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Analysis of Automated Highway System Risks and Uncertainties Vol. V

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ABSTRACT

This volume describes a risk analysis performed to help identify important Automated Highway System (AHS) deployment uncertainties and quantify their effect on costs and benefits for a range of AHS deployment scenarios. The analysis identified a suite of key factors affecting vehicle and roadway costs, capacities and market penetrations for alternative AHS deployment scenarios. A systematic protocol was utilized for obtaining expert judgments of key factor uncertainties in the form of subjective probability percentile assessments. Based on these assessments, probability distributions on vehicle and roadway costs, capacity and market penetration were developed for the different scenarios. The cost/benefit risk methodology and analysis provide insights by showing how uncertainties in key factors translate into uncertainties in summary cost/benefit indices.

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1. INTRODUCTION

This volume describes the "risk" analysis performed to help identify important Automated Highway System (AHS) deployment uncertainties and quantify their effect on costs and benefits for a range of AHS deployment scenarios. The cost/benefit risk analysis shows more formally how uncertainties in key implementation assumptions translate into uncertainties in summary cost/benefit indices. Although approximate, the risk analysis can provide planners with basic insights about the likelihood of realizing possible implementation cost and market penetration levels for alternative AHS deployment scenarios. These insights can also help direct further research efforts aimed at reducing the more acute uncertainties.

Specific Focus of Effort

Many types of uncertainties associated with requirements for successful AHS deployment have been identified.[1] These requirements include technological feasibility, institutional and interest group acceptance, willingness of auto makers to manufacture the required equipment, and willingness of customers to buy and maintain AHS equipped vehicles. In the volumes of this study, no attempt has been made to estimate the feasibility uncertainties of implementing AHS at either the technical or institutional level. Instead, the costs and benefits of alternative AHS scenarios have been analyzed under the assumption that system configurations perform as anticipated.

A key product of this study has been the creation of original AHS scenario cost estimates. These estimates for electronics costs and roadway infrastructure costs are developed in volumes 3 and 4 of this report for various deployment scenarios. However, each of the cost estimates developed is in the form of a single summary number or *best guess*. The risk analysis described in this volume develops *probability distributions* for the costs of each scenario. Unlike a single number or point estimate, these probability distributions quantify the range of uncertainty associated with scenario costs. The distributions specify the *risk* or likelihood that costs could turn out to be significantly higher (or lower) than estimated. The risk analysis shows how each scenario cost uncertainty range relates to the uncertainties in key cost estimation input parameters. In this way, the analysis helps to identify which parameters are most important to study further if reduction in uncertainty and risk is to be achieved. Besides costs, probability distributions are also developed for capacity gains and market penetration for selected scenarios.

Risk Analysis Overall Approach

The risk analysis was performed in four steps. These are outlined below.

Step 1: Selection of key cost/benefit factors for uncertainty assessment. Before costs and benefits of an AHS can be assessed, different AHS deployment scenarios are specified. Then models/judgments are used to quantify various kinds of costs and benefits that ensue given a specific AHS scenario.

In this step, key quantitative factors affecting the cost/benefits of particular AHS deployment scenarios were selected. The factors were chosen to be:

- comprehensive enough to address issues about which there may be significant uncertainty and/or concern.
- well-defined and meaningful to project team specialists.
- practical for addressing a variety of deployment scenarios.
- relatively few in number to make the overall analysis tractable.

In addition to *bottom-line* summary cost/benefit factors, we identified key "intermediate" parameters related to them which needed to be explicitly considered.

Step 2: Percentile estimates assessment for key factors. Percentile estimates for each key factor/parameter were assessed from individual specialists on the project team using formal subjective probability assessment techniques. In addition to formalizing parameter uncertainties quantitatively, these techniques help prevent the common pitfall of understating the uncertainty in knowledge that is present about key parameters. The estimates obtained were used to develop three-point probability (uncertainty) distribution approximations for each key parameter.

Activities in this step included implementing a formal interview protocol so that the assessment techniques were applied consistently and systematically for each factor. Assessments were conducted to exploit the common variables underlying different deployment scenarios and thus streamline the nature and number of assessments performed. Priority was placed on assessing factors that were intuitively felt to have the most significant uncertainty and greatest impact on cost/benefit results.

Three specialists provided the assessments used in this risk analysis. These individuals were selected because of their detailed knowledge of particular cost methodologies, cost/benefit factors, and scenario definitions used in this study, as well as AHS expertise. They were also very candid in their expression of uncertainty about factor estimates. As discussed in the conclusions to this report, these initial assessments from a small group of experts can help guide where assessments from additional experts would be especially helpful.

Step 3: Development of a simplified framework delineating relationships among intermediate and summary cost/benefit factors. This step developed formulas relating the intermediate parameters and *summary* cost/benefit factors with each other for each AHS scenario considered. The framework utilized the assessments and three-point distributions described in Step 2 as input to develop probability distributions on overall *bottom-line* cost/benefits.

Step 4: Framework implementation and risk analysis results. The framework from Step 3 was implemented on spreadsheet software for a personal computer. Tables and graphs of sensitivity analyses and probability distribution outputs were generated to highlight the likelihood of various cost/benefit results ensuing from different options. These results provide additional insight beyond that from using a *best guess* type (i.e., single point) estimate for each parameter, or from simplistic bounds obtained by using extremely optimistic or pessimistic estimates for all parameters. The risk analysis helps indicate which factors and uncertainties are most significant in influencing the relative desirability of an AHS option. While the modeling is of necessity approximate, basic insights obtained should help planners make projections of how likely it will be to realize various levels of costs and benefits from implementing an AHS.

Report Organization

The remainder of this report is organized as follows. Sections 2 through 5 describe each of the four steps of the risk analysis approach in more detail. Section 6 presents conclusions and recommendations for further study. The appendix contains formula details related to step 3 of the risk analysis approach.

2. SELECTION OF KEY COST/BENEFIT FACTORS FOR UNCERTAINTY ASSESSMENT

This section lists the various vehicle modification and roadway retrofit options considered by the risk analysis in defining alternative AHS implementation scenarios. Then, the factors selected for assessing scenario cost/benefit uncertainties are listed and discussed.

A list of factors was developed and organized into a structure for assessing cost/benefit uncertainties (see figure 1). The structure in figure 1 is discussed below.

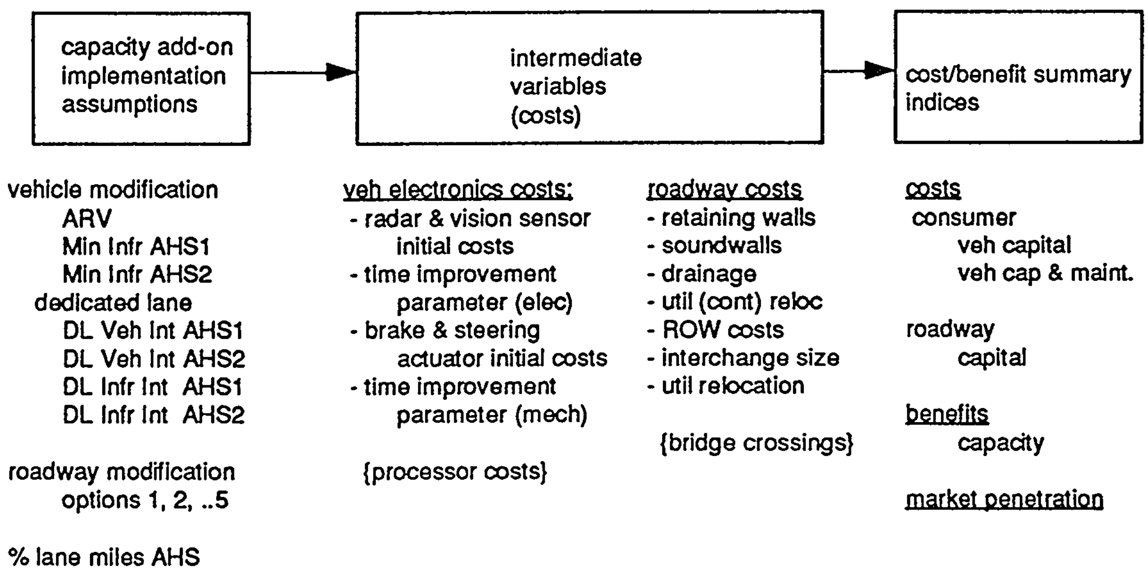


Figure 1. AHS options risk analysis: structure of factors for probability assessments.

Implementation Context Assumptions

Specific choices of roadway modification and vehicle modification assumptions define an AHS implementation scenario context. Cost and benefit estimates are different and calculated separately for each scenario. The vehicle modifications considered in the risk analysis are the seven vehicle options analyzed in volume 3. The roadway modifications considered are the five retrofit options analyzed in volume 4. Finally, assumptions about the percentage of freeway lane kilometers adapted for AHS relate to defining market penetration scenarios.

Basic vehicle types are the AHS ready vehicle (ARV), a fully automated vehicle but not able to automatically change lanes (AHS1), and a fully automated vehicle able to change lanes under full automation (AHS2). The seven vehicle modification options listed in figure 1 are very briefly summarized as follows (see volume 3 for more detail):

- ARV.
- AHS1 vehicle operating in mixed traffic with minimum infrastructure modification (Min Infr AHS1).
- AHS2 vehicle operating in mixed traffic with minimum infrastructure modification. (Min Infr AHS2).
- AHS1 vehicle operating on a dedicated AHS lane with vehicle intensive placement of sensing capabilities (DL Veh Int AHS1).
- AHS2 vehicle operating on a dedicated AHS lane with vehicle intensive placement of sensing capabilities (DL Veh Int AHS2).
- AHS1 vehicle operating on a dedicated AHS lane with more infrastructure intensive placement of sensing capabilities (DL Infr Int AHS1).
- AHS2 vehicle operating on a dedicated AHS lane with more infrastructure intensive placement of sensing capabilities (DL Infr Int AHS2).

The five roadway modification options are very briefly summarized as follows (see volume 4 for more detail):

- Option 1: all existing lanes remain but are automated.
- Option 2: one lane automated - three lanes conventional, buffer lane added between automated and conventional lane.
- Option 3: similar to Option 2 without buffer, on/off ramps added with bridge structure.
- Option 4: one lane automated, three lanes to remain conventional, separation by wide striping or rumble strips.
- Option 5: dedicated AHS structure - elevated.

Intermediate Variables

In the Electronics Cost Methodology (volume 3) and Roadway Infrastructure Cost Methodology (volume 4), models are described in which single number or point estimates are used for model parameters to compute summary costs for vehicle acquisition and maintenance, and roadway construction costs respectively for the different options under consideration. Tables 1 and 2 recap summaries of these model inputs and computations that were available at the time this risk analysis was performed. (*Note*: results in volumes 3 and 4 may differ from these tables reflecting changes to these computations that were made subsequent to this risk analysis. The risk analysis methodology and basic insights, however, should still be relevant.)

Table 1. Vehicle costs summary (point estimates).

Electronics procurement cost per vehicle, 2002 procurement, 1994 dollars, 1million unit market

E - Electronics component
M - Electro-mechanical component

					Dedicated Lanes			
			Mixed Traffic		Vehicle Intensive		Infrastructure Intensive	
		ARV	AHS1	AHS2	AHS1	AHS2	AHS1	AHS2
	VEHICLE	$1,045	$1,604	$1,905	$1,464	$1,742	$1,514	$1,672
SENSORS		$366	$584	$776	$570	$762	$570	$762
E	MULTI BEAM MILLIMETER RADAR	$306	$306	$306	$306	$306	$306	$306
E	MAGNETIC FIELD SENSOR	$60	$60	$0	$60	$0	$60	$0
E	VISION-BASED SENSOR	$0	$180	$360	$90	$270	$90	$270
M	RAIN/SNOW SENSOR	$0	$38	$38	$38	$38	$0	$0
M	BEACON EMITTERS	$0	$0	$0	$76	$76	$114	$114
E	MAG FIELD SENSOR W/ CODE	$0	$0	$72	$0	$72	$0	$72
INTELLIGENCE		$103	$481	$590	$355	$441	$332	$298
E	PROCESSOR/DIAGNOSTICS (1)	$103	$0	$0	$0	$0	$0	$0
E	PROCESSOR/DIAGNOSTICS (3.4)	$0	$0	$0	$0	$0	$0	$298
E	PROCESSOR/DIAGNOSTICS (4.0)	$0	$0	$0	$0	$0	$332	$0
E	PROCESSOR/DIAGNOSTICS (4.4)	$0	$0	$0	$355	$0	$0	$0
E	PROCESSOR/DIAGNOSTICS (5.9)	$0	$0	$0	$0	$441	$0	$0
E	PROCESSOR/DIAGNOSTICS (6.6)	$0	$481	$0	$0	$0	$0	$0
E	PROCESSOR/DIAGNOSTICS (8.5)	$0	$0	$590	$0	$0	$0	$0
COMMUNICATION		$0	$58	$58	$58	$58	$131	$131
E	ROAD TO VEHICLE (TMS), Receive	$0	$10	$10	$10	$10	$0	$0
E	VEHICLE/VEHICLE, FORE & AFT	$0	$48	$48	$48	$48	$48	$48
E	R/T, AUTO CHECK-IN AND CONTROL	$0	$0	$0	$0	$0	$83	$83
ACTUATORS		$513	$361	$361	$361	$361	$361	$361
M	BRAKE ACTUATOR	$190	$190	$190	$190	$190	$190	$190
M	THROTTLE ACTUATOR (Engine Control)	$19	$19	$19	$19	$19	$19	$19
M	STEERING ACTUATOR	$304	$304	$304	$304	$304	$304	$304
M	(LESS STD DIRECT DRIVE)	$0	($152)	($152)	($152)	($152)	($152)	($152)
INTERFACES		$63	$120	$120	$120	$120	$120	$120
E	STD ACTUATOR INTERFACE UNIT	$6	$6	$6	$6	$6	$6	$6
M	DRIVER INTERFACE UNIT	$57	$0	$0	$0	$0	$0	$0
M	DRIVE BY WIRE DRIVER INT UNIT	$0	$114	$114	$114	$114	$114	$114
v1	Brake & steering actuator	$494	$494	$494	$494	$494	$494	$494
v2	Radar & vision sensor (PRICE components)	$306	$486	$666	$396	$576	$396	$576
v3	Other electronics	$169	$605	$726	$479	$577	$529	$507
v4	Other electro-mechanical	$76	$19	$19	$95	$95	$95	$95

Table 1 (continued). Vehicle costs summary (point estimates).

Electronics 7 yr support cost per vehicle, 2002 procurement, 1994 dollars, 1million unit market

E - Electronics component
M - Electro-mechanical component

				Dedicated Lanes			
		Mixed Traffic		Vehicle Intensive		Infrastructure Intensive	
	ARV	AHS1	AHS2	AHS1	AHS2	AHS1	AHS2
VEHICLE	$571	$1,134	$1,399	$913	$1,106	$904	$922
SENSORS	$164	$221	$284	$197	$260	$198	$261
E MULTI BEAM MILLIMETER RADAR	$142	$142	$142	$142	$142	$142	$142
E MAGNETIC FIELD SENSOR	$22	$22	$0	$22	$0	$22	$0
E VISION-BASED SENSOR	$0	$56	$112	$28	$84	$28	$84
M RAIN/SNOW SENSOR	$0	$1	$1	$1	$1	$0	$0
M BEACON EMITTERS	$0	$0	$0	$4	$4	$6	$6
E MAG FIELD SENSOR W/ CODE	$0	$0	$29	$0	$29	$0	$29
INTELLIGENCE	$91	$565	$767	$368	$498	$336	$291
E PROCESSOR/DIAGNOSTICS (1)	$91	$0	$0	$0	$0	$0	$0
E PROCESSOR/DIAGNOSTICS (3.4)	$0	$0	$0	$0	$0	$0	$291
E PROCESSOR/DIAGNOSTICS (4.0)	$0	$0	$0	$0	$0	$336	$0
E PROCESSOR/DIAGNOSTICS (4.4)	$0	$0	$0	$368	$0	$0	$0
E PROCESSOR/DIAGNOSTICS (5.9)	$0	$0	$0	$0	$498	$0	$0
E PROCESSOR/DIAGNOSTICS (6.6)	$0	$565	$0	$0	$0	$0	$0
E PROCESSOR/DIAGNOSTICS (8.5)	$0	$0	$767	$0	$0	$0	$0
COMMUNICATION	$0	$5	$5	$5	$5	$27	$27
E ROAD TO VEHICLE (TMS), Receive	$0	$0	$0	$0	$0	$0	$0
E VEHICLE/VEHICLE, FORE & AFT	$0	$5	$5	$5	$5	$5	$5
E R/T, AUTO CHECK-IN AND CONTROL	$0	$0	$0	$0	$0	$22	$22
ACTUATORS	$311	$311	$311	$311	$311	$311	$311
M BRAKE ACTUATOR	$107	$107	$107	$107	$107	$107	$107
M THROTTLE ACTUATOR (Engine Control)	$5	$5	$5	$5	$5	$5	$5
M STEERING ACTUATOR	$199	$199	$199	$199	$199	$199	$199
M (LESS STD DIRECT DRIVE)	$0	$0	$0	$0	$0	$0	$0
INTERFACES	$5	$32	$32	$32	$32	$32	$32
E STD ACTUATOR INTERFACE UNIT	$4	$4	$4	$4	$4	$4	$4
M DRIVER INTERFACE UNIT	$1	$0	$0	$0	$0	$0	$0
M DRIVE BY WIRE DRIVER INT UNIT	$0	$28	$28	$28	$28	$28	$28
vm1 Total electronics	$259	$794	$1,059	$569	$762	$559	$577
vm2 Total electro-mechanical	$312	$340	$340	$344	$344	$345	$345

Table 2. Roadway costs summary (point estimates).

Roadway construction cost estimates

Item	Description	Option 2	Option 3	Option 5	Option 1	Option 4
1	Mass Earthwork	6,033,333	8,283,333	5,000,000	0	0
2	Retaining Walls	19,008,000	21,600,000	5,184,000	0	0
3	Bridges	65,877,200	83,886,800	148,320,000	0	0
4	Pavement	13,027,680	14,755,680	215,200	0	0
5	Soundwalls	6,652,800	6,652,800	0	0	0
6	Landscaping and Erosion Control	1,000,000	1,000,000	0	0	0
7	Pedestrian and Bicycle Facilities	250,000	400,000	100,000	0	0
8	Signalization and Lighting	3,000,000	5,250,000	2,250,000	0	0
9	Drainage & Creek Channel Improvements	5,000,000	7,700,000	2,250,000	0	0
10	Barrier and Guard Railing	2,084,000	2,309,000	225,000	0	0
11	Signage	3,000,000	3,450,000	450,000	150,000	150,000
12	Striping	2,000,000	2,180,000	380,000	0	528,000
13	Construction Support and Detours	17,084,000	21,584,000	21,584,000	2,220,000	1,120,000
14	Existing Facilities - Remove, salvage, etc	10,000,000	10,450,000	5,000,000	0	0
15	Utility Relocation incl in Construction Contract	15,000,000	15,900,000	5,000	0	0
16	Other Itemized Costs	5,880,000	6,255,000	3,255,000	3,520,000	880,000
	Subtotal	**174,897,013**	**211,656,613**	**194,218,200**	**5,890,000**	**2,678,000**
17	Mobilization	17,489,701	21,165,661	19,421,820	589,000	267,800
	Total Bid Level Cost	**192,386,714**	**232,822,274**	**213,640,020**	**6,479,000**	**2,945,800**
18	State Furnished Materials and Expenses	4,000,000	4,500,000	4,000,000	300,000	50,000
	Subtotal	**196,386,714**	**237,322,274**	**217,640,020**	**6,779,000**	**2,995,800**
19	Contingency	39,277,343	47,464,455	43,528,004	1,355,800	599,160
	Total Construction Cost	**235,664,057**	**284,786,729**	**261,168,024**	**8,134,800**	**3,594,960**
20	Land Acquisition	39,600,000	55,800,000	16,200,000	0	0
	Interchange area plus preservation	11,890,000	11,890,000			
21	Utility Relocation	25,000,000	27,250,000	2,250,000	0	0
	Total Right-of-way Cost	**76,490,000**	**94,940,000**	**18,450,000**	**0**	**0**
	Total Construction plus Right-of-way Cost	**312,154,057**	**379,726,729**	**279,618,024**	**8,134,800**	**3,594,960**

In discussions covering each model parameter shown in tables 1 and 2 with project team specialists, the variables shown in figure 1 (to be discussed shortly) were selected as focal points for assessing subjective probabilities. The specialists felt that these variables addressed issues for which there could be significant uncertainty. Other parameters in the cost models were felt to be essentially *deterministic* and did not require analysis beyond using point estimates.

A key study assumption is worth reiterating at this point. The study made no attempt to determine the feasibility of implementing AHS, at either the technical or institutional level. Instead, we have explored the costs and benefits of alternative AHS configurations, under the assumption that these configurations perform as anticipated. Thus in analyzing uncertainty related to vehicle costs for example, our emphasis was on assessing uncertainties about costs *given* the vehicle add-on equipment specifications. We did not analyze the uncertainty about whether the technical aspects of add-ons were adequate.

Vehicle electronics cost variables. The variables (and their mnemonics used in subsequent tables and graphs) chosen for uncertainty analysis were as follows:

1. Multibeam millimeter radar and vision-based sensor initial costs. While the cost of most items on table 1 were estimated using catalogue prices of similar items or actual engineering experience, these two key sensor costs were developed using a parametric cost prediction model (PRICE) where existing equivalent systems were not in production. Inputs to the PRICE model required forecasting such things as weight and technology for production subassemblies. An "initial cost multiplier" or *icm* variable was defined for PRICE estimated sensors (point estimate of 1) to quantitatively express uncertainty in these initial costs. (icm_elec)

2. Time improvement parameter (TIP) for electronics products. This is the yearly discount factor (point estimate of 20 percent or 0.20) used to model how economic competition lowers the initial cost of these products over time. (TIP_elec)

3. Brake and steering actuator initial costs. These electro-mechanical products were estimated from discussions with an owner/operator of a local automobile repair business rather than from catalogues. An "initial cost multiplier" or *icm* variable was defined for these actuators (point estimate of 1) to quantitatively express uncertainty in these initial costs. (icm_mech)

4. Time improvement parameter (TIP) for electro-mechanical products. This is the yearly discount factor (point estimate of 10 percent) used to model how economic competition lowers the initial cost of these products over time. (TIP_mech)

A 2002 vehicle procurement year and 1 million vehicle unit market were fixed for the risk analysis (although a limited sensitivity was performed assuming other procurement years). The use of cost reduction curves for calculating unit market costs was treated as a deterministic computation. (Discussions with the vehicle cost specialist indicated that reasonable changes in the choice of which cost reduction curve to use would not affect results significantly. The 100 thousand unit market implies costs approximately 7 percent greater than the 1 million unit market. Unit markets of still smaller size were considered of much less interest when taking into account the hoped for non-negligible market penetrations and the anticipated number of AHS ready vehicles required to utilize increased roadway capacity.)

Figure 1 lists processor/diagnostic costs in curly brackets to indicate that there emerged a contrary opinion from a different specialist elicited regarding the technical requirements of processing power of different AHS vehicle options relative to the ARV. Strictly speaking, since this risk analysis is confined to cost rather than technical requirement uncertainties, we used the processor assumptions described in volume 3. However, a limited sensitivity analysis was performed considering the contrary viewpoint.

Roadway cost variables. Most of the variables in table 2 were treated as deterministic based on information from standard sources or engineering experience. Analogous to vehicle costs, the specifications based directly on the nature of the specific example freeway selected (highway 101) were accepted in this risk analysis and not second-guessed as to how typical such situations might be elsewhere. (See, however, the topic of *bridge crossings* below.) However, even with this specific roadway, there were uncertainties deemed worth investigating in that it was possible to imagine potential significant cost changes if particular point estimate assumptions changed. The variables chosen for uncertainty analysis were as follows:

1. Retaining walls. The uncertainty revolved around what actual percentage of the project length (point estimate of 25 percent) would require retaining walls.

2. Sound walls. The uncertainty revolved around what actual percentage of the project length (point estimate of 30 percent) would require sound walls.

3. Drainage and creek channel improvements. These costs were computed for other options relative to Option 2. The latter had some uncertainty regarding the magnitude (point estimate of $5M) of the costs.

4. Utility relocation included in construction contract. The uncertainty revolved around what allowance (point estimate of $1.5M) per 1.6 km (1 mi) is appropriate.

5, Land acquisition right-of-way costs. The uncertainty revolved around what cost (point estimate of $25) per 0.093 square m (1 ft^2) is appropriate.

6. Land area required for interchanges. The uncertainty revolved around the size required (point estimate of 0.405 hectares (1 acre)).

7. Utility relocation. The uncertainty revolved around what allowance (point estimate of $25M) per 16 km (10 mi) is appropriate.

It turns out that none of these seven variables are relevant factors for Roadway Options 1 and 4 and thus these options are treated entirely using point estimates. (Their costs from table 2 are quite small relative to the other options shown.) Only Drainage/creek channel improvement and Land acquisition right-of-way cost uncertainties are germane for Option 5, and the risk analysis takes this into account.

The risk analysis did not address particular roadway cost items, most of which were not explicitly modeled in the Roadway Infrastructure Cost Methodology. These included:

- Inflation. As per the cost methodology assumption, no escalation or deflation was considered for roadway costs.

- Management costs. These costs were not available at the time the risk analysis was performed.

- Contingency costs were treated as a deterministic 20 percent multiplier on construction costs rather than with any probabilistic analysis (e.g., contingency costs were treated as pro forma parts of a contract protocol).

- Support building costs for roadway infrastructure were ignored.

- The requirement for queuing plazas for certain options was ignored. However, a *back-of-the-envelope* computation was elicited indicating that this cost would be approximately six million dollars.

Figure 1 lists *bridge crossing* modification costs in curly brackets. Table 2 indicates that such bridge costs (Item 3) represent a large component of construction costs. Technically speaking, however, such costs are not that uncertain for the *specific road segment* because the cost parameters for such construction averaged over a number of such crossings are well documented in cost handbooks. However, the specialist acknowledged that this particular roadway segment featured bridge crossings at a frequency of 1.5 to 2 per 1.6 km (1 mi) rather than a more typical one crossing per this distance. Because this cost item is so large in magnitude, we did a limited sensitivity analysis considering the cases where the bridge crossing costs for Options 2 and 3 were postulated to be 50 percent and 75 percent of their base case bridge costs.

Cost/Benefit Summary Indices

As shown in figure 1, five indices were selected as bottom-line summary factors.

Costs. Total *vehicle capital* (acquisition) costs and total *vehicle capital plus maintenance* costs in 1994 dollars were selected representing consumer related costs. Total roadway *capital construction plus right-of-way* costs in 1994 dollars were selected representing public related costs. (The risk analysis chose to ignore roadway electronics infrastructure capital and maintenance costs, and roadway maintenance costs as being much less significant in magnitude relative to the cost indices chosen). Probability distributions for summary cost indices were estimated using the intermediate cost variables. (No additional subjective probability assessments directly using any cost summary indices were required.)

Benefits. The focus for probability assessments was on the *capacity* (expressed in vehicles/hr/lane) that could be accommodated by the AHS1 and AHS2 vehicle options. For capacity, only the AHS1 and AHS2 distinction (i.e., manual versus automated lane changes) was considered relevant in these assessments. The focus on capacity relates especially to the following specific premise: whether or not an AHS should be built to relieve congestion in cities may hinge on whether the space savings aspect of AHS (due to higher capacity per lane) offset any cost increases that may come from more complicated construction or from installation of electronics in the vehicle or on the roadside.

The risk analysis did not formally consider other possible benefits from AHS such as energy savings, pollution reduction or improved safety. These are discussed in other volumes in this report. The safety aspect of AHS, however, was identified as a key factor affecting potential market penetration. Safety from a market penetration perspective is discussed below.

Market penetration. Subjective probability assessments of *market penetration* (defined as the percentage of registered vehicles consumers would equip for AHS2) were elicited for different market penetration scenarios. The market scenarios were defined by two parameters: acquisition cost in 1994 dollars of the vehicle electronics add-on (a $1000 and $2000 case were assessed), and the percent of freeway lane kilometers available for AHS2 operation (a 10 percent and 20 percent case were considered). A reference region for thinking about these assessments was the Los Angeles area assumed to involve a steady state future situation consisting of 10 million registered vehicles and 3600 lane kilometers of freeway. These assessments were used to develop overall distributions on market penetration considering both the uncertainty in actual vehicle acquisition costs and uncertainty in market penetration given such costs.

Finally, the potential impact of AHS safety on market penetration was considered as follows. The specialist felt that AHS as a new technology would not penetrate the market significantly unless it was shown to be safer vis-a-vis fatality accident rates than conventional alternatives. The market penetration issue was how much safer did AHS need to be for "market acceptance." Subjective probability assessments as to the fraction (less than 1) of the conventional fatality accident rate required for AHS acceptance were elicited.

In summary, section 2 described the factors that were chosen as focal points for the assessing of subjective probabilities. Uncertainty in these factors affects the uncertainty in the summary cost/benefit indices for the different vehicle and roadway modification AHS deployment scenarios.

3. PERCENTILE ESTIMATES ASSESSMENT FOR KEY FACTORS

This section describes the process by which subjective probability assessments were elicited from project team specialists. The results of this process in the form of percentile estimates were tabulated for each key factor identified in section 2. These percentile estimates are then used to develop three-point probability distribution approximations for each key factor.

Percentile estimates for each key factor were assessed from specialists using formal subjective probability assessment techniques.[2] In addition to formalizing parameter uncertainties quantitatively, these techniques help prevent common pitfalls such as understating the uncertainty in knowledge that is present about key parameters, and promote internal consistency.

The protocol used to perform these assessments for each of the factors described in section 2 consisted of the following sequence.

1. The specialist was asked to specify a level such that there was only a 5 percent probability the factor would be greater than this level (more loosely speaking, a level such that it would be surprising if the factor exceeded that level but not implausible). When asked to begin thinking of such a plausible higher level, the specialists elicited would often volunteer that the level they specified represented the 95th percentile before the assessor asked if that seemed appropriate.

2. The specialist was then asked (in a way analogous to the immediately preceding) to specify a level such that there was only a 5 percent probability the factor would be less than this level. This level represented the 5th percentile.

3. The specialist was then asked to specify a level such that the factor was just as likely to be above the level as below it. Levels between the 5th and 95th percentiles were successively suggested in a gradual "homing in" dialogue asking whether it was more likely for the factor to be above the level or below it. The process continued until the specialist felt that it would be an "even bet" that the factor would be above the final level suggested versus below that level. This level represented the 50th percentile. During this process, the specialist was reminded that it was not at all necessary for the point estimates described in the cost methodologies to be equated, say, with the 50th percentile. None of the specialists had any difficulty with this point.

4. The specialist was then asked, "given you *knew* the factor would be greater than the 50th percentile, what is the level for which it would be an even bet that the factor would be above it versus below it." This level represented the 75th percentile.

5. The specialist was then asked, "given you *knew* the factor would be less than the 50th percentile, what is the level for which it would an even bet that the factor would be above it versus below it." This level represented the 25th percentile.

6. The specialist was then asked if it represented a fair or even bet that the factor was between the 25th and 75th percentiles versus being outside this interval. For most of the assessments, the specialists answered yes to this question showing internal consistency. Occasionally, some slight adjusting of the 25th and 75th percentiles was performed to obtain this consistency.

7. The specialist was also asked if the 25th percentile divided the 5th to 50th percentile interval into approximately equally likely intervals. Technically, the 27.5 percentile would do this. But an affirmative answer to this question suggested that the assessed 5th percentile level was indeed reasonably close to that percentile as opposed to the known tendency for some subjects to state a less extreme percentile (like the 15th or 20th) but claim it represents the 5th percentile. The specialists responded with an affirmative confirmation to this consistency check.

8. The specialist was also asked if the 75th percentile divided the 50th to 95th percentile interval into approximately equally likely intervals. Technically, the 72.5 percentile would do this. But an affirmative answer to this question suggested that the assessed 95th percentile level was indeed reasonably close to that percentile as opposed to the known tendency for some subjects to state a less extreme percentile (like the 85th or 80th) but claim it represents the 95th percentile. The specialists responded with an affirmative confirmation to this consistency check.

The results of using this protocol for the factors described in section 2 are shown in table 3. A few comments now follow. The cost factors are as described above. For example, the electronics initial cost multiplier is just as likely to be above 1 as below 1. There is a 25 percent chance of it being below 0.85 (i.e., the actual initial 1994 dollar cost has a 25 percent chance of being less than 0.85 of the point estimate cost for the radar and vision-based sensors).

For the AHS1 capacity assessment, the specialist felt that without automated lane changing, the actual capacity realized by the system would hardly be better than a conventional system; that is, the manual lane changing problem would make it difficult to achieve capacity gains. The *Processor assessment* was elicited to reflect the specialist's contrary opinion about how to cost out processor requirements. It represents the 1994 initial dollar cost of a processor required for the radar of an ARV. For this specialist, this represented a *unit* of processing power with which other processor/diagnostic requirements could be scaled.

Table 3. Individual factor percentile assessments.

Subjective probability assessments

Vehicle electronics cost			Percentiles		
Electronics:	5th	25th	50th	75th	95th
Initial Cost Multiplier (icm)	0.5	0.85	1	1.25	2
Time improvement parameter	0.15	0.22	0.25	0.28	0.4

Notes: Electronics icm applies only to radar and vision-based sensors

Electro-mechanical:	5th	25th	50th	75th	95th
Initial Cost Multiplier (icm)	0.4	0.68	1	1.56	2.5
Time improvement parameter	0.05	0.09	0.12	0.14	0.2

Notes: Electro-mechanical icm applies only to brake actuator and steering actuator

Roadway costs						Notes
	5th	25th	50th	75th	95th	
Retaining Walls (02)	15%	24%	30%	38%	60%	% project length
Soundwalls (05)	20%	26%	33%	40%	50%	% project length
Drainage & Creek (09)	3	5.5	7	9	12	ratios to $5M
Utility Reloc incl (15)	0.8	1.2	1.5	1.8	2.5	$M/mi allowance
Land Acquisition (20)	15	24	30	34	50	$/sq. ft
Land Acquisition (20)	0.6	0.9	1	1.3	2	acres/interchange
Utility Relocation (21)	10	20	25	28	35	$M/10 mi allowance

Additional factors						Notes
	5th	25th	50th	75th	95th	
Capacity (AHS1)	1800		2200		2600	vehicles/lane/hr
Capacity (AHS2)	4500	5300	5800	6300	7500	vehicles/lane/hr
% registered veh - $1K/vehicle	15%	25%	35%	40%	50%	Market penetration
% registered veh - $2K/vehicle	5%	9%	12%	15%	20%	Market penetration
Processor "unit" base$	500		1500		3000	
Fatal accident rate	0.5				0.05	AHS/conventional

Notes: Market penetration assumes 20% of freeway lane miles are AHS.
A criticial mass for penetration is 10% of freeway lane miles.

Vehicle cost add-on:	$500	$1,000	$1,500	$2,000	$3,000	
median % registered vehicles	50%	20%	10%	5%	2%	AHS freeway - 10%
median % registered vehicles		35%		12%		AHS freeway - 20%

The *Fatality accident rate* improvement requirement assessment indicated that there is only a 5 percent chance of market acceptance (i.e., non-rejection) on the safety issue if an AHS has only 0.5 (one-half) the fatality accident rate of the conventional alternative. There is a 95 percent chance of acceptance on the safety issue if the AHS fatality accident rate is 0.05 (one-twentieth) the conventional alternative.

Finally, the last few lines of the table outline a functional relationship between market penetration and the two parameters of vehicle electronics acquisition cost and percentage of available AHS freeway lane kilometers. For example, given 10 percent of the freeway lane kilometers are available for AHS, market penetrations are 20 percent of the registered vehicles for a $1,000 add-on cost and 5 percent for a $2,000 add-on.

Given the percentile estimates shown in table 3, we used the three-point Pearson-Tukey discrete probability distribution to approximate the uncertainty in each factor for purposes of calculating the *mean* and *variance* of individual factors and functions of these factors.[3,4] The Pearson-Tukey or PT three-point approximation replaces the actual probability density function of any continuous factor as defined above with the following three-point discrete probability distribution:

probability (x) = 0.185	if x = 5th percentile.
probability (x) = 0.63	if x = 50th percentile.
probability (x) = 0.185	if x = 95th percentile.
probability (x) = 0	otherwise.

This three-point PT approximation has been shown to give excellent results in estimating the mean and variance for a wide variety of probability distributions for uncertain factors and functions of those factors.[3,4] This approximation is also superior to other suggested *universal* three-point approximations in this regard, and even suggested five point approximations. (It is also superior to simulating in most cases unless the number of simulations becomes enormous. It also gives reproducible results not dependent on a simulation random starting seed.)

As will be elaborated on in sections 4 and 5, the assessed percentile points were used subsequently as follows:

- all percentiles are used in a sensitivity analysis diagram (called a tornado diagram) to show how the cost methodology point estimate summary indices would change as each single factor is varied from its 5th through its 95th percentile level. The percentiles are also interesting in their own right for insight about what uncertainty is present in the current state of knowledge for each factor.

- the PT approximations are used to estimate the mean and variance of summary indices and to derive probability distributions on the summary indices. Although the 25th and 75th percentiles are not part of the PT approximation, they were still useful indirectly by: a) helping to provide an approximate consistency check that the 5th and 95th percentiles were reasonably assessed as described in the protocol above, and b) helping to provide an approximate consistency check on the summary indices distributions derivation by means of an alternate calculation (described in the appendix).

In summary, the main results of section 3 are the subjectively assessed percentile estimates for each key factor as shown in table 3. The protocol for obtaining these assessments allowed project specialists for vehicle costs, roadway modification costs, and capacity/market penetration respectively to systematically quantify their judgmental uncertainty about these factors. The percentile estimates were then used to develop approximate three-point discrete probability distributions for each factor. These three-point distributions provide the mechanism for ultimately deriving the probability distributions on the cost/benefit summary indices

4. DEVELOPMENT OF A SIMPLIFIED FRAMEWORK DELINEATING RELATIONSHIPS AMONG INTERMEDIATE AND SUMMARY COST/BENEFIT FACTORS

This section describes how the three-point approximations developed in section 3 are used to develop probability distributions for the cost/benefit summary indices. Formulas were developed relating intermediate and summary cost/benefit facts. Most of the relationships concern those between the various cost parameters that go into computing overall vehicle electronics add-on and roadway capital/construction cost summary indices. These will be described first followed by relationships involving market penetration estimation. Also discussed are formulas relating the means and variances of individual factors to those of the summary indices, and formulas for the probability distribution derivation for the summary indices.

Vehicle Capital Costs (Electronics Add-On)

Table 1 shows the summary point estimate computation for the electronics add-on package to a vehicle for the different vehicle options. As described in volume 3, each cost component was arrived at by estimating an initial cost and then applying time improvement factors and unit production cost reduction factors to arrive at the result such as that shown in table 1. At the bottom of the first page of table 1 are the summation of the individual capital cost components separated into four groupings (labeled v1 through v4).

The risk analysis developed a formula to take as input the summary four grouping figures in table 1, infer original initial cost estimates, and then recompute a summary figure based on alternative estimates for the four factors involving uncertainty described in section 2 for vehicle electronics cost. The formula developed is:

Vehicle capital costs =

$$(v2*icm\_elec+v3)*e\_init*(1-TIP\_elec)^n+$$
$$(v1*icm\_mech+v4)*m\_init*(1-TIP\_mech)^n \qquad (1)$$

where:

- v1, v2, v3, v4 are the four groupings at the bottom of table 1 for vehicle procurement;
- icm_elec, TIP_elec, icm_mech, TIP_mech are the four factors regarding vehicle costs for which percentiles were assessed in table 3;
- n equals the number of years over which the time improvement parameter or TIP operates (e.g., n=8);
- $e\_init = 1/(1-0.2)^n$ (= 5.96 for n=8) is the factor for computing the initial cost before any TIP_elec was considered;
- $m\_init = 1/(1-0.1)^n$ (= 2.323 for n=8) is the factor for computing the initial cost before any TIP_mech was considered.

When initial point estimates for the uncertain factors are inserted in this formula, the results shown in table 1 are obtained. This formula allows alternate estimates (selected assessed percentile points, for example) to be used instead of the original point estimates to help compute risk analysis results (e.g., means variances, tornado diagram points) as shown in section 5.

Vehicle Capital and Maintenance Costs (Electronics Add-On)

The second page of table 1 shows the summary vehicle seven year support (maintenance) costs. At the bottom of the second page of table 1 are the summation of the maintenance individual cost components separated into two groupings (labeled vm1 and vm2).

The risk analysis developed a simplified formula to take as input the summary two grouping maintenance figures in table 1, and then recompute a summary figure based on alternative estimates for the two TIP parameters involving uncertainty described in section 2 for vehicle electronics cost. The formula developed is:

Vehicle maintenance costs =
$vm1*ma\_e\_init*(1-TIP\_elec-0.06)^n+$
$vm2*ma\_m\_init*(1-TIP\_mech-0.06)^n$

where:
$ma\_e\_init = 1/(1-0.26)^n$ (= 11.12 for n=8) is the factor for computing the electronics maintenance cost before any TIP_elec was considered
$ma\_m\_init = 1/(1-0.16)^n$ (= 4.034 for n=8) is the factor for computing the electro-mechanical maintenance cost before any TIP_mech was considered

When initial point estimates for the uncertain factors are inserted in this formula, the results shown in table 1 are obtained. (The formula is based on a purely empirical relationship that was noticed in which adding the term of 0.06 to the two original TIPs in a discounting-like formula seemed to reproduce reasonably well the calculations of support costs available at the time of this risk analysis.) This formula allows alternate estimates (selected assessed percentile points, for example) to be used instead of the original point estimates to help compute risk analysis results (e.g., means variances, tornado diagram points) in Step 4. This formula allows alternate estimates (selected assessed percentile points, for example) to be used instead of the original point estimates to help compute risk analysis results (e.g., means variances, tornado diagram points) in section 5.

In implementing the maintenance formula to obtain capital and maintenance costs, the maintenance term related to electronics costs was simply added to the electronics cost term of formula (1) while the electro-mechanical maintenance cost term was added to the electro-mechanical cost term. The reason for separating the capital and maintenance *electronics* and *electro-mechanical* costs into two distinct terms to be summed is related to mean and variance computations discussed below.

Roadway Capital Costs (Total Construction Plus Right-of-Way)

The formulas for deriving these costs are all documented in volume 4 of this report. The formulas include how the factors identified for uncertainty analysis are used to compute the cost *items* 2, 5, 9, 15, 20 and 21 shown in table 2. These formulas were implemented so that the cost items in table 2 could be recomputed depending on factor level assignments.

The bottom-line cost figure (total construction plus right-of-way cost) in table 2 can be viewed as coming from summing: items 1 through 16 each multiplied by the factor 1.32 (1.1*1.2 to include mobilization and contingency), item 18 multiplied by 1.2, and items 20 and 21. Item 20 or land acquisition is a combination of purchasing right-of-way along the route and land for interchanges. Both the amount of interchange land and its price affect the cost of the interchange property purchased.

Capacity

The percentiles for this index were directly assessed (see table 3) for AHS1 and AHS2 and required no further computation or analysis. Comments on the uncertainty about AHS2 capacity in relation to the uncertainty about AHS vehicle add-on costs are presented in section 5.

Market Penetration

The risk analysis developed simplified formulas relating the mean and standard deviation of market penetration to vehicle capital costs and percentage of available AHS freeway kilometers. The mean market penetration (for capital costs greater than $500) equals:

$$\max((34\% - 73\% * \log_{10}(\text{capital cost in \$K})),0) \text{ for 20\% AHS availability} \quad (2)$$

$$\max((12\% - 50\% * \log_{10}(\text{capital cost in \$K})),0) \text{ for 10\% AHS availability} \quad (3)$$

The market penetration was assumed to be normally distributed about the mean, with a standard deviation equal to 0.33*mean for each case. (See section 5 below for how particular coefficients/fits were estimated from the assessed data.)

These formulas were derived by postulating a simple linear relationship between the log of capital costs and mean market penetration, and then solving the linear relationship exactly using the estimated means for the 20 percent AHS availability and point estimates for the 10 percent AHS availability based on assessments in table 3 for the $1,000 and $2,000 capital cost cases respectively. Although coarse, the formulas do give plausible numbers and seemed suitable for the very approximate analysis for which they were employed in section 5.

Mean and Variance Calculations

Means and variances of factors and functions of factors were estimated using the PT three-point approximations as follows:

Individual factors:

$$\text{mean} = 0.185^*(\text{5th percentile} + \text{95th percentile}) + 0.63^*(\text{50th percentile}) \quad (4)$$

$$\text{variance} = 0.185^*((\text{5th percentile})^2 + (\text{95th percentile})^2) + 0.63^*(\text{50th percentile})^2 - \text{mean}^2 \quad (5)$$

(The standard deviation or std is equal to the square root of the variance.) Note that the variance equals the mean of the square minus the square of the mean (e.g., see reference 5 for statistical formulas).

All the factors for which percentiles were assessed in table 3 are assumed to be mutually *probabilistically independent* (heuristically, being told the level of one variable does not change the uncertainty distributions for the other variables).

Cost elements which are functions of a unique single factor:

The mean and variance of such a cost element is obtained by using the *cost* corresponding to (i.e., computed using) each factor percentile, in place of those percentiles in formulas (4) and (5). The cost elements having this property are roadway cost items 2, 5, 9, 15, and 21 in table 2.

Elements which are functions of two independent factors:

For vehicle costs, the electronics cost is a function of icm_elec and TIP_elec, while the electro-mechanical cost is a function of icm_mech and TIP_mech. For roadway costs, total land acquisition costs (item 20 in table 2) is a function of the acquisition price and the interchange area required. For these cases, the PT approximation is first used to derive the probabilities for each possible combination of factor levels. The mean and mean of the square (and from them the variance) of the cost element is then computed using the cost corresponding to each percentile combination and the following combination *weights*:

	Factor A:	5th	50th	95th
Factor B	5th	0.185*0.185	0.185*0.63	0.185*0.185
	50th	0.63*0.185	0.63*0.63	0.63*0.185
	95th	0.185*0.185	0.185*0.63	0.185*0.185

The nine combination probabilities come directly from the assumption that the factors are probabilistically independent (e.g., *given* the 5th percentile on Factor B, the same probability distribution is expected for the 5th, 50th and 95th percentiles on Factor A as originally assessed).

Summary cost indices which are sums of independent random variables:

Once the means and variances of the cost elements described above have been computed, for our case where these cost elements are probabilistically independent of each other we can compute:

overall mean = sum of the means
overall variance = sum of the variances

Thus for the vehicle costs, the mean is the sum of the means of the electronic and electro-mechanical costs (assumed to be independent of each other) and the variance is the sum of their respective variances. For roadway costs, the computed means for items 2, 5, 9, 15, 20 (in total) and 21 are substituted into the sum in table 2 to compute an overall mean. The overall variance is equal to: 1.32*1.32 *(sum of the variances of items 2, 5, 9 and 15) + (sum of the variances of item 20 (in total) and item 21). We need to multiply the variance of the indicated items by 1.32 squared because the variance of a constant times a variable is the constant squared times the variance of the variable. This properly takes into account the effect of the mobilization and contingency multipliers on the variance of the roadway costs.

Market penetration:

This represents a case where *conditional* on a vehicle cost, we get a distribution on the market penetration percentage and we must then *integrate* this over possible vehicle capital costs to arrive at an overall mean and variance for market penetration. Computationally, this case turns out to be very similar to the two-factor combination *matrix*. We first develop a separate PT three-point approximation for the summary vehicle capital cost. Now, however, the market penetration percentiles are not independent of the capital cost percentiles. But, we have a relationship giving the market *mean* conditional on any given vehicle capital cost, namely formulas (2) and (3). For a given cost, the market penetration 5th, 50th and 95th percentiles are (*mean* - 1.645* std), *mean* and (*mean* + 1.645* std) when a normal distribution is assumed. Using this relationship, we compute a total of nine market penetrations (three each for the 5th, 50th and 95th vehicle capital cost percentiles) and compute the mean and mean of the square with the matrix weights shown previously.

Deriving an Overall Distribution on a Summary Index Given Its Mean and Variance

Finally, after computing the mean and variance of a summary index using the PT approximations, we fit these parameters to an overall distribution. In this analysis, we have chosen a lognormal distribution (so-called because the log of the variable is distributed normally) for this fit. (See reference 5 for details of the lognormal distribution). This distribution is reasonable for the summary indices for the following reasons:

- it is the distribution having the maximum entropy (least assumed "information content") when all that is known about a variable is its mean, variance, and that it is nonnegative.[6]
- for a *coefficient of variation* or COV (the ratio of a variable's standard deviation to its mean) that is small (e.g., less than 0.2), the lognormal and normal distributions are very similar and so for sums of variables having this property, one does not really lose the advantage of sums of variables sometimes being well approximated by a normal distribution if one uses a lognormal instead.
- for a coefficient of variation that is somewhat larger, the lognormal captures the property that is often present of there being a distinct skew to the right, which is not well modeled using a normal distribution.

The algorithm for fitting a lognormal proceeds as follows:[5]

1. the *sigma* parameter = sqrt(ln(1+COV^2)), where sqrt means square root.
2. the *mu* parameter = ln(mean) - 0.5*$sigma^2$, where ln means natural log.

To compute any percentile of the lognormal, one uses the percentile points of the "underlying" normal distribution and exponentiates them. For example,

5th percentile = exp(mu - 1.645*sigma), where exp(x) mean e^x
50th percentile = exp(mu)
95th percentile = exp(mu + 1.645*sigma)

The lognormal fit was felt to be the best way to estimate the so-called credibility interval (5th to 95th percentile range) of the summary indices, because it is a commonly-used flexible distribution and it is based on the mean and variance which can be computed somewhat robustly using the PT three-point approximations. However, as a partial check on the credibility interval computations for the summary indices, we made use of other approximations, which are not as good as the PT, but could at least provide a check. These check calculations, described in the appendix, gave 5th, 50th and 95th percentile results very similar to that of the lognormal.

The lognormal distribution has the property that the ratio of the 95th to the 50th percentile is equal to the ratio of the 50th to the 5th percentile. This ratio is equal to exp(1.645*sigma). For example, if sigma were equal to 0.67, the preceding ratio is equal to approximately 3. In relative terms, the credibility interval is sometimes characterized in terms of this ratio (e.g., a "factor of 3" about the median).

In summary, the main result of section 4 is the development of a quantitative framework for relating the factors about which uncertainties have been assessed to the summary cost/benefit indices of interest. This framework contain formulas that calculate how the summary indices change in response to changes in the input factors. Using these formulas, the framework derives lognormal probability distributions on the summary cost/benefit indices based on the subjectively assessed percentiles of the input factors. The lognormal distributions allow for the calculation of uncertainty ranges (credibility intervals) in the summary indices as a indication of the *risk* due to uncertain knowledge.

5. FRAMEWORK IMPLEMENTATION AND RISK ANALYSIS RESULTS

This section describes the results of implementing the framework described in section 4 for different AHS scenarios. The first results presented are sensitivity analyses showing how each cost summary index point estimate changes in response to changes of individual factor inputs across their credibility ranges. These sensitivity analyses provide insight as to which factors have the most effect on the uncertainty in the cost summary indices. Then the derived overall distributions for the cost summary indices, as well as capacity and market penetration are presented and discussed along with their credibility intervals. These overall distributions represent the key results of the risk computations. Finally, selected additional sensitivity analyses are presented with regard to market penetration. The simplified framework from section 4 was implemented using EXCEL spreadsheet software. Graphs and tables of sensitivity analysis and risk analysis distribution outputs were generated to highlight the likelihood of various costs/benefit levels ensuing from different AHS alternatives. Presented below are the main results of the risk analysis computations.

Tornado Diagrams for Summary Cost Indices

The first kind of risk analysis result explores how the summary point estimate costs would change when each factor is set at its 5th, 25th, 50th, 75th and 95th percentile (or *fractile*) respectively while all the other factors remain at their originally assigned point estimates. When such cost variations are sorted by the cost spread from 5th to 95th percentile and then plotted from top to bottom, a type of "tornado" diagram (see for example, references 4 and 7) is produced. The tornado diagrams described here indicate the relative sensitivity of summary cost indices for each AHS option to variations of each factor individually over its *credibility* interval. Figure 2 shows tornado diagrams for the seven vehicle options, while figure 3 shows tornado diagrams for the three roadway retrofit options that were not considered deterministic.

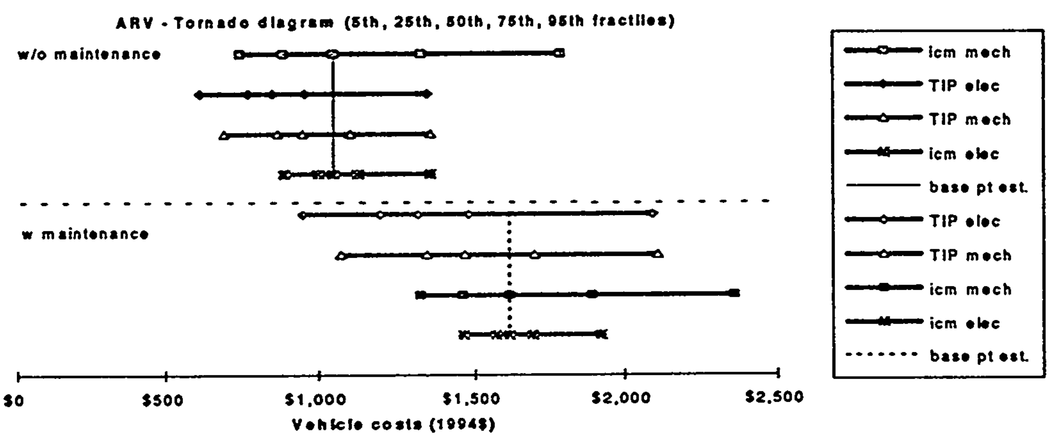
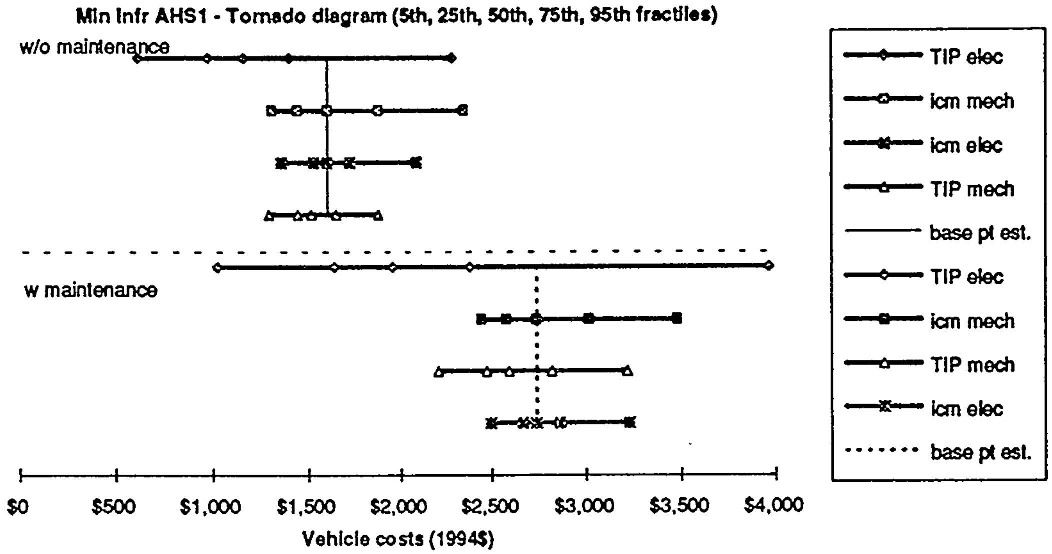


Figure 2. Vehicle cost tornado diagrams.



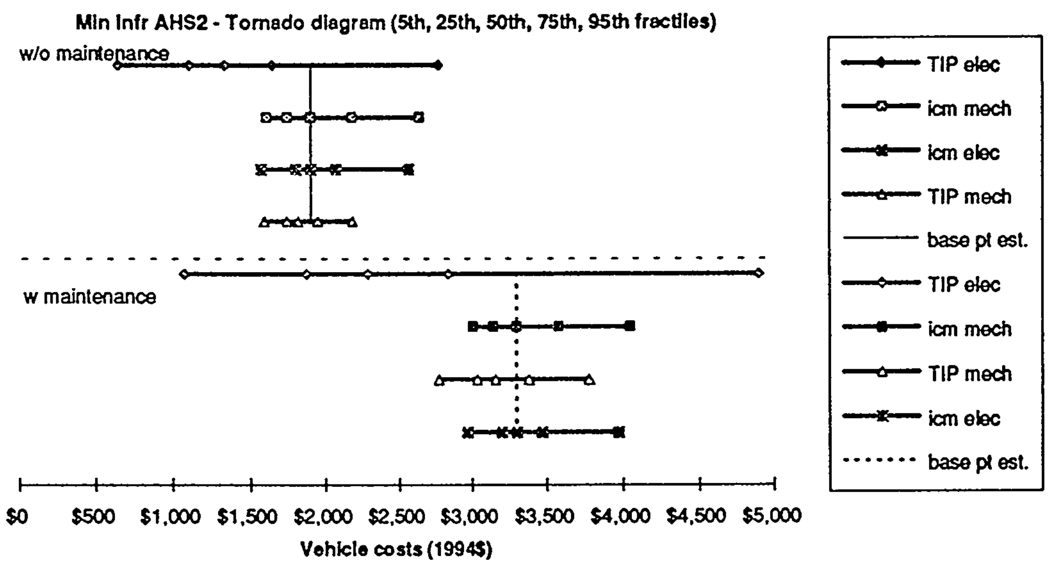
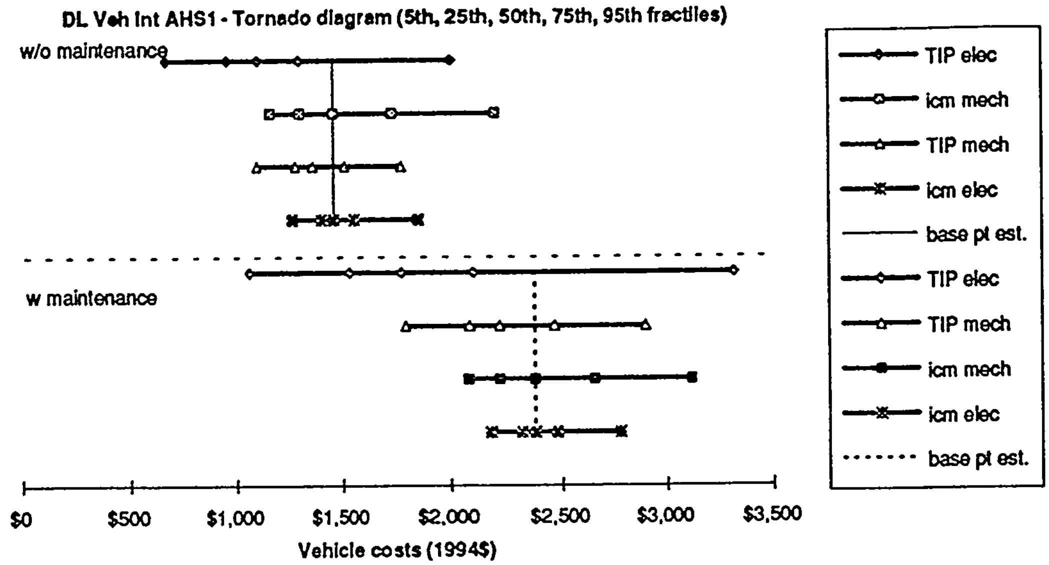


Figure 2 (continued). Vehicle cost tornado diagrams.



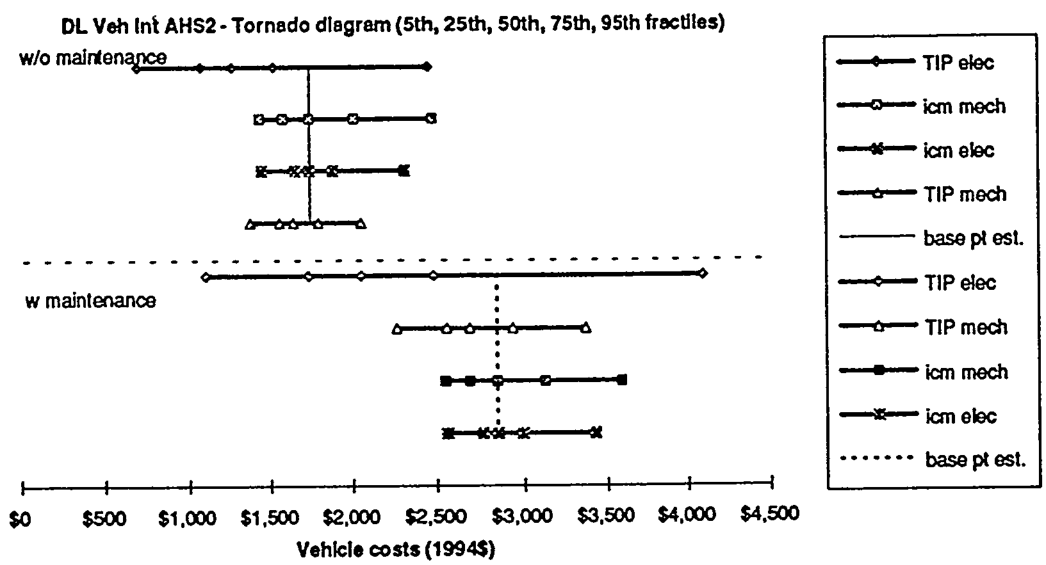
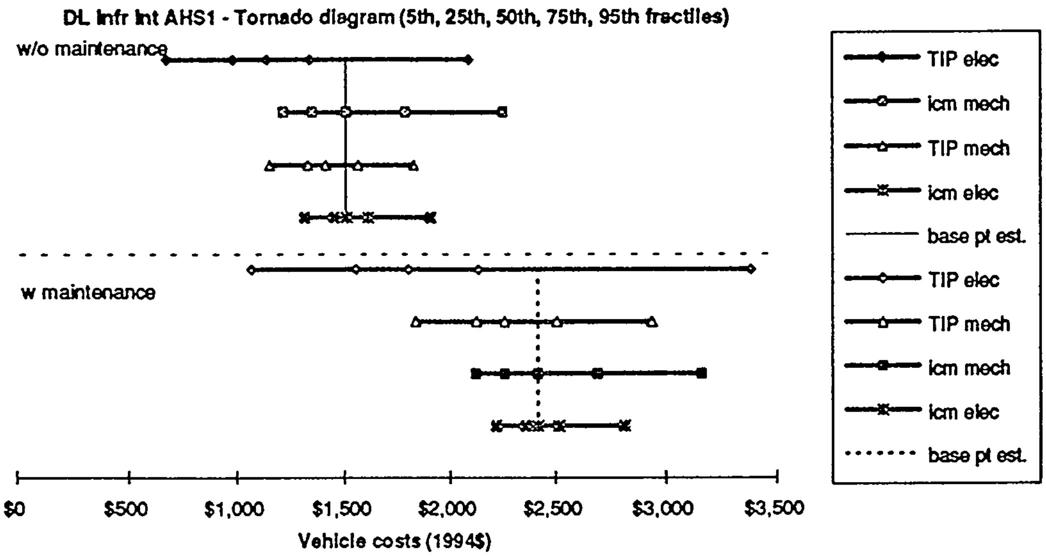


Figure 2 (continued). Vehicle cost tornado diagrams.



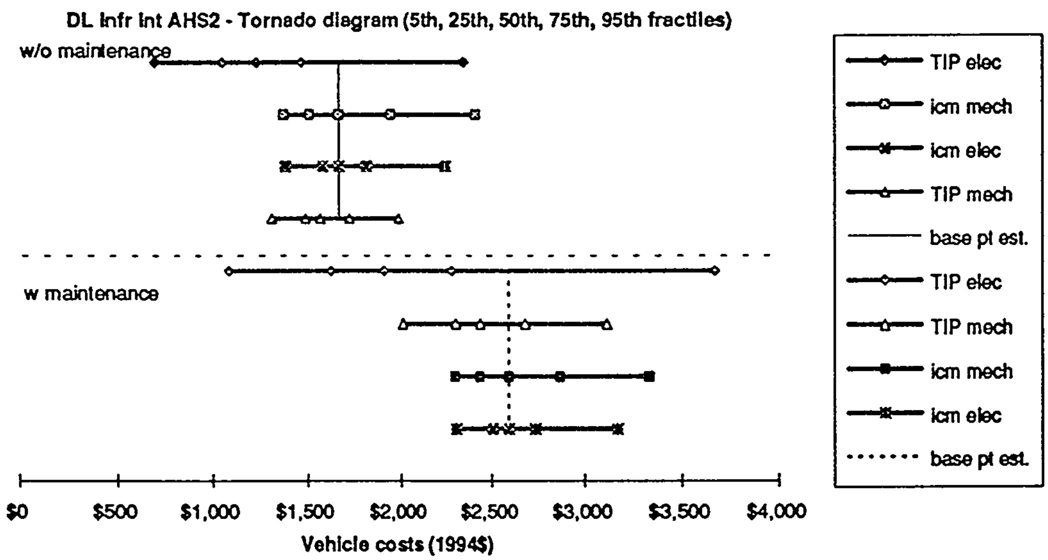
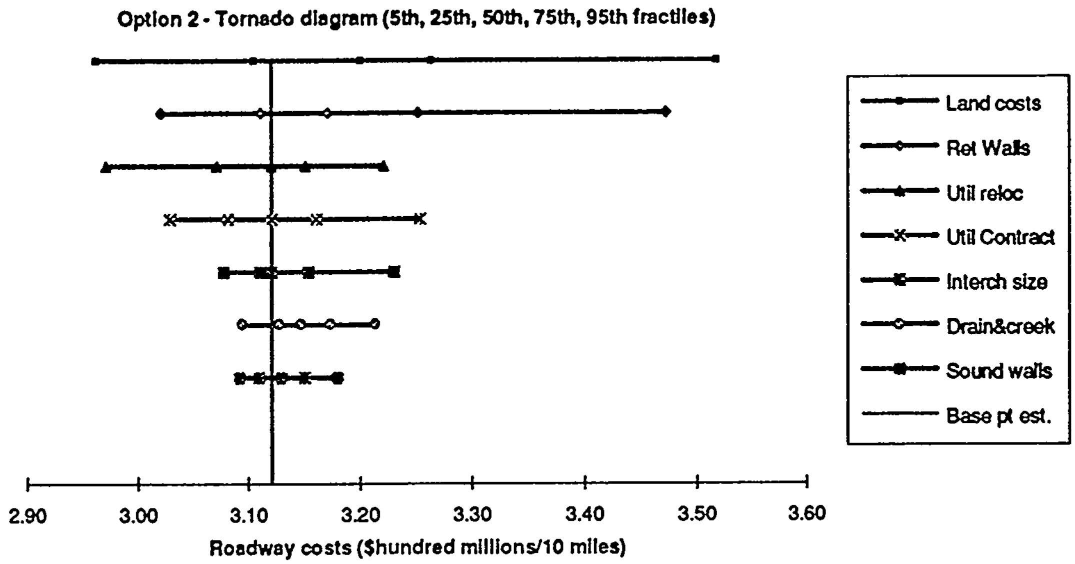


Figure 2 (continued). Vehicle cost tornado diagrams.



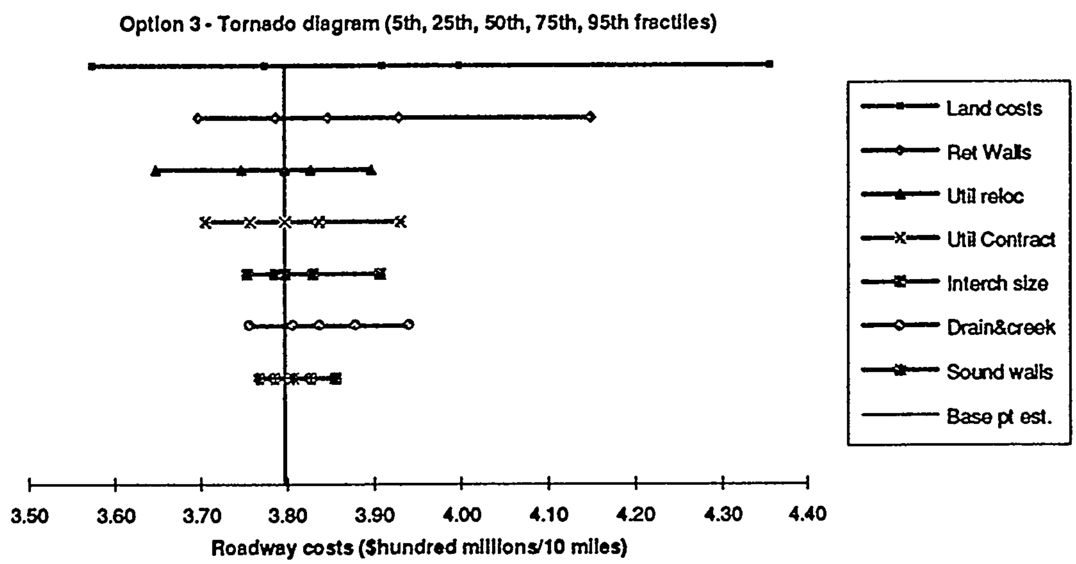


Figure 3. Roadway cost tornado diagrams.

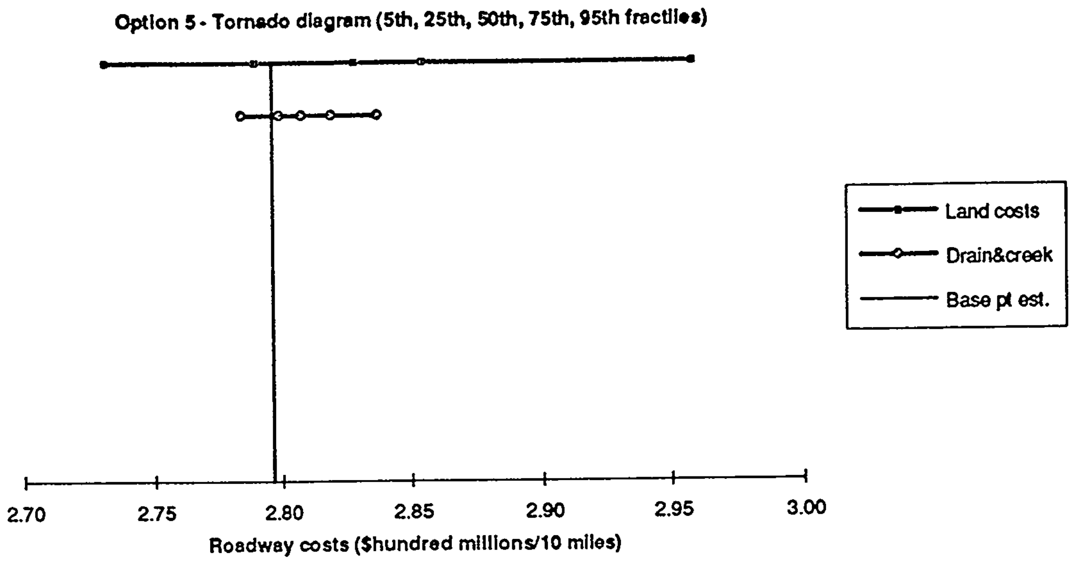


Figure 3 (continued). Roadway cost tornado diagrams.

The vehicle cost spreads in figure 2 come about from a combination of both the degree of uncertainty in each factor listed and the importance of that factor to vehicle costs. The TIP_elec parameter appears at the top of the tornado diagrams for all cases shown except the ARV procurement costs. As can be seen from the cost summary groupings at the bottom of table 1, for the ARV case, the actuator components represent a larger fraction of the procurement costs than in the case of more advanced vehicle alternatives where non-electro-mechanical components represent a much larger fraction. The level assigned to the TIP_elec parameter can have a significant effect on vehicle cost estimates. This factor is relatively critical in estimating if truly dramatic electronics cost reductions over time can occur due to economic competition. The TIP_elec *point estimate* of 20 percent was such that the tornado diagrams show significant possibilities of lower costs (relative to the base point estimate) as this parameter is varied.

The roadway costs results in figure 3 come about from a combination of both the degree of uncertainty in each factor listed and the importance of that factor to roadway retrofit costs. Land costs appear at the top of the tornado diagrams shown for all cases. The point estimates for several factors were such that significant possibilities of higher costs (relative to the base point estimates) are indicated in the tornado diagrams as these factors are varied.

Overall Distributions on Summary Cost Indices

Tables 4 and 5 show features of the derived distributions on the summary cost indices. As indicated in the relationships developed in section 4, these are the lognormal distributions that aggregate the uncertainties on *all* the relevant intermediate cost parameters into a probability distribution for each summary cost index. Figures 4 and 5 show selected key features in graphical terms. Vehicle and roadway results are discussed separately below.

Vehicle costs. Table 4 shows that for vehicle costs, the initial point estimates are larger than the estimated means. This is because the TIP parameters used in the point estimates were relatively conservative when compared to the probability assessments. (The point estimates were both near the 25th percentiles of the factor assessments, and the higher the TIP factors are, the lower the cost.) The standard deviations (or std) are considerable, typically about 50 percent of the means in table 4, for example.

The 5th, 50th and 95th percentiles on vehicle costs are shown given the lognormal parameter fit (mu and sigma) to the estimated mean and standard deviation as described in section 4. They are graphed along with the mean in figure 4 and span the so-called *credibility intervals* for vehicle costs. As explained in section 4, the lognormal can be used to compute any percentile of the overall vehicle cost distribution desired. The credibility intervals are relatively large. In relative terms as described in section 4, the credibility intervals for vehicle costs are approximately a "factor of 2.2" about the median.

Roadway retrofit costs. Table 5 shows that for roadway costs, the point estimates are smaller than the estimated means. This is because particular factor point estimates were relatively *optimistic* when compared to the probability assessments. (The point estimate for land costs for example, was below the 25th percentile of the factor assessment, and the higher the price of land, the higher the overall cost.) The standard deviations (or std) are less than 10 percent of the means in table 5. The main contribution to the *variance* term comes from the land cost uncertainties and amounts to 62 percent, 72 percent and 95 percent for Options 2, 3 and 5 respectively. Retaining wall uncertainties provide 23 percent and 16 percent of the variance for Options 2 and 3 respectively.

The 5th, 50th and 95th percentiles on roadway costs are shown given the lognormal parameter fit (mu and sigma) to the estimated mean and standard deviation as described in section 4. They are graphed along with the mean in figure 5 and span the so-called *credibility interval* for roadway costs. The relative symmetry of the credibility intervals around the median (especially when compared to vehicle costs) reflect a relatively small coefficient of variation (std less than 10 percent of the mean).

Table 4. Probability distributions on summary vehicle costs.

Vehicle costs (excluding maintenance)

				Dedicated Lanes			
		Mixed Traffic		Vehicle Intensive		Infrastructure Intensive	
	ARV	AHS1	AHS2	AHS1	AHS2	AHS1	AHS2
Point estimate	**$1,045**	**$1,604**	**$1,905**	**$1,464**	**$1,742**	**$1,514**	**$1,672**
Grand mean	**$926**	**$1,315**	**$1,536**	**$1,227**	**$1,431**	**$1,261**	**$1,383**
Grand variance	2.28E+05	4.82E+05	7.05E+05	3.75E+05	5.47E+05	3.97E+05	5.09E+05
Grand std	**$478**	**$694**	**$840**	**$612**	**$740**	**$630**	**$714**
Lognormal distribution fit							
5th percentile	$370	$515	$581	$505	$571	$519	$552
50th percentile	$823	$1,163	$1,348	$1,097	$1,272	$1,129	$1,229
95th percentile	$1,830	$2,629	$3,125	$2,384	$2,831	$2,452	$2,733
mu	6.713	7.059	7.206	7.001	7.148	7.029	7.114
sigma	0.486	0.496	0.511	0.472	0.486	0.472	0.486

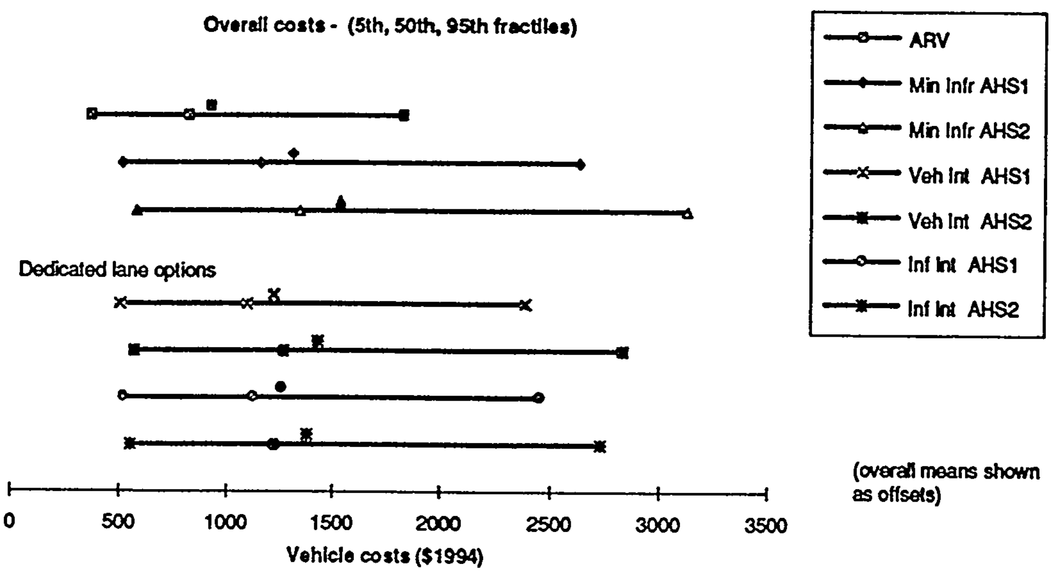
Vehicle costs (including maintenance)

				Dedicated Lanes			
		Mixed Traffic		Vehicle Intensive		Infrastructure Intensive	
	ARV	AHS1	AHS2	AHS1	AHS2	AHS1	AHS2
Point estimate	**$1,616**	**$2,738**	**$3,304**	**$2,377**	**$2,848**	**$2,418**	**$2,594**
Grand mean	**$1,379**	**$2,160**	**$2,563**	**$1,920**	**$2,258**	**$1,949**	**$2,083**
Grand variance	3.79E+05	1.16E+06	1.83E+06	7.94E+05	1.23E+06	8.22E+05	9.98E+05
Grand std	**$615**	**$1,078**	**$1,351**	**$891**	**$1,109**	**$907**	**$999**
Lognormal distribution fit							
5th percentile	$624	$890	$1,004	$842	$943	$853	$888
50th percentile	$1,259	$1,933	$2,267	$1,742	$2,026	$1,767	$1,878
95th percentile	$2,538	$4,198	$5,121	$3,602	$4,354	$3,660	$3,970
mu	7.138	7.567	7.726	7.463	7.614	7.477	7.538
sigma	0.426	0.471	0.495	0.442	0.465	0.443	0.455

Table 5. Probability distribution on summary roadway costs.

Total Construction plus Right--of--way Cost

	Option 2	Option 3	Option 5	Option 1	Option 4
Point estimate	**312,154,057**	**379,726,729**	**279,618,024**	**8,134,800**	**3,594,960**
Grand mean	**337,205,209**	**410,174,749**	**284,755,314**	**8,134,800**	**3,594,960**
Grand variance	8.61E+14	1.23E+15	5.09E+13	not applic.	not applic.
Grand std	**29,341,525**	**35,139,038**	**7,131,361**		
Lognormal distribution fit					
5th percentile	291,212,218	355,050,514	273,178,695		
50th percentile	335,935,853	408,677,830	284,666,058		
95th percentile	387,528,031	470,405,090	296,636,473		
mu	19.632	19.828	19.467		
sigma	0.087	0.086	0.025		



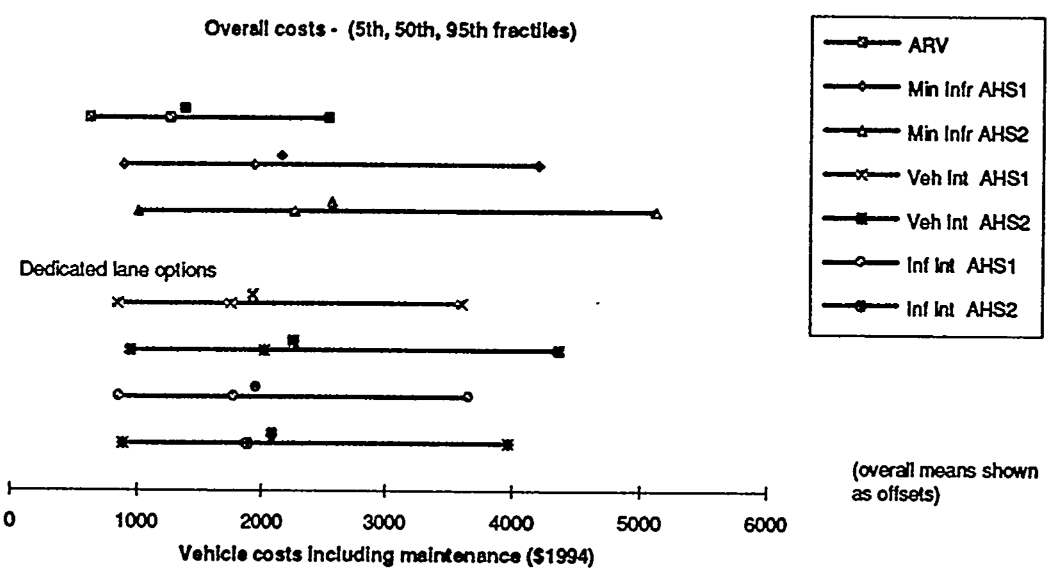


Figure 4. Vehicle cost credibility intervals.

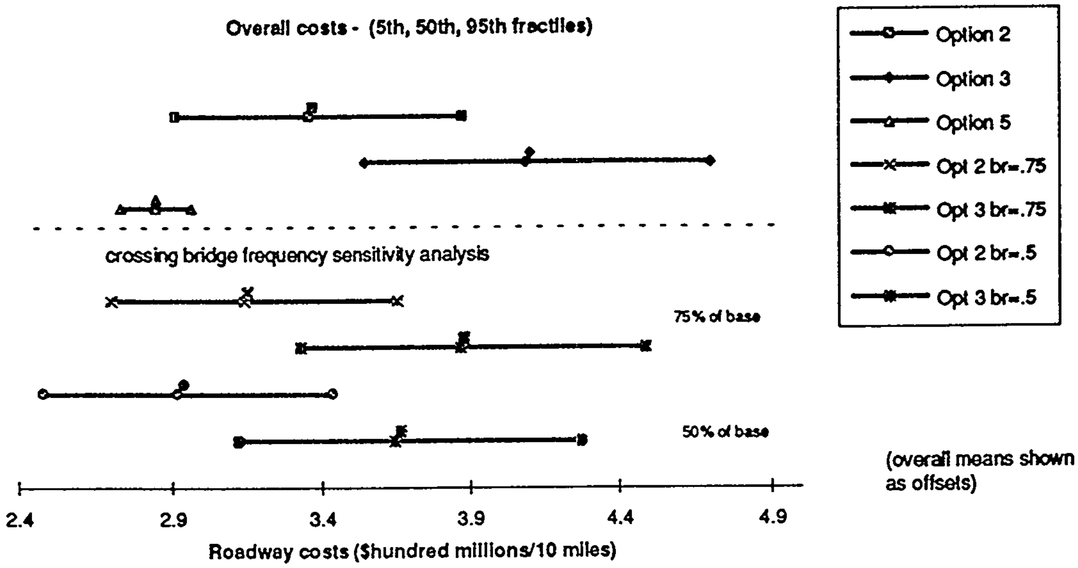


Figure 5. Roadway retrofit cost credibility intervals and bridge crossing sensitivity analysis.

Figure 5 also shows the sensitivity analysis done for different assumptions regarding the number of bridge crossings that may be more typical for a retrofit. The base case clearly shows Option 5 to be cheaper than Option 2, even when the *quite optimistic* 5th percentile costs are compared for all options. With only 75 percent of the base case bridge crossings assumed, Option 2 could be competitive with Option 5 if quite optimistic outcomes (the 5th percentile cost estimate) occurred for both options. However, with 50 percent of the base case bridge crossing assumed, the means of Options 2 and 5 are close. With optimistic outcomes (the 5th percentile cost estimates), Option 2 is cheaper while for pessimistic outcomes (the 95th percentile cost estimates), Option 5 is cheaper. Thus in this last case, it is no longer clear whether Option 2 or 5 would be the best, and land costs would clearly strongly influence the relative attractiveness of Option 2 versus Option 5.

Overall Distribution on Capacity

Using the percentiles directly assessed for the capacity (vehicles/hr/lane) resulting from an AHS2 deployment as shown in table 3, we computed a mean and standard deviation and also fit a lognormal distribution. The percentile inputs and rounded results are summarized in table 6.

Table 6. Probability distribution on capacity.

	5th	25th	50th	75th	95th
Capacity (AHS2)	4500	5300	5800	6300	7500
Capacity: Lognormal fit	4500	5200	5800	6400	7500

Lognormal parameters	Mean	Std	mu	sigma
Capacity (AHS2)	5874.00	917.51	8.67	0.16

One of the properties of the lognormal distribution is that the ratio of two independent lognormally distributed variables is also distributed lognormally with parameters:

mu = mu of the numerator - mu of the denominator and
sigma = sqrt (sigma² of the numerator + sigma² of the denominator).

If we consider the ratio of vehicle costs (see table 4) to capacity, we notice that the sigma parameters for AHS2 vehicles are close to 0.5. Substituting 0.5 and 0.16 for the numerator and denominator sigmas into the formula immediately above, we find that the sigma parameter for the ratio is about 0.525 or essentially the same as would have been the case if we ignored uncertainty in capacity altogether. Given these assessments, if a cost/benefit focus is on vehicle costs per capacity, the vehicle cost uncertainty *dominates* the uncertainty about capacity. (This result would also be obtained if the original capacity percentile assessments were expressed in terms of AHS capacity gains versus conventional capacity; e.g., if conventional capacity were fixed at 2000, such percentiles would be 2.25, 2.65, 2.9, 3.15 and 3.75).

Overall Distribution on Market Penetration

Table 7 recaps market penetration percentile assessment input, and relational assumptions described in section 4 above. The specialist's judgments involved comparing the perceived consumer value of the benefits obtained from the vehicle cost add-on in view of other historical add-ons and their market penetrations (e.g., ABS brakes).

The market penetration fractile inputs are not too asymmetrical about the estimated means. The ratio of the standard deviation to the mean was also similar in both the $1,000 and $2,000 add-on cases (0.31 and 0.37 respectively). Given these observations, the assumption of a market penetration normally distributed about its mean with a standard deviation equal to 0.333 of its mean was made.

Table 7. Market penetration modeling data and calibration.

Overall distribution on market penetration (MP in % of registered vehicles) calculations
Subjective percentile assessments recap: (COV = ratio of std to the mean)

	percentiles							
MP given vehic add-on cost	5th	25th	50th	75th	95th	mean	std	COV
% registered veh - $1K/vehicl	15%	25%	35%	40%	50%	34%	11%	0.31
% registered veh - $2K/vehicl	5%	9%	12%	15%	20%	12%	5%	0.37

Notes: Market penetration assumes 20% of freeway lane miles are AHS.
A criticial mass for penetration is 10% of freeway lane miles.

Vehicle cost add-on:	$500	$1,000	$1,500	$2,000	$3,000	
median % registered vehicles	50%	20%	10%	5%	2%	AHS freeway - 10%
median % registered vehicles		35%		12%		AHS freeway - 20%

Assumptions based on assessments above:
MP is distributed normally with std = 0.333* mean (COV assumed equal to 0.333)

	a	b	AHS freeway
log (mean MP) = a+ b* log (vehicle add-on cost in $K)			
(a, b parameters based on vehicle cost $1K & $2K assessments)	20%	-50%	10% lane miles
	34%	-73%	20% lane miles

Model fit illustration: Vehicle add-on cost in thousands of dollars

	0.50	1.00	1.50	2.00	3.00	4.00	AHS freeway
Mean MP =	35%	20%	11%	5%	0%	0%	10% lane miles
Mean MP =	56%	34%	21%	12%	0%	0%	20% lane miles

The computation of overall means and variances of market penetration for the different AHS2 options was then performed and a lognormal distribution fit done using the relationships described in section 4. The resulting market penetration distributions reflect both the uncertainty in vehicle capital costs and the uncertainty in market penetration given those costs. The market penetration distributions and results are summarized in table 8.

These results can be interpreted in view of the cost estimates shown in table 4 and the market penetration assessments in table 7. The AHS2 options have similar 50th percentile costs, namely, $1348, $1272, and $1229 respectively in table 4. For 20 percent lane kilometer AHS implementation, table 7 indicates a mean (equal to the median for a normal distribution) market penetration of about 27 percent (about halfway between 21 percent and 34 percent in the last line of the table) if costs were exactly $1250. Thus we *expect* a ball park number of about 27 percent or so for the options and indeed this is the case with mean market penetrations ranging from 25 to 28 percent. Similarly for the 10 percent lane kilometer case, the ball park expectation is about 15 percent which is also close to the actual result.

Table 8. Probability distribution on market penetration for AHS2 vehicle options.

20 percent AHS lane kilometers case

Lognormal fit	Mean	Std	mu	sigma	Market penetration prob <5%	prob 5-20%	prob >20%
Minimum infrastructure	25%	18%	-1.60	0.66	0.02	0.48	0.50
Vehicle intensive (DL)	26%	18%	-1.53	0.63	0.01	0.44	0.55
Infrastru intensive (DL)	28%	19%	-1.48	0.61	0.01	0.41	0.59

10 percent AHS lane kilometers case

Lognormal fit	Mean	Std	mu	sigma	Market penetration prob <5%	prob 5-20%	prob >20%
Minimum infrastructure	14%	11%	-2.18	0.69	0.12	0.67	0.21
Vehicle intensive (DL)	15%	12%	-2.10	0.67	0.09	0.68	0.23
Infrastru intensive (DL)	16%	12%	-2.06	0.66	0.08	0.67	0.25

Rather than show the 5th, 50th and 95th market penetration percentiles, table 8 instead indicates the probability that any option would have less than 5 percent (*very small*), between 5 percent and 20 percent (*small*) and greater than 20 percent (*moderate* to *high*) market penetration, using the lognormal distribution fits. The results show that for the 20 percent lane kilometer case, there is about a 0.50 probability of moderate to high market penetration in contrast to about a 0.25 probability for the 10 percent lane kilometer case. These probabilities reflect the relatively large coefficients of variation (std about 70 percent of the mean) for the AHS2 option market penetration summary indices.

Selected Sensitivity Analyses in Relation to Market Penetration

Some limited sensitivity analyses were performed regarding parameters not formally selected for analysis but still mentioned in section 3 above.

Procurement year for vehicle. If the interval over which time improvement factors or TIPs is shortened from the base case of eight years (year 2002) to fewer years such as four, the vehicle capital costs rise significantly. We would expect market penetrations to suffer accordingly. The four year case was run and the results for the lowest cost option were:

- for the 20 percent lane kilometer case, penetration probabilities of 0.83 for very small, 0.15 for small and 0.02 for moderate to high.

- for the 10 percent lane kilometer case, penetration probabilities of 0.93 for very small, 0.06 for small and 0.01 for moderate to high.

Processor costs for AHS2 options. A contrary opinion to the base case processor needs assumptions for AHS2 vehicles assumed the following relative processing requirements in terms of processing units:

ARV	1.3	processing power units
Minimum infrastructure vehicle	46.4	processing power units
Vehicle intensive dedicated lane	18	processing power units
Infrastructure intensive dedicated lane	15.4	processing power units

An analysis was run assuming the *optimistic* $500 per processing power unit (5th percentile) shown in table 3, instead of the original processor costs. The market penetration results for the lowest cost option (Infrastructure intensive vehicle) were:

- for the 20 percent lane kilometer case, penetration probabilities of 0.10 for very small, 0.62 for small and 0.28 for moderate to high.

- for the 10 percent lane kilometer case, penetration probabilities of 0.38 for very small, 0.53 for small and 0.09 for moderate to high.

A similar case was run where the unit base processor cost was assumed to be $1500 or the 50th percentile in table 3. The results for that case were:

- for the 20 percent lane kilometer case, penetration probabilities of 0.77 for very small, 0.19 for small and 0.04 for moderate to high.

- for the 10 percent lane kilometer case, penetration probabilities of 0.88 for very small, 0.11 for small and 0.01 for moderate to high.

If processor requirements and therefore costs were much higher than assumed in the base case risk analysis, the estimated probability of market penetration being moderate to high is significantly lower. It is useful to reiterate at this point that the market penetration model is somewhat coarse. Nevertheless, the base case AHS2 options for vehicles have a great deal of processing built into the vehicle rather than the roadway. Potentially, this choice of technology direction could strongly impact market penetration if such processing proved to be more expensive than originally envisioned. The possible progress in both simultaneously reducing costs and increasing the capabilities of computers, however, is noted in the costing methodology of volume 3. The costing methodology uses the conservative assumption of having only considered cost reduction trends and not capability increases as well. If capability advances were to be considered as well, perhaps even heavier processing requirements than originally assumed for AHS2 vehicles would still give probability of market penetration results similar to the base case risk analysis described in this report.

In summary, section 5 presented the main results obtained from implementing the risk analysis framework. Tornado sensitivity analysis diagrams indicated that the time improvement parameter for electronics (TIP_elec) is a key factor affecting the uncertainty of vehicle costs, and that land costs are a key factor affecting the uncertainty in roadway costs for most scenarios. Overall distributions on the cost/benefit summary indices were described and summarized in tables 4 and 5 for costs, table 6 for capacity and table 8 for market penetration. These distributions reflect risk/uncertainty via the size of the *credibility intervals* (the range spanning the 5th to 95th percentiles) for the summary indices. The market penetration distribution also produces probability estimates of *very small*, *small* and *moderate to high*, market penetrations as a function of vehicle capital cost and AHS freeway lane availability.

6. CONCLUSIONS

Major Findings

A cost/benefit risk analysis can help provide insights by showing more formally how uncertainties in key factors translate into uncertainties in summary cost/benefit indices. The focus of this risk analysis was on quantifying such uncertainty by developing probability distributions for vehicle and roadway costs, capacities and market penetrations for alternative AHS deployment scenarios. These distributions specify the risk or likelihood that costs, capacities or market penetrations could turn out to be significantly higher (or lower) than specified by a single summary best guess number or point estimate.

For vehicle costs, the risk analysis identified four key factors and systematically elicited the subjective probability judgments of a project team specialist to quantify uncertainties about these factor levels. The risk analysis revealed that the *time improvement parameter* for electronics products (the yearly discount factor used to model how economic competition lowers the initial cost of these products over time) is the most important of the factors in its effect on vehicle cost uncertainties. A lognormal distribution for vehicle costs was derived with a coefficient of variation (or ratio of the standard deviation to the mean) around 50 percent for the different scenarios. Vehicle cost percentile levels (e.g., 5th percentile level indicating a 5 percent chance of being less than that level) were tabulated to indicate a credibility interval ranging from the 5th to 95th percentiles. The ratio of the 95th to 50th (median) percentile was typically about 2.2.

For roadway modification costs, the risk analysis identified seven key factors and systematically elicited the subjective probability judgments of a project team specialist to quantify uncertainties about these factor levels. The risk analysis revealed that *land cost* is the most important of the factors in its effect on roadway cost uncertainties. A lognormal distribution for roadway costs was derived with a coefficient of variation (or ratio of the standard deviation to the mean) of less than 10 percent for the different scenarios. Special sensitivity analysis revealed that the cost comparison between a dedicated AHS elevated structure (Option 5) versus one lane automated with an added buffer lane (Option 2) depended strongly on land costs and the frequency of freeway bridge crossings per unit roadway length. Given a crossing frequency 50 percent reduced from the base case (which may have been atypically large), land cost uncertainties are significant enough so that either alternative could turn out to be the cheaper one.

For resulting capacity of an AHS2 implementation, the credibility interval (5th to 95th percentile range) was assessed to be 4500 to 7500 vehicles/lane/hr with a median estimate of 5800. If a cost/benefit focus is on the ratio of vehicle cost to capacity, the uncertainty in the vehicle cost in this analysis dominates the uncertainty in capacity in its effect on the uncertainty in the ratio.

A coarse model relating market penetration to vehicle acquisition cost and AHS freeway availability was calibrated. The uncertainties in both vehicle costs, and the market penetration given vehicle costs led to significant uncertainties in the market penetration for AHS2 scenarios. For 20 percent AHS freeway availability, the probability of moderate to high market penetration (greater than 20 percent of registered vehicles) was around 50 percent . For 10 percent availability, the probability of moderate to high market penetration was 25 percent.

In summary, it is not surprising that the risk analysis in this volume indicates notable uncertainties in indices such as vehicle costs and market penetration. At this point in time of AHS development, a risk analysis which did not show much uncertainty would not be very credible.

Although approximate, cost methodologies and risk analyses using expert judgment can help provide insight into the conditions necessary for having significant market penetration of AHS. The risk analysis methodology presented here is especially pragmatic. It utilizes a systematic protocol for obtaining expert judgments in the form of percentile assessments. It then develops tractable approximations and distribution fits based on these assessments to estimate probability distributions of interest such as those on vehicle and roadway costs and market penetrations. The relationships and approximations are used to estimate key features (such as means and variances) in a sound and effective manner even though the subjective input data by nature can not be extensive. The methodology is also generic and flexible enough to be applicable to other AHS problems having features similar to the one analyzed here.

Recommendations for Future Study

Because AHS technologies still require considerable research, the implementation scenarios analyzed in this study are somewhat speculative, and represent a best guess based on the state-of-the-art today. From this perspective, the implications of the numerical results of this risk analysis should not be overemphasized in suggesting directions for future study. However, the most important factors affecting uncertainty in cost/benefit indices in this risk analysis make intuitive sense to study further for decreasing uncertainties about costs and market penetration. More detailed modeling of the time improvement factor for electronics technologies, and market penetration as a function of vehicle costs and perceived benefits could help reduce some of the more significant uncertainties that relate especially to consumer cost.

Another way of reducing estimation uncertainty about key factors is to pool subjective probability judgments from multiple experts.[8] In this study, single specialists were elicited for each factor because of their special familiarity with the particular cost methodologies, scenario definitions, and time and budget constraints. It is also prudent to first develop a risk analysis methodology that works well with single expert assessments before moving on to the more complicated multiple expert aggregation techniques. Further study into which of these techniques is best suited to pool assessments from multiple experts in the AHS problem context would be desirable.

APPENDIX

This appendix contains further technical detail related to section 4 of this volume concerned with the development of a simplified framework delineating relationships among intermediate and summary cost/benefit factors. The discussion below concerns the development of a check on the lognormal distribution fit to cost/benefit summary indices.

The lognormal fit was felt to be the best way to estimate the so-called credibility interval (5th to 95th percentile range) of the summary indices, because it is a commonly-used flexible distribution and it is based on the mean and variance which can be computed somewhat robustly using the PT three-point approximations. However, as a partial check on the credibility interval computations for the summary indices, we made use of other approximations, which are not as good as the PT, but could at least provide a check. These approximations use formulas for:

- the mean in terms of the 5th and 95th percentiles and the *mode* (most likely value).
- the standard deviation (or std) in terms of the 5th and 95th percentiles alone.

We computed the *mode* of the summary index by using the *computed modes* of each factor for which uncertainty was assessed as factor *point* estimates. The factor modes were developed as follows. If a factor's 25th and 75th percentiles were symmetric about the 50th, we used the 50th percentile as the mode. Otherwise, we used the midpoint of the shortest of the *roughly* equiprobable four intervals: 5th-25th, 25th-50th, 50th-75th, 75th-95th. The formulas for the summary index percentiles in terms of the index mean, mode and standard deviation are:

5th percentile = (2.95*mean - 3.25*std - 0.95*mode)/2
50th percentile = 0.721*mean + 0.279*mode
95th percentile = (2.95*mean + 3.25*std - 0.95*mode)/2

(These formulas are derived from approximations cited in reference 3. While it need not follow that the mode of the summary index is obtained by using the modes of the factors, we hoped this approximation would at least provide a ball-park check on the lognormal fit.) The 5th, 50th and 95th percentile results for the summary indices using both the lognormal and mode methods were very similar with the 50th and 95th percentiles being quite close. Given both methods, the risk analysis advocates using the lognormal as the more confident fit, that is also partially checked with an approximate fit that was done without any distribution assumptions.

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