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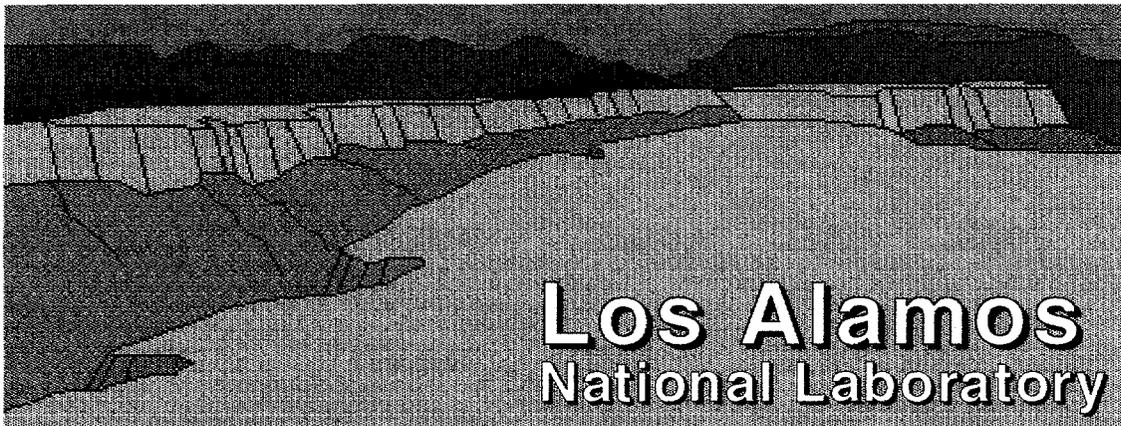
Characterizing Size Dependence of Ceramic-Fiber Strength Using Modified Weibull Distribution

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Characterizing Size Dependence of Ceramic-Fiber Strength Using Modified Weibull Distribution

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Abstract

The strengths of ceramic fibers have been observed to increase with decreasing fiber diameter and length. The traditional single-modal Weibull distribution function can only take into account one type of flaw, which makes it inappropriate to characterize the strength dependence of both the diameter and the length since ceramic fibers usually have both volume and surface flaws which affect the strength dependence in different ways. Although the bi-modal Weibull distribution can be used to characterize both volume and surface flaws, the mathematical difficulty in its application makes it undesirable. In this paper, the factors affecting fiber strength are analyzed in terms of fracture mechanics and flaw formation. A modified Weibull distribution function is proposed to characterize both the diameter dependence and the length dependence of ceramic fibers.

1. Introduction

Ceramic fibers and whiskers have been increasingly used as reinforcements for advanced composite materials [1-4]. The mechanical properties of these reinforcements significantly affect the strength of the composite materials. In order to make full use of the reinforcing potential of ceramic fibers in composite design, it is essential to understand and accurately characterize their mechanical properties. Brittle fractures have been observed in most advanced ceramic fibers under tensile stress. It is a well known fact that the strengths of ceramic fibers are size dependent [5]. As the diameter or gauge length decreases, the strength of ceramic fibers increases [6-8]. In addition, experimental strengths of brittle ceramic fibers display a range of values for a given configuration.

The traditional single-modal Weibull distribution function [9] has been widely used to characterize the dependence of the brittle-fiber strength on gauge length [10 - 13]. However, brittle ceramic fibers usually have both volume and surface flaws which affect the size dependence of fiber strength in different ways. Therefore, it is inappropriate to characterize the strength dependence of both the diameter and the length using the traditional single-modal Weibull distribution. Modified bi-modal Weibull distributions can be used to characterize the strength dependence of ceramic fibers on both diameter and gauge length when two distinct flaw populations are presumed to exist. However, it is very difficult to determine the four parameters of the bi-modal Weibull distribution from the experimental data [1, 10, 14]. Since the bi-modal Weibull distribution cannot be integrated, repeated numerical integration is needed to determine the four parameters that fit the experimental data best by trial and error, which makes it unsuitable for practical application. In addition to its complexity and difficulty, the bi-modal Weibull distribution also provides little insight into the actual failure mechanisms which act on the ceramic fibers.

This paper analyzes several proposed mechanisms which could affect the fiber strength. A modified Weibull distribution is proposed to characterize the strength dependence of ceramic fibers on both diameter and gauge length. Comparison with experimental data demonstrates the superiority of the proposed modified Weibull distribution.

2. Strength Affecting Factors

There are three factors which affect the fiber strength. The first one is fiber volume, to which the probability of encountering a critical-size volume flaw is assumed to be proportional in the traditional single-modal Weibull distribution; the second factor is statistical fracture mechanics; and the third factor is the effect of fiber diameter on the volume flaw density. The first factor has been well characterized by the traditional single-modal Weibull distribution, and therefore will not be discussed here. The Second and the third factors are discussed in the following sections.

2.1 Statistical fracture strength

Consider a fiber of radius b containing a central penny-shaped crack of radius a . The crack is in a plane normal to the longitudinal axis. Under a tensile force F , the stress intensity factor can be expressed as [15]:

$$K_I = \left[\frac{2}{\pi} (1 + 0.5q - 0.625q^2) + 0.268q^3 \right] \sqrt{1-q} \frac{F\sqrt{a}}{\pi(b^2 - a^2)} \quad (1)$$

where $q = a/b$, is the ratio of crack radius to fiber radius, K_I is the stress intensity factor. Substituting $F = \pi b^2 \sigma$ into Eq. 1 and rearranging yields:

$$K_I = \frac{\sigma\sqrt{b}}{G(q)} \quad (2)$$

where

$$G(q) = \frac{(1+q)\sqrt{1-q}}{\left[2(1+0.5q-0.625q^2)/\pi+0.268q^3\right]\sqrt{q}} \quad (3)$$

Assuming the fracture toughness of the fiber is K_{Ic} , the fiber strength can be derived from Eq. 2 as:

$$\sigma_f = K_{Ic}G(q)b^{-1/2} \quad (4)$$

It can be seen from Eq. 4 that for a fixed crack radius to fiber radius ratio, q , the fiber strength increases with decreasing fiber radius. However, calculations show that, for a fixed crack radius, the fiber strength decreases with decreasing fiber radius. In the ceramic fibers, the probability of encountering a crack of a certain size is statistical in nature. The fiber strength is determined by the largest crack existing in the fiber.

To examine the effect of statistical fracture mechanics on fiber strength, first we will assume the probability of encountering a crack of size a per unit fiber volume can be characterized by a function $\rho(a)$ disregarding the fiber radius, *i.e.* the crack size distribution is not affected by the fiber radius. Of course, this is only an assumption. However, it is necessary to make this assumption so that we can isolate and investigate the effect of statistical fracture mechanics without the complication of crack density variation with fiber diameter, which will be discussed in a later section. The probability that a is the largest crack radius size in the fiber, $P(a)$, can be expressed as:

$$P(a) = \rho(a) \left[1 - \int_a^b \rho(a) da \right] \quad (5)$$

The expectation of largest crack radius (*i.e.* the value of a at which $P(a)$ has a maximum) can be obtained by setting

$$\frac{dP(a)}{da} = 0 \quad (6)$$

and solving it for \bar{a} . The expectation of ceramic fiber strength can be calculated by substituting $\bar{q} = \bar{a}/b$ into Eq. 4.

To calculate fiber strength, first we need to find a function $\rho(a)$, which satisfies three requirements of "real" crack size distributions. First, $\rho(a)$ has to increase with decreasing a ; Second, $\rho(a)$ has to approach 0 as a approaches ∞ ; Third, it has to satisfy the normalization condition: $\int_0^{\infty} \rho(a) da = 1$. Assume $\rho(a)$ takes the form

$$\rho(a) = \lambda e^{-\lambda a} \quad (7)$$

where λ can be considered as a crack size distribution parameter. This function satisfies all three requirements mentioned above, and can describe a wide range of crack size distributions with different λ values (see Fig. 1). \bar{q} can be derived following the procedures mentioned above as

$$\bar{q} = -\frac{1}{\lambda b} \ln \frac{1}{2} (1 + e^{-\lambda b}) \quad (8)$$

The above statistical fracture strength analysis shows that the fiber strength is affected not only by the fiber radius b but also by the statistical crack size distribution parameter λ .

2.2. Effects of fiber diameter on defects population

In the analysis of the last section, the crack size density distribution is assumed to be unaffected by scale, but this may be a poor assumption in reality. During the production of ceramic fibers, cracks form to release global or local stress and to lower the system energy. Undesirable internal stresses generated during fiber processing are usually caused by thermal stress, local chemical variations, or structural inhomogeneity.

As the fiber diameter decreases, it is likely that the fiber will be subjected to lower processing stresses because thermal, chemical and mechanical equilibrium can be achieved more easily. Since stress is the driving force for crack formation, less crack per unit volume of fiber is proposed for fibers with smaller diameter. On the other hand, the fiber surface area to volume ratio increases with decreasing fiber diameter, and therefore, the surface flaws per unit volume of fiber may increase as a result of abrasive contact damage following the synthesis.

3. Characterization of Ceramic Fiber Strength

As discussed in the strength analysis and the introduction, it is inadequate to characterize the diameter dependence and the gauge-length dependence of experimental ceramic fiber strength data with the traditional single-modal Weibull distribution function, and a bi-modal Weibull distribution is undesirable because of its complexity and mathematical difficulty. Therefore, to adequately characterize the size dependence of brittle ceramic fibers, one has to find a function which can take into account not only the statistical nature of the fiber strength but also the three strength affecting factors discussed in the strength analysis. This may be accomplished by incorporating the strength affecting factors into the Weibull distribution. To do this, let us examine the general form of the Weibull distribution:

$$F(\sigma) = 1 - \exp(-\alpha_v \sigma^{\beta_v} V) \quad (9)$$

where $F(\sigma)$ is the probability that a brittle fiber will fail under stress σ , α_v and β_v are Weibull distribution parameters associated with fiber volume, and V is the fiber volume under stress. It can be seen from Eq. 9 that the probability of fiber failure increases with increasing fiber volume, V . Fiber volume is used because it is assumed in the Weibull distribution that the probability of encountering a critical size flaw is proportional to the

fiber volume. However, it is the probability of encountering a critical size flaw that determines fiber failure. Therefore, a more general form of Weibull distribution is proposed that uses the probability of encountering a critical size defect, P , instead of the fiber volume. Thus, we can write the probability-based Weibull distribution as

$$F(\sigma) = 1 - \exp(-\alpha_p \sigma^{\beta_p} P) \quad (10)$$

where α_p and β_p are Weibull distribution parameters related to P .

P is proportional to the fiber length L since the flaw density is not affected by fiber length. However, P may not be proportional to the fiber cross-section area $\pi d^2/4$.

Let us assume

$$P = CLd^{e'} \quad (11)$$

where C and e' are constants and d is the fiber diameter. Substituting Eq. 11 into Eq. 10 yields

$$F(\sigma) = 1 - \exp(-C\alpha_p \sigma^{\beta_p} Ld^{e'}) \quad (12)$$

Eqs. 4 and 8 account for fiber strength that is affected by fracture mechanics and the crack size distribution parameter λ . Therefore, we can also incorporate the results of Eqs 4 and 8 into Eq. 12. Of the two Weibull distribution parameters, α_p determines the average fiber strength and β_p determines the fiber strength scattering. Since Eq. 4 is only related to average fiber strength, we can incorporate Eq. 4 into Eq. 12 by multiplying α_p with another parameter α_s :

$$F(\sigma) = 1 - \exp(-C\alpha_s \alpha_p \sigma^{\beta_p} Ld^{e'}) \quad (13)$$

where α_s can be written as

$$\alpha_s = [\sqrt{2}K_{Ic}G(\bar{q})d^{-1/2}]^{-\beta} \quad (14)$$

K_{Ic} and d are as defined before, \bar{q} is defined by Eq. 8 and $G(\bar{q})$ is defined by Eq. 3.

Substituting Eq. 14 into Eq. 13 and rearranging it yield

$$F(\sigma) = 1 - \exp\{-\alpha[G(\bar{q})]^{-\beta} \sigma^\beta L d^e\} \quad (15)$$

where $\beta = \beta_p$ and

$$\alpha = C\alpha_p 2^{-\beta/2} K_{Ic}^{-\beta} \quad (16)$$

$$e = e' + \beta/2 \quad (17)$$

The average fiber strength can be obtained from Eq. 15 as [16]:

$$\bar{\sigma} = AG(\bar{q})L^{-m}d^{-n} \quad (18)$$

where

$$m = 1/\beta \quad (19)$$

$$n = -e/\beta \quad (20)$$

$$A = \alpha^{-1/\beta} \Gamma\left(1 + \frac{1}{\beta}\right) \quad (21)$$

There are four unknown constant in Eq. 18: A , λ , m and n . λ is inexplicitly present because it is used to calculate \bar{q} . The four constants can be obtained by fitting Eq. 18 into experimental data using the least square method. The parameters in the modified Weibull distribution (Eq. 15) can be calculated from the four constants using Eqs. 19-21.

Although Eqs. 15 and 18 are much easier to use in comparison with the bi-modal Weibull distribution, it is still cumbersome to calculate its parameters from experimental data. To further simplify the Eqs. 15 and 18, let us approximate $G(\bar{q})$ as follows

$$G(\bar{q}) = Bd^{h'} \quad (22)$$

where B and h' are constants. We choose an exponent function to approximate $G(\bar{q})$ because it yields a very simple form of modified Weibull distribution. The validity of this approximation is shown in Fig. 2, in which $G(\bar{q})$ is approximated as $G(\bar{q}) = 1.094d^{0.456}$ for $\lambda = 0.5$. It will also be shown later (Fig. 3) that the form of Eq. 24 resulting from this approximation fits the experimental fiber strength data well. Substituting Eq. 22 into Eq. 15 and rearranging it yield:

$$F(\sigma) = 1 - \exp(-\alpha' \sigma^\beta L d^h) \quad (23)$$

where $\alpha' = \alpha B^{-\beta}$ and $h = e - \beta h'$. Now, the average fiber strength as a function of fiber diameter and length can be derived from Eq. 23 as [16]:

$$\bar{\sigma} = \alpha'^{-1/\beta} \Gamma\left(1 + \frac{1}{\beta}\right) L^{-1/\beta} d^{-h/\beta} = KL^{-m'} d^{-n'} \quad (24)$$

where K , m' and n' are constants.

Eq. 24 is identical to the empirical equations obtained by Bayer and Cooper [17, 18], which explains why their empirical equations fit into experimental data well.

However, there is a major difference between the modified Weibull distribution function derived here and the empirical equations of Bayer and Cooper. Their empirical equations can only calculate the average fiber strength while the modified Weibull distribution can characterize both the fiber strength and its statistical nature.

The values of K , m' and n' in Eq. 24 can be easily obtained by fitting into experimental data, and the parameters for the modified Weibull distribution can be calculated as

$$\beta = \frac{1}{m'} \quad (25)$$

$$h = \beta n' \quad (26)$$

and

$$\alpha' = \left[\frac{1}{K} \Gamma \left(1 + \frac{1}{\beta} \right) \right]^\beta \quad (27)$$

The modified Weibull distribution (Eqs. 23 and 24) can be used to characterize strengths of both unpolished and polished ceramic fibers. For example, for surface unpolished and polished A type (axis orientation: $\langle 11\bar{2}0 \rangle$ and $\langle 10\bar{1}0 \rangle$) sapphire fibers [17, 18], $\bar{\sigma} = 720L^{-0.39}d^{-0.56}$ and $\bar{\sigma} = 1019L^{-0.21}d^{-0.03}$ are obtained from the experimental data, respectively. The parameters for the modified Weibull distribution can be calculated as $\beta \approx 2.564$, $h \approx 1.436$ and $\alpha' \approx 3.48 \times 10^{-8}$ for unpolished fibers, and $\beta \approx 2.564$, $h \approx 1.436$ and $\alpha' \approx 3.48 \times 10^{-8}$ for polished fibers.

It should be pointed out that Eqs. 23 and 24 are only approximations to Eqs. 15 and 18. For accurate characterization of ceramic fiber strength, Eqs. 15 and 18 should be used. However, in most cases, Eqs. 23 and 24 are quite good approximations. For example, both Eqs. 18 and 24 were used to fit the experimental data for unpolished

alumina fiber [19] as shown in Fig. 3. It can be seen that both equations fit the data well, which proves the validity of the approximation in Eq. 22.

Conclusions

The traditional volume-based single-modal Weibull distribution is inadequate to characterize both diameter and length dependence of ceramic fiber strength. The factors affecting the strength of ceramic fibers include fiber volume, statistical fracture mechanics and a crack density variation with fiber diameter. These factors are incorporated into a modified Weibull distribution function, which can be used to statistically characterize the size dependence of strength for both polished and unpolished ceramic fibers. The modified Weibull distribution function shows good fit with the experimental strength data of ceramic fibers.

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Caption

Fig. 1 A wide range of crack size distribution functions ($\rho(a) = \lambda e^{-\lambda a}$) can be created with different λ values.

Fig. 2 Approximation of $G(\bar{q})$ with $G(\bar{q}) = 1.094d^{0.456}$ for $\lambda = 0.5$.

Fig. 3 The experimental strength data for alumina fibers can be fitted pretty well with both Eq. 18 (with parameters $\lambda = 0.5$, $AL^m = 41824$ and $n = 0.9176$) and Eq. 24 ($\bar{\sigma} = 54315d^{-1.014}$). The two equations have little difference in this case.

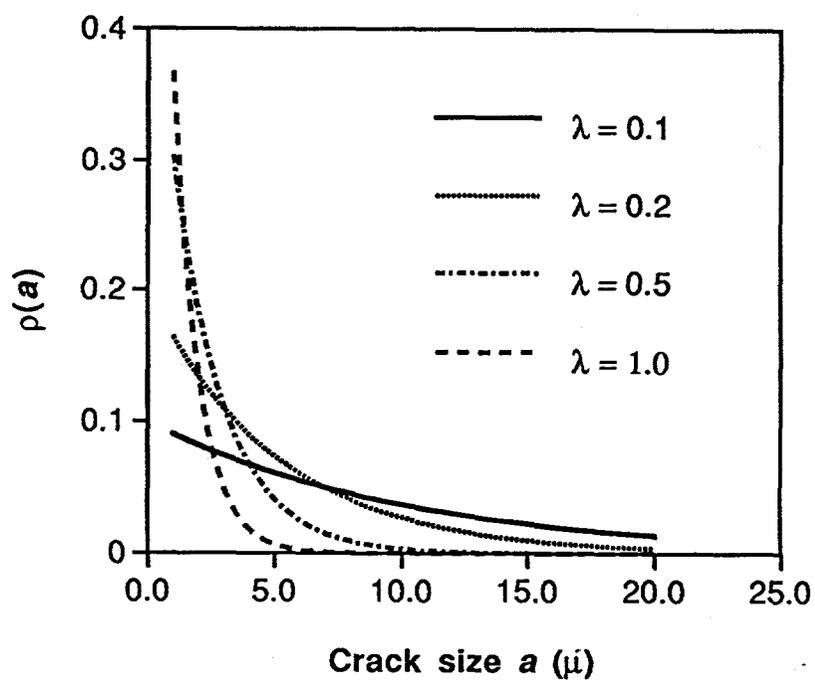


Fig. 1. Y. T. Zhu, W. R. Blumenthal & B. L. Zhou

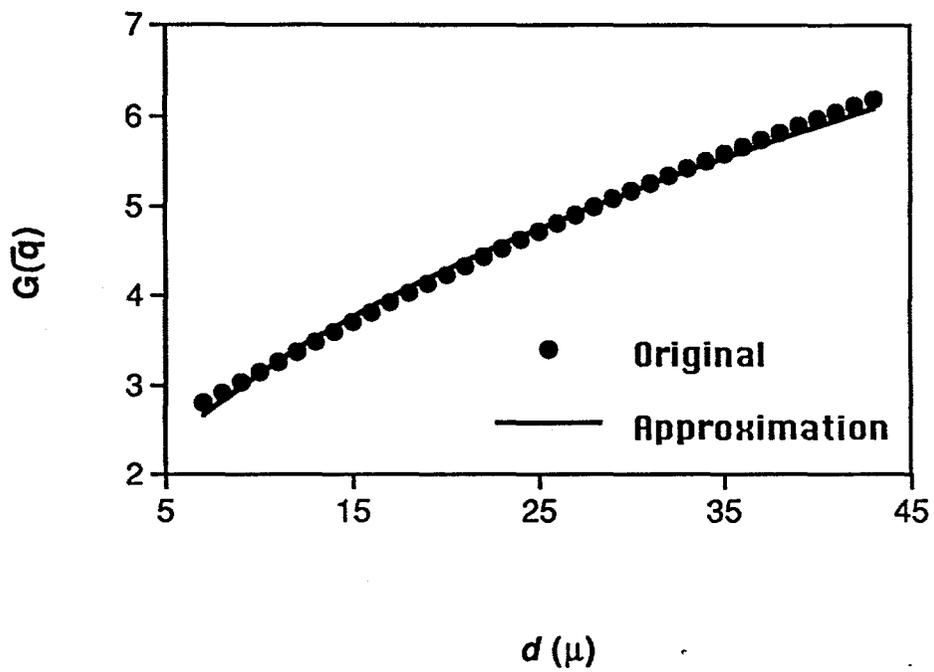


Fig. 2. Y. T. Zhu, W. R. Blumenthal & B. L. Zhou

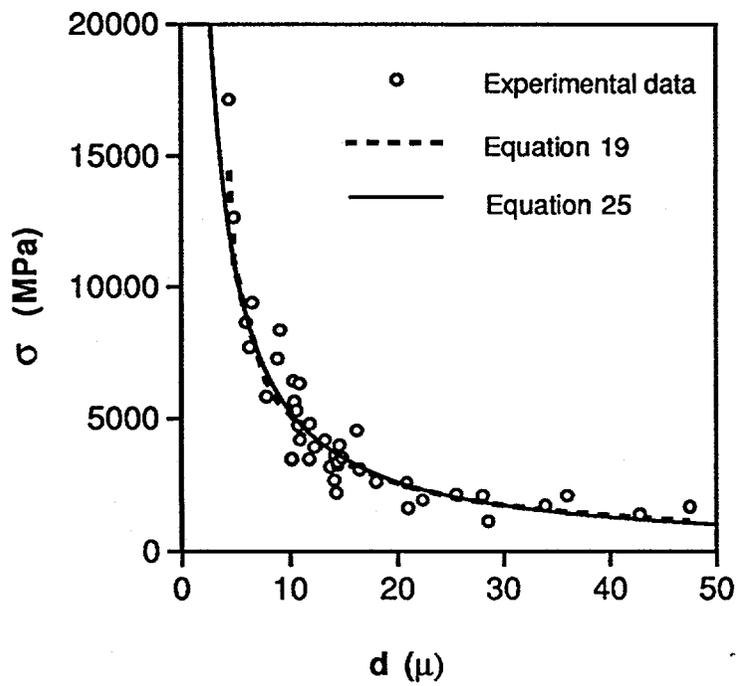


Fig. 3. Y. T. Zhu, W. R. Blumenthal & B. L. Zhou