Fermi National Accelerator Laboratory

FERMILAB-Pub-90/198-A
September 1990

INFLATIONARY AXION COSMOLOGY

Michael S. Turner

NASA/Fermilab Astrophysics Center
Fermi National Accelerator Laboratory
Batavia, IL 60510-0500

and

Departments of Physics and Astronomy & Astrophysics
Enrico Fermi Institute
The University of Chicago
Chicago, IL 60637-1433

Frank Wilczek

Institute for Advanced Study
School of Natural Sciences
Princeton, NJ 08540

Abstract. If Peccei-Quinn (PQ) symmetry is broken after inflation, the initial axion angle is a random variable on cosmological scales; based on this fact, estimates of the relic-axion mass density give too large a value if the axion mass is less than about 10^{-6} eV. This bound can be evaded if the Universe underwent inflation after PQ symmetry breaking and if the observable Universe happens to be a region where the initial axion angle was atypically small, $\theta_1 \lesssim (m_a/10^{-6}\,\mathrm{eV})^{0.59}$. We show consideration of fluctuations induced during inflation severely constrains the latter alternative.

Submitted to the **Phys. Rev. Lett.**

Peccei-Quinn (PQ) symmetry with its attendant pseudo-Nambu-Goldstone boson—the axion—remains the most attractive and promising solution to the strong-CP problem.[1] Moreover, the axion arises naturally in supersymmetric and superstring models. The axion mass and PQ symmetry breaking scale are related by

$$m_a \simeq \frac{\sqrt{z}}{1+z}\frac{f_\pi m_\pi}{(f_a/N)} \simeq \frac{0.62\,\mathrm{eV}}{(f_a/N)/10^7\,\mathrm{GeV}}, \tag{1}$$

where f_a is the PQ symmetry breaking scale, $z \simeq 0.56$ is the ratio of the up to down quark masses, f_π and m_π are pion decay constant and mass, and N is the color anomaly of PQ symmetry. At present there is little theoretical guidance as to the key parameter: the PQ symmetry breaking scale f_a (or equivalently the axion mass).

Relic axions arise due to three distinct cosmological mechanisms: (i) thermal production[2]—for an axion of mass greater than about 10^{-4} eV axions thermalize shortly after the QCD transition and, today, like neutrinos, should have a relic abundance of order $30\,\mathrm{cm}^{-3}$; (ii) the "misalignment" mechanism[3]—at high temperatures the dependence of the free energy upon the axion field becomes very weak and thus the value of the axion field will in general not coincide with the low-temperature, CP-conserving minimum; due to this misalignment the axion field is set in motion around the time of the QCD transition; and (iii) axionic string decay[4]—since PQ symmetry breaking involves the spontaneous breakdown of a global $U(1)$ symmetry, strings are produced; they decay by radiating (among other things) axions. While the thermal population of axions dominates for axion masses greater than about 10^{-2} eV, there are strong constraints on axions in this parameter regime due to the potential observability of their decays and to the effect they would have upon the evolution of red giant stars.[1] Thermal axions can contribute at most 10% of critical density. (Relic thermal axions of mass 3 to 8 eV have not been ruled out and recently a search for them has been performed[5].)

For axion masses greater than about 10^{-2} eV misalignment and axionic string decay are the dominant production processes, and sufficient numbers of axions can be produced to provide closure density. The importance of axionic string decay is still a matter of intense debate. It seems to be agreed that axion production through this mechanism is somewhere between being comparable to and about 100 times more important than the misalignment mechanism[4]. Here we are mainly interested in the case where the Universe inflated either before or during PQ symmetry breaking. Since axionic strings do not survive a period of inflation, our considerations are largely independent of this issue.

Let us briefly recall the misalignment mechanism of axion production, ignoring at first the possibility of inflation. The free energy of the vacuum depends upon the axion field only because this field modulates the phase of the instanton amplitude. At low temperatures the free energy has a maximum value of about $\Lambda_{\rm QCD}^4$, is periodic in the "axion angle" $\theta \equiv a/(f_a/N)$, and is minimized at a value of $\theta = 0$. The mass of the axion is determined by the curvature of the free energy at $\theta = 0$ and is given approximately by Eq. (1). At high temperatures instanton effects are strongly suppressed, and for $T \gg \Lambda_{\rm QCD}$ the free energy is essentially independent of the axion field. Thus, when PQ symmetry breaking occurs ($T \sim f_a$), no value of the axion angle is singled out dynamically, and one expects that the value of the axion angle in different causally distinct regions will be randomly distributed between $-\pi$ and π. Averaging will occur over a distance scale corresponding to the horizon size at the Peccei-Quinn transition, which is negligibly small cosmologically. Thus the primæval energy density associated with the misalignment of the axion field should be of order $\Lambda_{\rm QCD}^4$, and close to uniform. Further, when the axion mass exceeds $3H$ ($H \sim T^2/m_{\rm Pl}$ is the expansion rate and $m_{\rm Pl} = 1.22 \times 10^{19}$ GeV is the Planck mass) the axion field will begin to relax toward $\theta = 0$. Because it has no efficient way to shed energy, the field is left oscillating. The energy density in oscillations of the axion field behaves as nonrelativistic matter during the subsequent evolution of the Universe, and may be interpreted in particle language as a gas of zero-momentum axions. These relic axions, if present, behave like cold dark matter and are a promising dark matter candidate.

The contribution of these axions to the present mass density of the Universe is estimated to be[3]

$$\Omega_a h^2 \simeq 0.13 \times 10^{\pm 0.4} \Lambda_{200}^{-0.7} f(\theta_1^2) \theta_1^2 (m/10^{-5}\,{\rm eV})^{-1.18}, \qquad (2)$$

where Ω_a is the fraction of critical density contributed by axions, the present value of the Hubble constant $H_0 = 100h\,{\rm km\,sec^{-1}\,Mpc^{-1}}$, $\Lambda_{\rm QCD} = \Lambda_{200} 200\,{\rm MeV}$, and θ_1 is the initial misalignment angle. The function $f(\theta_1^2)$ accounts for anharmonic effects, and is of order unity (and specifically $f \to 1$ for $\theta_1 \ll 1$). The $10^{\pm 0.4}$ factor is an estimate of theoretical uncertainties—e.g., in the temperature dependence of the axion mass. Other possible physical process which may affect the result are discussed in Refs. 6. Note that closure density in axions is achieved for a mass somewhere between 10^{-6} and 10^{-4} eV.

The unusual dependence of the axion energy density upon the axion mass is easily understood. Regardless of the value of the axion mass, the energy density associated with the initial misalignment of the axion field is of order $\Lambda_{\rm QCD}^4$; once the axion field starts to oscillate that energy density red shifts as R^{-3} (R is the cosmic scale factor). The axion field begins to oscillate when the axion mass $m_a(T) \simeq 3H$: For smaller masses the axion oscillations begin later, and the energy density trapped in the misalignment of the axion field is diminished less.

Since the initial misalignment angle θ_1 is a random variable, at the time of PQ symmetry breaking the value of θ_1 will be different and uncorrelated in different causally distinct regions of the Universe. In the absence of inflation, these different regions are very small, and today the Universe is comprised of a very large number of regions that each had a different value of θ_1. To obtain the average axion energy density, one uses the *rms* average of θ_1, which is just $\pi/3$, in Eq. (2). In this circumstance the limit to $\Omega_a h^2$ based upon the age of the Universe,[7] $\Omega_a h^2 \lesssim 1$, implies a lower bound to the axion mass: $m_a \gtrsim 10^{-6}\,\mathrm{eV}$, or an upper bound to f_a: $f_a/N \lesssim 10^{13}\,\mathrm{GeV}$. The upper bound to f_a is significantly below two interesting energy scales: the unification scale of $10^{14}\,\mathrm{GeV}$ to $10^{16}\,\mathrm{GeV}$ and the Planck scale, a somewhat disappointing fact. (However there are models, particularly those involving supersymmetry breaking in a "hidden sector", where the geometric mean of the weak symmetry breaking scale and the Planck mass, $\sqrt{m_{\mathrm{Pl}} m_{\mathrm{weak}}} \approx 10^{11}\mathrm{GeV}$ is the fundamental symmetry-breaking scale.)

If the Universe inflated before or during PQ symmetry breaking the fluctuations in the axion field take an entirely different form. While the average of θ_1^2 over many causally separate volumes is still $\pi/3$, the practical relevance of this fact is nil, because the entire presently observable Universe lies within one causal region where θ_1 is constant.

A number of authors[8] have pointed out that axion masses smaller than $10^{-6}\,\mathrm{eV}$ (PQ symmetry breaking scales greater than $10^{13}\,\mathrm{GeV}$) could be consistent with $\Omega_a h^2 \lesssim 1$, provided that θ_1 was sufficiently small:

$$\theta_1 \lesssim (m_a/10^{-6}\,\mathrm{eV})^{0.59}. \tag{3}$$

In this case, then, we would be living in a rare, axion-poor region of the Universe. If the Universe did indeed undergo inflation, the fundamental laws of physics do not determine θ_1. Despite its cosmic import the local value of this parameter is an "historical accident," and can only be determined through direct measurement of $\Omega_a h^2$ and m_a.

One might then be left with the impression that if the Universe underwent inflation any value of f_a can be tolerated, provided that θ_1 is appropriately small. Values of $f_a/N \gtrsim 10^{13}\,\mathrm{GeV}$ are only improbable, not forbidden. The main purpose of this *Letter* is to point out that additional, very important constraints emerge when fluctuations in the axion field that arise during inflation are taken into account.

Quantum fluctuations in the axion field have two important effects: (i) They set an absolute minimum to θ_1, the effective misalignment angle to be used in the energy estimate Eq. (2)—it can be no smaller than the size of its fluctuations; and (ii) They give rise to isocurvature axion perturbations,[9] i.e., fluctuations in the local axion-to-photon ratio. At late times ($t \gg t_{EQ} \simeq 4.4 \times 10^{10} h^{-4}\,\mathrm{sec}$), these evolve into density perturbations of the same magnitude, leading to fluctuations in the temperature of the cosmic microwave background radiation (CMBR). Isocurvature axion fluctuations have been discussed elsewhere in detail;[10] we will now review the relevant points.

For simplicity, suppose that PQ symmetry is broken when a complex scalar field $\vec{\phi}$ which carries PQ charge takes on a vacuum expectation value $\langle\vec{\phi}\rangle = f_a \exp(i\theta)/\sqrt{2}$, which minimizes its scalar potential $V(\vec{\phi}) = \lambda(|\vec{\phi}|^2 - f_a^2/2)^2$. The flat, θ degree of freedom corresponds to the axion. In de Sitter space all minimally coupled, massless scalar fields have quantum fluctuations characterized by amplitude $H_I/2\pi$ where H_I is the expansion rate during inflation. For the axion field these fluctuations lead to fluctuations in θ:

$$\delta\theta = \frac{H_I}{2\sqrt{2}\pi f_a} \simeq 4\times 10^{-5}\left(\frac{M_{14}^2}{N}\right) m_{-6}, \tag{4}$$

where $m_{-6} \equiv m_a/10^{-6}\,\mathrm{eV}$, the value of the Hubble constant during inflation has been written as $H_I^2 = 8\pi M^4/3m_{\rm Pl}^2$, M^4 is the false-vacuum energy, and $M_{14} \equiv M/10^{14}\,\mathrm{GeV}$.[11]

Fluctuations in the axion angle θ lead to constraints to the possible values of the misalignment angle θ_1. First, it is not possible for θ_1 to be smaller than the fluctuations in θ; thus

$$\theta_1 \gtrsim \delta\theta \simeq 4\times 10^{-5}\left(\frac{M_{14}^2}{N}\right) m_{-6}. \tag{5}$$

Next, fluctuations in the misalignment angle ultimately result in density perturbations of the same amplitude, which, on the largest angular scales ($\gg 1^\circ$), lead to anisotropies of the CMBR temperature. For $\Omega_a \simeq 1$, $\delta T/T \sim \delta\rho_a/\rho_a \sim \delta\theta/\theta_1$. If Ω_a is smaller than unity, i.e., if some species other than axions provides closure density, then the magnitude of this effect is reduced by a factor Ω_a: $\delta T/T \simeq \Omega_a(\delta\theta/\theta_1)$. On large angular scales the CMBR temperature is remarkably smooth, as evidenced by the absence of a quadrupole anisotropy at the level,[12] $\delta T/T \lesssim 3\times 10^{-5}$. This results in the following additional constraint to the initial misalignment angle θ_1:

$$\theta_1 \lesssim 0.1\left(\frac{N}{M_{14}^2}\right) m_{-6}^{0.18}. \tag{6}$$

The three constraints to the initial misalignment angle θ_1—Eqs. (3, 5, 6)—based upon the conditions $\Omega_a h^2 \lesssim 1$, $\theta_1 \gtrsim \delta\theta$, and $\delta T/T \lesssim 3\times 10^{-5}$ are illustrated in Fig. 1. They imply constraints upon the axion mass. First, to ensure that fluctuations in the misalignment angle do not lead to overproduction of axions the axion mass must satisfy:

$$m_a \lesssim 3\times 10^4\left(\frac{N}{M_{14}^2}\right)^{2.4}\ \mathrm{eV}. \tag{7a}$$

Unless the vacuum energy density is very large, $M/\sqrt{N} \gtrsim 10^{16}$ GeV, this condition is not very interesting. Moreover, it will be superseded by the next constraint.

Second, to ensure that the fluctuations in the axion density do not lead to excessive fluctuations in the temperature of the CMBR, the axion mass must satisfy:

$$m_a \lesssim 10^{-2} \left(\frac{N}{M_{14}^2}\right)^{2.4} \text{ eV}. \tag{7b}$$

For $M/\sqrt{N} \gtrsim 7 \times 10^{14}$ GeV, this condition implies that the axion mass *must* be less than about 10^{-6} eV. In chaotic inflation[13] the value of M is about 8×10^{15} GeV, implying that the axion mass must be *less* than about 10^{-11} eV, corresponding to a symmetry breaking scale greater than 10^{18} GeV.

These limits are general. If we assume that axions do in fact provide closure density, then to avoid excessive CMBR anisotropies we must require the stronger condition

$$m_a \lesssim 10^{-8} \left(\frac{N}{M_{14}^2}\right)^{2.4} \text{ eV}. \tag{7c}$$

This is perhaps the most stringent and interesting condition. In chaotic inflation it precludes the axion from being the dark matter. If $(N/M_{14}^2)^{2.4} \gtrsim 1$, one must require a very small axion mass, and correspondingly a very small misalignment angle, to achieve closure density in axions. (And of course an even smaller and less likely angle, to achieve a smaller density!) Current[14] and proposed[15] cosmic axion searches have focused on the mass interval 10^{-6} eV to 10^{-4} eV. To the extent our considerations make the low mass axion plus inflation option less attractive they further emphasize the importance of this accessible interval, which is motivated by the "no inflation after PQ breaking" case.

Let us summarize. In the absence of inflation, or if the PQ transition occurs after inflation, axions are cosmologically overproduced unless the axion mass is more than about 10^{-6} eV. If the Universe underwent inflation after the PQ transition it might seem that a very low mass axion is only improbable and that closure density of axions could be achieved for any value of the axion mass less than about 10^{-4} eV, provided that the misalignment angle is sufficiently small.[8] However, here we have shown that when one takes into account the effect of the quantum fluctuations in the axion field that arise during inflation—and the resulting CMBR temperature fluctuations—closure density of axions is only consistent for values of the axion mass *less* than about $10^{-8}\,(N/M_{14}^2)^{2.4}$ eV. Based upon the isotropy of the CMBR alone, it follows that in the inflationary case the axion mass must be less than about $10^{-2}\,(N/M_{14}^2)^{2.4}$ eV, whether or not axions provide closure density.

This work was supported by the NASA (at Fermilab through grant NAGW-1340), the DOE (at Chicago and Fermilab), and by the NSF (at Princeton) and was initiated at the

Nobel Symposium on the Birth and Early Evolution of the Universe, held at Graftavallens, Sweden, June 1990. MST thanks the Aspen Center for Physics for its hospitality.

References

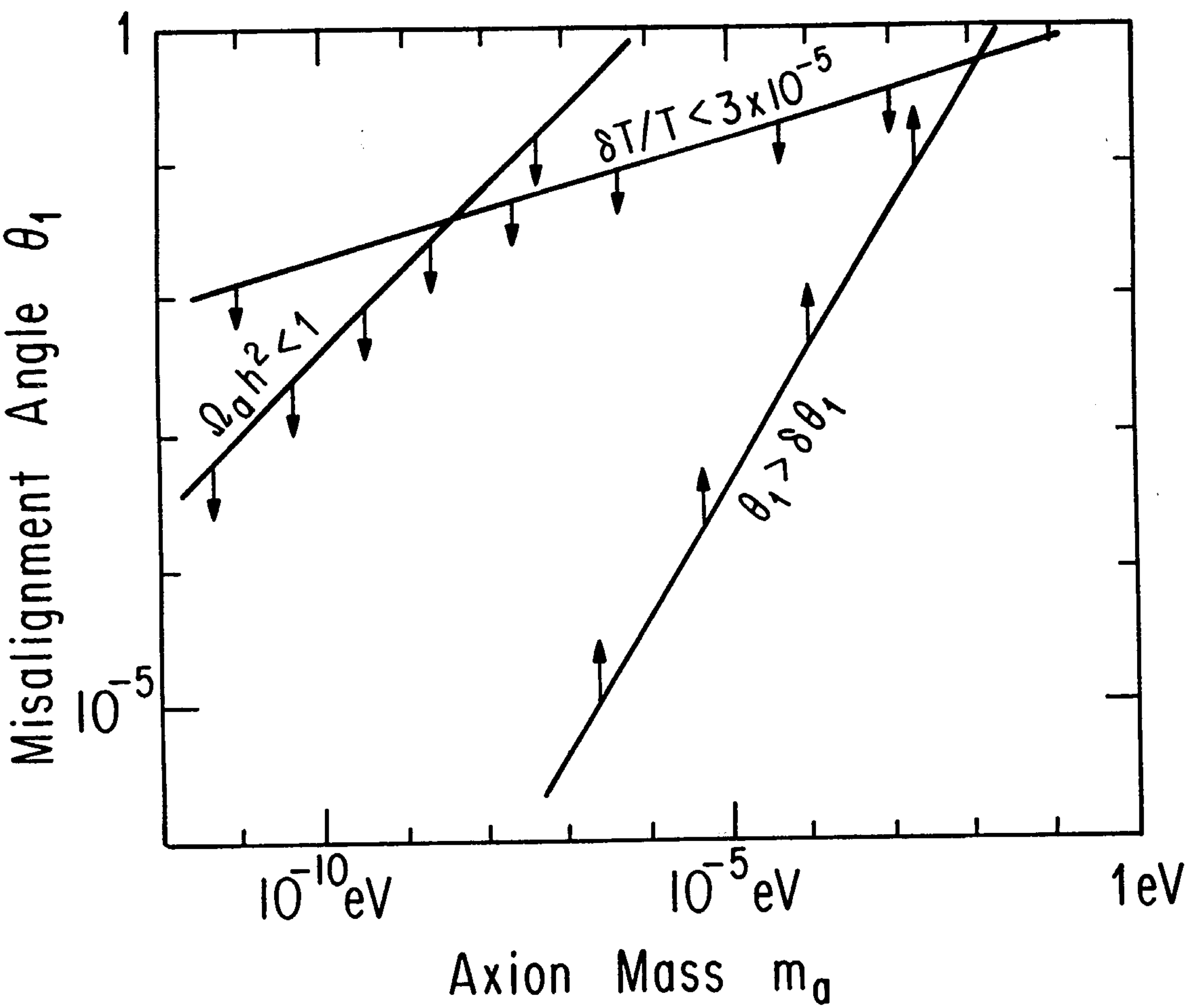
1. R.D. Peccei and H.R. Quinn, *Phys. Rev. Lett.* **38**, 1440 (1977); F. Wilczek, *Phys. Rev. Lett.* **40**, 279 (1978); S. Weinberg, *Phys. Rev. Lett.* **48**, 223 (1978). For a recent review of the axion see R.D. Peccei, in *CP Violation*, edited by C. Jarlskog (WSPC, Singapore, 1989); for a recent review of the astrophysical and cosmological constraints to the axion see, M.S. Turner, *Phys. Rept.*, in press (1990).
2. M.S. Turner, *Phys. Rev. Lett.* **59**, 2489 (1987).
3. J. Preskill, M. Wise, and F. Wilczek, *Phys. Lett. B* **120**, 127 (1983); L. Abbott and P. Sikivie, *Phys. Lett. B* **120**, 133 (1983); M. Dine and W. Fischler, *Phys. Lett. B* **120**, 137 (1983); M.S. Turner, *Phys. Rev. D* **33**, 889 (1986).
4. R. Davis, *Phys. Lett. B* **180**, 225 (1986); D. Harrari and P. Sikivie, *Phys. Lett. B* **195**, 361 (1987); R. Davis and E.P.S. Shellard, *Nucl. Phys.* **B324**, 167 (1990); A. Dabholkar and J.M. Quashnock, *Nucl. Phys.*, in press (1990); C. Hagmann and P. Sikivie, *Nucl. Phys.*, in press (1990).
5. T. Kephart and T. Weiler, *Phys. Rev. Lett.* **58**, 171 (1987); M.S. Turner, *Phys. Rev. Lett.* **59**, 2489 (1987); M. Bershady, M.T. Ressell, and M.S. Turner, in preparation (1990).
6. W. Unruh and R. Wald, *Phys. Rev. D* **32**, 831 (1985); M.S. Turner, *Phys. Rev. D* **32**, 843 (1985); T. Degrand, T.W. Kephart, and T.J. Weiler, *Phys. Rev. D* **33**, 910 (1986); P.J. Steinhardt and M.S. Turner, *Phys. Lett. B* **129**, 51 (1983); G. Lazarides, D. Panagiotakapoulos, and Q. Shafi, *Phys. Lett. B* **192**, 323 (1987); K. Yamamoto, *Phys. Lett. B* **161**, 289 (1985); S. Dimopoulos and L. Hall, *Phys. Rev. Lett.* **60**, 1899 (1988).
7. E.W. Kolb and M.S. Turner, *The Early Universe* (Addison-Wesley, Redwood City, CA, 1990), pp. 57-58.
8. S.-Y. Pi, *Phys. Rev. Lett.* **52**, 1725 (1984); M.S. Turner, *Phys. Rev. D* **33**, 889 (1986); A.D. Linde, *Phys. Lett. B* **201**, 437 (1988).
9. D. Seckel and M.S. Turner, *Phys. Rev. D* **32**, 3178 (1985); A.D. Linde, *Phys. Lett. B* **158**, 375 (1985).
10. D. Seckel and M.S. Turner, *Phys. Rev. D* **32**, 3178 (1985).
11. If the value of $|\vec{\phi}\,|$ evolves during inflation, then the amplitude of the fluctuations in θ_1 is larger, as f_a should be replaced by the instantaneous value of $|\vec{\phi}\,|$; see, Ref. 10 for

further discussion.

12. D.J. Fixsen, E.S. Chang, and D. Wilkinson, *Phys. Rev. Lett.* **50**, 620 (1983); P.M. Lubin, et al., *Astrophys. J.* **298**, L1 (1985); A.A. Klypin, et al., *Sov. Astron. Lett.* **13**, 104 (1987).
13. A.D. Linde, *Phys. Rev. B* **129**, 177 (1983); for a detailed discussion of this and other inflationary models see, e.g., Ref. 7, Ch. 8.
13. S. DePanfilis et al., *Phys. Rev. Lett.* **59**, 839 (1987); *Phys. Rev. D* **40**, 3153 (1989); C. Hagmann, P. Sikivie, N.S. Sullivan, and D.B. Tanner, *Phys. Rev. D (RC)*, in press (1990); *Rev. Sci. Instrum.* **61**, 1076 (1990); S. Inagaki et al., in *Proceedings of the Workshop on Cosmic Axions*, edited by C. Jones and A. Melissinos (WSPC, Singapore, 1990). Many, if not most, of the issues and experiments involving cosmic axions are discussed in *Proceedings of the Workshop on Cosmic Axions*, edited by C. Jones and A. Melissinos (WSPC, Singapore, 1990).
14. P. Sikivie et al., *Proposal to the DOE and LLNL for an Experimental Search for Dark Matter Axions in the* $0.6 - 16\,\mu$eV *Mass Range* (submitted 20 July 1990).

Figure Caption

FIGURE 1: Summary of the three constraints to the initial misalignment angle θ_1 for $M_{14}^2/N = 1$. The first constraint follows from requiring that axions not "overclose" the Universe; the second from requiring θ_1 to be larger than the quantum fluctuations in θ_1; and the third from requiring that fluctuations in θ do not lead to excessive, large-angle fluctuations in the temperature of the CMBR. Each constraint divides the θ_1–m_a plane into allowed (indicated by arrows) and forbidden regions.



Misalignment Angle θ1
1
10^-5
Ωa h^2 < 1
δT/T < 3 x 10^-5
θ1 > δθ1
10^-10 eV
10^-5 eV
1eV
Axion Mass ma