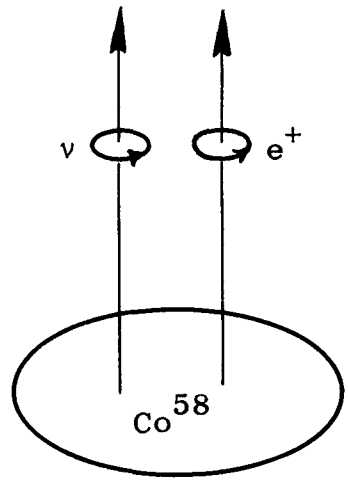


Diagram B

Since positron emission proceeds through the Hermitian conjugate terms in the interaction: $(\psi_n^+ O_i^+ \psi_p)(\psi_\nu^+(1 - \gamma_5)O_i^+ \psi_e)$, one would expect it to be associated with emission of a ν. An analysis similar to the above would then show that the e^+ should be right-handed in the extreme relativistic limit with S, T or P

coupling; or if $v_e/c < 1$, with S and T coupling, e^+ should have a polarization $(+v_e/c)$. That is, we expect $p \rightarrow n + e_R^+ + \nu_R$. In a $J \rightarrow J - 1$ (no) or $J \rightarrow J$ (no) ($|M_F| << |M_{GT}|$ β^+-transition with oriented nuclei, therefore, we would expect to have emission in the forward direction. (See Diagram B). Such an effect was observed in the Co^{58} $J = 2 \rightarrow J = 2$ transition, although it was not as pronounced as in Co^{60}. (The transition $\Delta J_z = 0$ introduces no asymmetry while $\Delta J_z = +1$ gives the forward asymmetry).

<u>μ Decay in the Two-Component Theory</u>

$$\mu^{\pm} \rightarrow e^{\pm} + \nu + \bar{\nu}$$

The interaction Hamiltonian for μ^- decay may be written with just two terms

$$H = g_s(\psi_e^\dagger O_s \psi_\nu)(\psi_\nu^\dagger O_s \psi_\mu) + g_v(\psi_e^\dagger O_v \psi_\nu)(\psi_\nu^\dagger O_v \psi_\mu).$$

μ^+ decay is described by the Hermitian conjugate Hamiltonian. ψ_e and ψ_μ are the e^- and μ^- spinors, respectively. Because of the constraint $(\gamma_5 + 1)\psi_\nu = 0$, the A and P terms are identical with the V and S terms. The T term is identically zero. For consider the (12) component:

$$(\psi_e^\dagger \gamma_4\gamma_1\gamma_2\psi_\nu)(\psi_\nu^\dagger \gamma_4\gamma_1\gamma_2\psi_\mu) = (\psi_e^\dagger \gamma_4\gamma_1\gamma_2\gamma_1\gamma_2\gamma_3\gamma_4\psi_\nu)(\psi_\nu^\dagger \gamma_1\gamma_2\gamma_3\gamma_4\gamma_4\gamma_1\gamma_2\psi_\mu)$$

$$= -(\psi_e^\dagger \gamma_3\psi_\nu)(\psi_\nu^\dagger \gamma_3 \psi_\mu) = -(\psi_e^\dagger \gamma_4\gamma_4\gamma_3\psi_\nu)(\psi_\nu^\dagger \gamma_4\gamma_4\gamma_3\psi_\mu),$$

which just cancels the (43) component. Similar calculations hold for the other components of T.

The energy and angular distribution of e^- from polarized μ^- predicted by this theory is then

$$dN_e = \frac{x^2}{2\pi} \left[(3 - 2x) + \xi \cos \emptyset \, (1-2x) \right] dx \, d\Omega_e \qquad (4)$$

where $x = \frac{p}{p_{max}}$, $\xi = \frac{|g_s|^2 - 4|g_v|^2}{|g_s|^2 + 4|g_v|^2}$; $\emptyset = \measuredangle(\vec{s}_\mu, \vec{p}_e)$

so that $D(\emptyset)d\Omega_e = d\Omega_e \, / \, 4\pi \, [1 - \xi/3 \cos \emptyset \,]$.

In obtaining this result we have set $m_e = 0$, an approximation which is certainly justified throughout most of the electron spectrum.

By an argument analogous to that used in the case of Co^{60} decay, we can infer (since $(v_e/c) \approx 1$, except for a relatively small segment of $e^- - \nu - \bar{\nu}$ phase space) that scalar coupling leads to left-handed electrons and vector coupling to right-handed electrons, throughout most of the electron energy range. Since ψ_R is orthogonal to ψ_L, there is no interference between S and V. We may use these facts to acquire an understanding of the expression for dN_e given above.

At maximum energy ($x = 1$), the ν and $\bar{\nu}$ are both emitted in a direction opposite to that of the electron. Hence, the spins of ν and $\bar{\nu}$ add to zero, and the only way there can be conservation of total angular momentum when the electron is left-handed ($g_v = 0$) is to have the electron emitted in the ''backward'' direction, relative to the μ^- spin, (Diagram C). Similarly, if the electron is right-handed, ($g_s = 0$), it must be emitted in the ''forward''

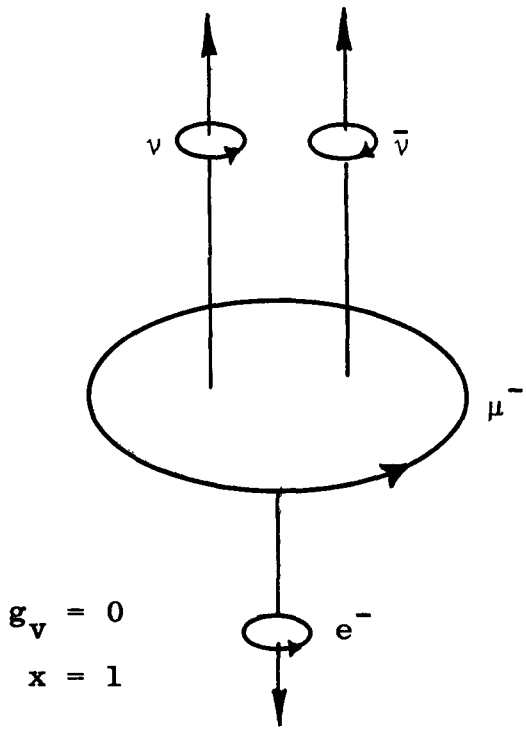


Diagram C

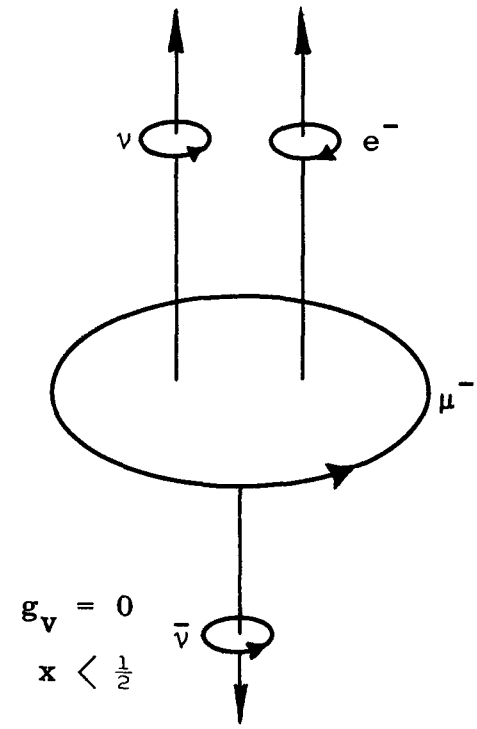


Diagram D

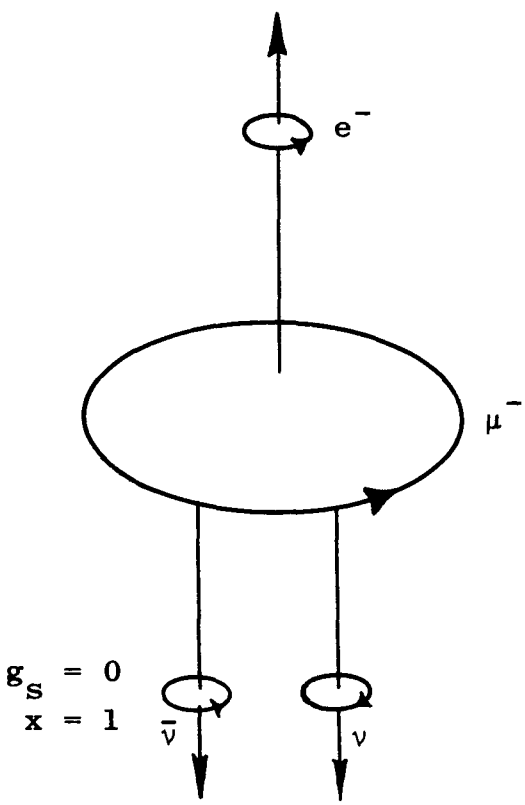


Diagram E

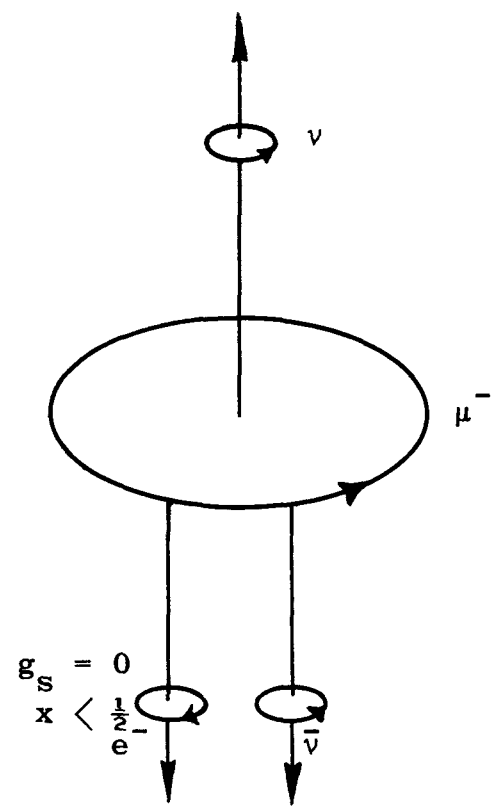


Diagram F

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direction (Diagram E). This is consistent with the fact that, for $x = 1$, equation (4) reduces to

$$dN_e = \begin{cases} \frac{1}{2\pi}(1 - \cos \emptyset)\, dx\, d\Omega_e & \text{(no forward emission)} \quad \text{for } \xi = +1,\ g_v = 0 \\ \frac{1}{2\pi}(1 + \cos \emptyset)\, dx\, d\Omega_e & \text{(no backward emission)} \quad \text{for } \xi = -1,\ g_s = 0. \end{cases}$$

On the other hand, when $x < 1/2$, the directions of ν and $\bar{\nu}$ are opposed, and their spins add up to one unit. The only possible way to have this one unit combine with the 1/2 unit of the electron to give the original μ^--spin, is to have the one unit in the ''forward'' direction. That is, the right-handed neutrino, ν, is emitted forward, the left-handed anti-neutrino, $\bar{\nu}$, backward. Then, since the spin of the electron must be directed in the backward direction, a left-handed electron ($g_v = 0$) must be emitted in the forward direction, parallel to ν (Diagram D); a right-handed electron ($g_s = 0$) in the backward direction, parallel to $\bar{\nu}$ (Diagram F). These results are consistent with the behavior of dN_e for $x < 1/2$, and for $\xi = +1$ ($g_v = 0$), $\xi = -1$ ($g_s = 0$):

$$dN_e \propto \begin{cases} 1 + A \cos \emptyset & \text{For } g_v = 0 \\ 1 - A \cos \emptyset & \text{For } g_s = 0, \end{cases}$$

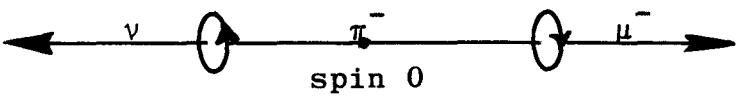
where $A = 1 - 2x / 3 - 2x$ so that $0 \leq A < 1/3$ in the range $0 \leq x < 1/2$. That is, in this energy range, forward emission is favored for scalar coupling, backward emission for vector coupling. (We consider here for $x < 1/2$ only the special case that all momenta $\vec{p}_e$, $\vec{p}_\nu$, $\vec{p}_{\bar{\nu}}$ are parallel or anti-parallel which is not the only configuration possible. Thus the asymmetry is not perfect.)

In the μ-decay experiments, the μ's were obtained from the decay of π-mesons. Thus, before we can draw any conclusions from these experiments we must investigate the π-μdecay.

<u>π-μ decay</u>

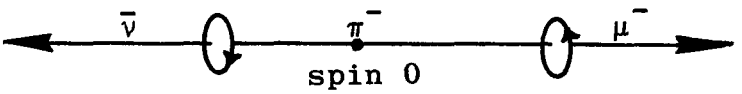
In pion decay, we must try to choose between two alternatives:

(1) $\pi^- \rightarrow \mu^- + \nu$.



This clearly leads to right-handed μ^- in the center of mass system of the pion.

(2) $\pi^- \rightarrow \mu^- + \bar{\nu}$.



In this case the μ^- is left-handed.

In the subsequent μ^--decay, we observe the angular correlation between the direction of the emitted electron and the direction of forward motion of the μ^-. (See Diagram G). The μ^- are stopped in carbon and they yield an angular distribution of electrons, (integrated over the energy spectrum)

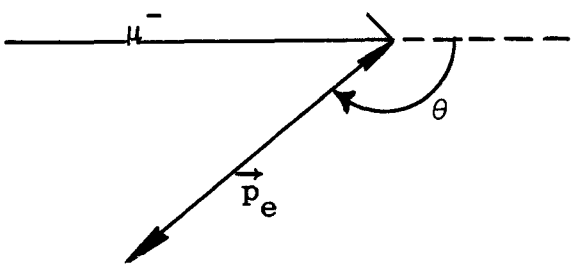


Diagram G

$$D(\theta)d\Omega = \frac{1}{4\pi}(1 + \alpha \cos\theta)d\Omega,$$

$$\theta = \measuredangle(\vec{p}_e, \vec{p}_\mu)$$

where α is found to be ~ -0.3 . That is, backward emission relative to the direction of the μ^- is preferred. (With materials other than carbon, a greater depolarization of the μ, and hence a smaller value of α, are found.) The assumption

$|\xi| = 1$ fits this value for α very well, but since the angles θ and $\emptyset$ might be supplementary instead of equal, we can conclude only that $\xi = +1$, whence $g_s = 0$, or $\xi = -1$, whence $g_v = 0$.

A direct measurement of the polarization of μ is of interest for at least two reasons:

(1) Suppose that we should find opposite polarizations of the muons in the two decays

$$\text{(a).} \quad \pi^+ \rightarrow \mu^+ + \nu$$
$$\text{(b).} \quad \pi^- \rightarrow \mu^- + \bar{\nu},$$

as the two-component neutrino theory would imply. We could then conclude, independently of any detailed theory of the neutrino, that there is non-invariance under charge conjugation, C, as we have defined it. For according to our definition of C, C-invariance would require μ^- in (b) to have the same polarization as μ^+ in (a). In the Wu-Ambler and Frauenfelder experiments with Co^{60}, the non-invariance of P was established directly, independently of any detailed theory of the interaction. We had to make use of the theory, however, (see Lecture 3) to indirectly deduce the violation of C-invariance.

(2) If $\pi^- \rightarrow \mu^- + \bar{\nu}$ were found to be the case, we could speak meaningfully of conservation of leptons. The already established transition $Co^{60} \rightarrow Ni^{60} + e^- + \bar{\nu}$ demands that if we call e^- a lepton then $\bar{\nu}$ is an anti-lepton. Now if two neutrinos, or two anti-neutrinos were emitted in the μ-decay, the energy distribution would be zero at $E = E_{max}$, which is in contradiction to the experiment. Hence, we say $\mu^- \rightarrow e^- + \nu + \bar{\nu}$.

Conservation of leptons then defines μ^- as a lepton and thus requires $\pi^- \rightarrow \mu^- + \bar{\nu}$.

Orear and Harris plan to measure the polarization of μ in π-decay by deflecting $\vec{p}_\mu$ without changing $\vec{s}_\mu$, by means of multiple small angle coulomb scattering. The emerging μ's will then have transverse components of polarization, which can be determined by a subsequent large angle, spin dependent, coulomb scattering.

A direct measurement of the polarization of e^-, of course, would enable us to distinguish between $\xi = \pm 1$. Then

e^- = left-handed $\Longrightarrow g_V = 0 \Longrightarrow \xi = +1 \Longrightarrow \mu$ is right-handed in π-decay (i.e. $\pi^- \rightarrow \mu^- + \nu$).

e^- = right-handed $\Longrightarrow g_S = 0 \Longrightarrow \xi = -1 \Longrightarrow \mu^-$ is left-handed in π-decay (i.e. $\pi^- \rightarrow \mu^- + \bar{\nu}$).

The polarization state of e^- depends only on $g_V = 0$ or $g_S = 0$ and is independent of the polarization state of μ.

<u>Status of Experimental Evidence for the Two Component Theory.</u>

(1) Oriented Co^{60} decay experiment. Wu, Ambler <u>et al.</u> report a rough value for the asymmetry parameter, β, ($\alpha = \beta \langle J_z \rangle / J$ of -0.7, et electron velocities $(v_e / c) \sim 0.65$. The assignment $C_A = C_A' = 0$, $C_T = -C_T'$ leads (see Lecture 2) to a predicted value of $\beta = (-v/c) \simeq -0.65$. Thus the present rough experimental values favor the two-component theory ($C' = -C$) with small axial vector coupling.

(2) $\pi - \mu - e$ decay experiments. We have seen that the angular

distribution integrated over the energy spectrum gives a fit with the two component theory and determines $\xi \approx \pm 1$. Michel has shown that the energy spectrum, integrated over all angles can be expressed in terms of a single parameter, ρ:

$$dN = 4x^2[3(1-x) + 2/3\ \rho\ (4x-3)]\ dx. \qquad (5)$$

ρ is called the Michel parameter and is a function of the various coupling constants. The most recent experiments by Crowe on the spectra, fit (5) with a value $\rho_{exp} = 0.64 \pm 0.02$. Now if we integrate equation (4) over all angles, we note that the terms proportional to ξ drop out, and on comparison with (5) we find that $\rho = 0.75$, which does not agree with ρ_{exp}. By assuming the worst possible combination of experimental errors, one might be able to push the experimental value up to a maximum of 0.68. At the same time, it has been estimated that higher order radiative corrections to the two-component theory and mass effects could lower $\rho_{theor.}$ by 1 or 2%. Nevertheless, this discrepancy cannot yet be considered as a serious objection to the two-component theory.

(3) The neutrino capture experiment of Cowan and Reines at Los Alamos. The measured capture cross section is found to be less than 1/2 of the cross section predicted by the two-component theory. This, however, cannot be considered a defect of the two-component theory, for if we assume that the ν's from the pile are polarized (which is the most reasonable assumption, from the knowledge of the Co^{60} experiments), then a detailed-balancing argument alone allows us to infer from the β-decay (using the known experimental

result that the emitted ν are polarized) that σ for the inverse process could be about twice the experimental value of Cowan and Reines. That is, we conclude that if detailed balancing is valid, then the Co^{60} experiment is in contradiction with the Los Alamos experiment, independent of any detailed theory.

The Universal Fermi Interaction.

Prior to the discovery of the slowly decaying strange particles, piople tried to explain β-decay and μ-decay on the basis of a universal interaction coupling for Fermi fields. It was hoped that $C_i(\beta\text{-decay}) = g_i(\mu\text{-decay})$, for all i. However, the new, weak processes have properties similar to these old ones. They cannot be easily incorporated into a universal Fermi interaction and the possible usefulness of such an attempt has become less clear. In spite of this fact, we shall see what conclusions regarding the μ- and β-Hamiltonians can be drawn from the experiments, within the framework of the two component theory of the neutrino.

The β-Hamiltonian would seem to consist predominantly of C_S and C_T terms. Only g_S and g_V terms are present in the μ-Hamiltonian. Since $H_T^{(\mu)} = 0$, we can set $g_T = C_T$ without really having said anything. From the fact that

$$\xi = \frac{|g_S|^2 - 4|g_V|^2}{|g_S|^2 + 4|g_V|^2} = \pm 1$$

we have concluded that either $g_S = 0$ or $g_V = 0$. If $\xi = 1$, whence

$g_V = 0$, all would be well provided $g_S = C_S$. But experiments show that $|g_S|^2$ and $|C_S|^2$ differ by a factor of 3.

If $\xi = -1$, whence $g_S = 0$, $g_V \neq 0$, it would appear that all were immediately lost. But this is not so. If we consider a new Hamiltonian $H'^{(\mu)}$, where the electron is coupled to the anti-neutrino instead of to the neutrino:

$$H'^{(\mu)} = \sum_i g_i' \, (\psi_e^+ O_i \, \psi_{\bar{\nu}})(\psi_{\bar{\nu}}^+ \, O_i \, \psi_\mu) + \text{hermitian conjugate}$$

We again would find that only the g_V' and g_S' terms would contribute. But now $\xi = \dfrac{4|g_V'|^2 - |g_S'|^2}{4|g_V'|^2 + |g_S'|^2}$ and so, with $\xi = -1$, $g_V' = 0$, $g_S' \neq 0$.

But once again there would be a factor of 3 difference between $|g_S'|^2$ and $|C_S|^2$.

The factors of 3 might be accounted for by considering the effect of the strong π-meson coupling in β-decay. This coupling has no effect on the μ-decay.

We note that H is consistent with the idea of conservation of light fermions (e^-, μ^- and ν), while H' is in contradiction with the concept. However, if H is right then e^- is left-handed in μ^--decay which implies $\pi^- \rightarrow \mu^- + \nu$, thus violating the conservation of lepton concept. The whole thing is certainly not satisfactory.

Unsettled Questions.

We have seen that even though parity and charge conjugation are

violated in the two-component neutrino theory, we still have a certain uniformity, so that, in a sense, we might say that the sorld has become simpler. There remain, however, two important questions.

(1) Is T (or CP) conserved?

We discussed this problem in Lecture 3. We say there that to test violation of T we had to look for terms proportional to Z in the electron-neutrino angular correlation $(\vec{p}_e \cdot \vec{p}_\nu)$ or in the asymmetry of the angular distribution of electrons from polarized nuclei. The absence of such effects would be inconclusive, for they might indicate merely that C_A and C_A' are zero. Experiments to look for a Z-independent part of a $(\vec{s}_n \cdot \vec{p}_e \times \vec{p}_\nu)$ term in neutron decay are being planned at Chalk River. We recall, that if we do not detect such a term in the recoil experiment from polarized nuclei, it would mean (with $C_A = 0$) that $\mathrm{Im}(C_S C_T^* + C_S' C_T') = 0$. In the two-component theory $C_S' = -C_S$ and $C_T' = -C_T$, so that $2\ \mathrm{Im}\ (C_S C_T^*) = 0$. Thus C_T and C_S are in phase and may be taken to be real, since the results of any experiments depend only on the relative phase of C_T and C_S. We would thus have a rigorous proof of invariance under T. (On the other hand, if we do not require $C_S' = -C_S$ and $C_T' = -C_T$, C_S and C_T could be out of phase in such a way that $\mathrm{Im}\ (C_S C_T^* + C_S' C_T'^*) = 0$).

If the $s_n \cdot (\vec{p}_e \times \vec{p}_\nu)$ experiment were to show that T is invariant, then by SLP, CP is invariant. Thus we should have chosen P' = CP as the proper symmetry operator from the beginning and when we invarted space, we should have immediately replaced particle by antiparticle as well. In this way we will have obtained a simple

connection between left and right and at the same time retain the symmetry between past and future.

On the other hand, if T is shown to be not invariant, then CP will also not be invariant. That is, the transformation: $R \rightarrow L$, particle $\rightarrow$ antiparticle will bring us to a nonexistent state. If we consider this asymmetry to be essentially a local effect, we might still maintain an overall symmetry by assuming there are two different kinds of protons p_R, p_L, two different kinds of neutrons n_R, n_L, etc.

(2) The θ-τ problem.

Although the θ-τ puzzle was the motivation for the recent investigations on parity, the problem of understanding the decay of K-mesons is still unsolved. It is possible to take two opposite points of view with respect to this problem:

(a) Since it has been shown that parity is not conserved in some decay processes, we can assume that it is violated for θ- and τ-decay as well. Then τ and θ are different modes of decay of one particle characterized by a single mass, spin and lifetime. That is, there is no longer a problem and it may not be necessary to ask further questions.

(b) We can assume that parity is violated only in processes involving a neutrino. Then τ and θ are two different particles. Trying to decide the question of the number of K-particles, Steinberger and collaborators have examined pictures of events $\pi^- + p^+ \rightarrow \Sigma^- + K^+$ in bubble chambers. They looked for short-lived K-mesons. They found no evidence for such a particle.

It is important to notice that, if the spin of K is 0, the only way of detecting parity violation in the τ and θ decay is precisely the one that has already been observed; namely, the decay into channels of different parities. The only quantities that can be observed are the momenta of the final pions, $\vec{p}_i$. Since only two independent momenta are available, it is impossible to form a pseudoscalar quantity from them.

On the other hand, if the K-meson has a spin, the correlation between the polarization plane and the plane of the decay pions should provide additional possibilities of testing parity conservation. Corresponding experimental tests for hyperon decays are in progress.

Now that we have obtained some knowledge of conservation of I_z and its violation, conservation of P and its violation and invariance under C and its violation, we might ask in light of these similarities: Can all weak interactions be understood as manifestations of a more fundamental law?