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Title: **DIRECT FIBER STRENGTHENING IN THREE DIMENSIONAL RANDOM-ORIENTED SHORT-FIBER COMPOSITES**

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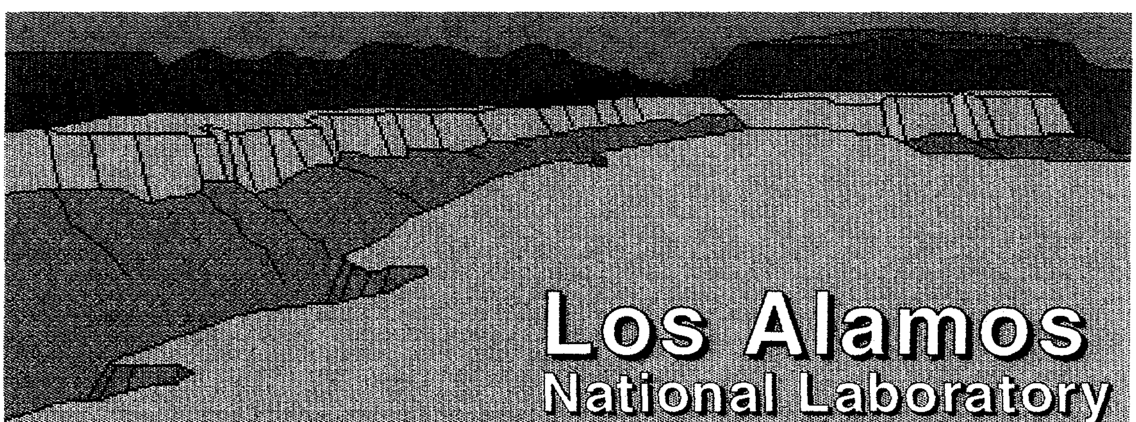

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Direct Fiber Strengthening in Three Dimensional Random-Oriented Short-Fiber Composites

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ABSTRACT

A theory for direct fiber strengthening in random-oriented short-fiber composites is developed. The theory adopts a maximum load composite failure criterion and takes into account the fiber orientation effect on the probability of a fiber being intercepted by a specimen cross-section. The strain and load of short-fibers with different inclination angles with respect to the loading direction were first calculated, and their contribution in carrying load toward the composite load direction was integrated to give the total load. The fibers with smaller inclination angles bear greater stress and break first. This load is then transferred to fibers with larger inclination angles. Direct fiber strengthening component of the composite strength was calculated from the maximum total load these short fibers can carry. The present theory predicts a much greater direct short-fiber strengthening than does previous theories, and provides useful information for composite design and strength assessment.

1. INTRODUCTION

Composites reinforced with randomly-oriented short fibers have become increasingly popular in recent years [1, 2], and the strength of such composites is one of the most important properties. In order to predict the strength of a randomly-oriented short-fiber-reinforced composite, it is essential to understand the strengthening potential of short fibers. Although theories have been developed to predict the strength of composites having continuous or discontinuous fibers with an unidirectional-orientation[3-7], the strength of composites with randomly oriented short fibers has not been well studied. The models of Chen [8] and Halpin and Kardos [9] are among the earliest works on the strength of composites reinforced by randomly oriented short fibers. They treated the composite as a stack of unidirectional-short-fiber reinforced laminae bonded together at different angles, which is unrealistic. In addition, these two theories do not provide any clear relationship between the composite strength and the component properties since they rely on the experimental failure strength and strain data of the unidirectional laminae. Friend [1, 10] proposed an empirical strength equation for randomly oriented short-fiber reinforced metal matrix composites. Due to its empirical nature, his equation can only be applied to particular alloy matrix composite systems. For example, his model seems to agree with experimental data of some aluminum alloy matrix composites, but does not explain the high strength of composites with pure aluminum matrix.

Fukuda *et al* [11] was the first who developed a theory to predict the strength of composites with the three-dimensional-oriented short fibers. However, their theory predicts a low fiber strengthening contribution to the composite strength and does not fit experimentally observed composite strength data. The theory by Zhu *et al* [12] overcomes a few shortcomings of other works. It takes into account the influences of the thermal stress, short-fiber dispersion hardening, dislocation density in matrix and the non-homogeneous deformation on the composite strength. However, it also has certain drawbacks. First, it failed to consider the probability variation of intercepting fibers with different inclination angles by a planar across-section. This results in its prediction of lower direct fiber strengthening potential. Second, the composite failure strain included in its calculation of fiber strengthening can only be obtained experimentally or estimated.

The objective of this paper is to develop a theory which can overcome the deficiencies of the theories mentioned above. In this model, the probability variation of fibers being intercepted by a cross-section with varying fiber inclination angles is taken into account in the calculation of direct fiber strengthening, and the maximum total load is adopted as the composite failure criterion.

2. STRENGTH THEORY

The strengthening mechanisms in short-fiber-reinforced metal and polymer matrix composites include: direct short-fiber strengthening[12], short-fiber dispersion hardening [12], thermally induced matrix work hardening and residual thermal microstress [13-17]. The direct short-fiber strengthening is a major strengthening mechanism and is what we are going to investigate in this paper.

The following assumptions are made for simplicity: (1) All fibers in the composite have the same tensile strength, which is a reasonable assumption and has been widely used in many other theories; (2) All the fibers have the same length and are randomly oriented; And (3) strong bonding exists between the fiber and the matrix.

Fibers with a smaller inclination angle θ toward the loading direction (see Fig. 1) bear larger stress and break first during a tensile loading. Since the fiber usually has higher Young's modulus than the matrix, these broken short-fibers shift their previously carried load through the matrix to fibers with larger inclination angles, which have not reached their ultimate strength yet. Assuming that θ_0 is the critical inclination angle within which every fiber is broken, *i.e.*. fibers with the inclination angle θ_0 bear a stress equal to their ultimate strength and are just about to break, the total load carried by the short fibers can be calculated by integrating the loads carried by all those unbroken short fibers and is a function of θ_0 . Representing the total load by a function $P(\theta_0)$, the maximum of $P(\theta_0)$ can be considered as the total load the short-fiber carries at composite failure and be used to calculate the direct fiber strengthening.

Before deriving $P(\theta_0)$, we need first to calculate the load carried by a fiber with an inclination angle θ. Shown in Fig. 1 is a fiber with an inclination angle $0 \le \theta \le \pi/2$ in a composite sample. Under load P in x_3 direction, the composite strain, ε_c, in the loading direction is

$$\varepsilon_c = \varepsilon_{33} \tag{1}$$

and the strain in x_1 and x_2 directions are

$$\varepsilon_{11} = \varepsilon_{22} = -\nu\varepsilon_{33} \tag{2}$$

where ν is the Poisson's ratio of the composite.

To calculate the strain in a fiber with inclination angle θ, we rotate the coordinate system around x_1 axis clockwise by an angle of θ (see Fig. 1). The transformation matrix A is

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \tag{3}$$

where $a_{ij} = \cos\alpha_{ij}$, and α_{ij} is the angle between y_i and x_j. The strain in y_3 direction (along the fiber) can be calculated as

$$\varepsilon_{33}^{y}(\theta) = \sum_{i=1}^{3} a_{3i} \sum_{j=1}^{3} a_{3j}\varepsilon_{ij} = \varepsilon_{33}\left(\cos^2\theta - \nu\sin^2\theta\right) \tag{4}$$

Substituting Eq. 1 into Eq. 4 yields

$$\varepsilon_{33}^{y}(\theta) = \varepsilon_c\left(\cos^2\theta - \nu\sin^2\theta\right) \tag{5}$$

By the definition of the critical inclination angle θ_0, fibers with θ_0 will be at their failure strain and stress, therefore

$$\varepsilon_{33}^{y}(\theta_0) = \varepsilon_f = \varepsilon_c\left(\cos^2\theta_0 - \nu\sin^2\theta_0\right) \tag{6}$$

where ε_f is the fiber failure strain. Substituting Eq. 6 into Eq. 5 yields

$$\varepsilon_{33}^{y}(\theta) = \frac{\varepsilon_f\left(\cos^2\theta - \nu\sin^2\theta\right)}{\cos^2\theta_0 - \nu\sin^2\theta_0} \tag{7}$$

At $\theta < \theta_0$, $\varepsilon_{33}^{y}(\theta)$ calculated with Eq. 5 will be larger than ε_f, and so the fiber is broken and can no longer carry load, *i.e.* the load carried by a fiber with an inclination angle $\theta < \theta_0$ is

$$f(\theta) = 0 \tag{8}$$

The strain in the fibers with $\theta \geq \theta_0$ can be considered approximately equal to $\varepsilon_{33}^{y}(\theta)$. The load carried by these fibers, therefore, is

$$f(\theta) = E_f a_f \varepsilon_{33}^{y}(\theta) \tag{9}$$

where a_f is the fiber cross-sectional area and E_f is the fiber Young's modulus. Considering $f_0 = E_f \varepsilon_f a_f$, where f_0 is the maximum load a fiber can carry, and substituting Eq. 7 into Eq. 9, we get

$$f(\theta) = \frac{f_0\left(\cos^2\theta - \upsilon\sin^2\theta\right)}{\left(\cos^2\theta_0 - \upsilon\sin^2\theta_0\right)} \tag{10}$$

Setting $f(\theta) = 0$ in Eq. 10 yields

$$\sin^2 \theta_f = 1/(1+\upsilon) \tag{11}$$

where θ_f can be calculated from Eq. (11). From Eq. (10), it can be seen that $f(\theta)$ is positive if $\theta < \theta_f$, which means tensile load in the fiber. But, if θ is lager than θ_f, $f(\theta)$ will be negative due to the Poisson constriction. Since fibers usually has higher Young's modulus than the matrix, fibers should always have higher resistance to deformation than the matrix. Therefore, those fibers under the Poisson constriction will also make positive contribution toward the composite strength. Based on the above argument, the absolute value of $f(\theta)$ should be used for the calculation of average load contribution per fiber toward the loading direction.

With these discussion , the load carried by fibers with different inclination angles can be summarized as

$$f(\theta) = \begin{cases} 0 & \theta < \theta_0 \\ \dfrac{f_0(\cos^2\theta - \upsilon\sin^2\theta)}{\cos^2\theta_0 - \upsilon\sin^2\theta_0} & \theta_0 \le \theta < \theta_f \\ -\dfrac{f_0(\cos^2\theta - \upsilon\sin^2\theta)}{\cos^2\theta_0 - \upsilon\sin^2\theta_0} & \theta_f \le \theta < \dfrac{\pi}{2} \end{cases} \tag{12}$$

To obtain the total load, $P(\theta)$, at a specimen cross-section perpendicular to the loading direction (hereafter referred as cross-section A as indicated in Fig. 2), the orientation distribution of fibers cut by the cross-section as a function of inclination angle θ, $n_c(\theta)$, is also needed. Defining the fiber orientation distribution in the volume of a specimen as $n_v(\theta)$, we can obtain $n_c(\theta)$ from $n_v(\theta)$ by taking into account the following two factors: First, the probability of a fiber being cut by the cross-section A changes with the inclination angle of the fiber, and second, not every fiber intercepted by cross-section A bears load (*e.g.* when the fiber end is cut).

It is assumed that short fibers are initially randomly-oriented. The short fibers tend to align up along the loading direction during tensile loading. But the alignment is so insignificant that it can be ignored [12]. Therefore, the short fibers can be approximately considered to remain randomly-oriented during the loading, and the fiber orientation distribution in the volume of the specimen can be expressed as [11, 12]

$$n_v(\theta) = N \sin\theta \tag{13}$$

where N is the total number of short fibers in the composite specimen.

Assuming the effective load-carrying length of a short fiber is l_e, which can be expressed as

$$l_e = l - l_c/2 \tag{14}$$

where l and l_c are the short fiber length and the average critical fiber length, respectively, the projected effective fiber length on the loading direction can be calculated as

$$l_p = l_e \cos\theta = (l - l_c/2)\cos\theta \tag{15}$$

$n_c(\theta)$ can be calculated from $n_v(\theta)$ and l_p as

$$n_c(\theta) = n_v(\theta) l_p / L \tag{16}$$

where L is the composite specimen length. It should be noted that the critical fiber length is also a function of θ. But, the average critical fiber length l_c is used here for simplicity. The total number of fibers in the composite specimen, N, can be calculated by the following equation

$$N = \frac{LAV_f}{la_f} \tag{17}$$

where A is the sample cross-section area and V_f is fiber volume fraction. Substituting Eqs. 13, 15 and 17 into Eq. 16 yields

$$n_c(\theta) = \frac{V_f A}{a_f}\left(1 - \frac{l_c}{2l}\right)\sin\theta\cos\theta \tag{18}$$

Knowing $n_c(\theta)$, we can calculate $P(\theta_0)$ as

$$P(\theta_0) = \int_{\theta_0}^{\pi/2} n_c(\theta) f(\theta)\cos\theta d\theta \tag{19}$$

Substituting Eqs. 11, 12 and 18 into Eq. 19 and integrating yields

$$P(\theta_0) = \frac{f_0 V_f A}{15a_f}\left(1 - \frac{l_c}{2l}\right) g(\theta_0) \tag{20}$$

where

$$g(\theta_0) = \frac{\left[3(1+v)\cos^5\theta_0 - 5v\cos^3\theta_0 + 4v^{5/2}/(1+v)^{3/2}\right]}{\left(\cos^2\theta_0 - v\sin^2\theta_0\right)} \tag{21}$$

It can be seen from Eq. 20 that the maximum of $P(\theta_0)$ can be obtained as

$$\left[P(\theta_0)\right]_{\max} = \frac{f_0 V_f A}{15a_f}\left(1 - \frac{l_c}{2l}\right)\left[g(\theta_0)\right]_{\max} \tag{22}$$

The θ_0 at which $g(\theta_0)$ equals $\left[g(\theta_0)\right]_{\max}$ can be obtained by setting

$$dg(\theta_0)/d\theta_0 = 0 \tag{22}$$

and solving it for θ_0, which yields

$$\theta_0 = 0 \tag{23}$$

Equation 23 indicates that once the fibers parallel to the loading direction fails, catastrophic failure will occur for random-oriented short-fiber composites. Plots of $g(\theta_0)$ as a function of θ_0 (Fig. 3) also confirms that $g(\theta_0)$ is at its maximum at $\theta_0 = 0$. Therefore, we have

$$\left[g(\theta_0)\right]_{\max} = g(0) = \frac{(3-2\nu)(1+\nu)^{3/2} + 4\nu^{5/2}}{(1+\nu)^{3/2}} \tag{24}$$

Substituting Eq. 24 and $f_0 = a_f \sigma_f$ into Eq. 20 yield

$$\left[P(\theta_0)\right]_{\max} = \frac{A\sigma_f V_f\left[(3-2\nu)(1+\nu)^{3/2} + 4\nu^{5/2}\right]}{15(1+\nu)^{3/2}}\left(1 - \frac{l_c}{2l}\right) \tag{25}$$

where σ_f is the fiber strength. The direct short-fiber strengthening can be calculated as

$$\sigma_c^f = \frac{\left[P(\theta_0)\right]_{\max}}{A} = \frac{\sigma_f V_f\left[(3-2\nu)(1+\nu)^{3/2} + 4\nu^{5/2}\right]}{15(1+\nu)^{3/2}}\left(1 - \frac{l_c}{2l}\right) \tag{26}$$

where σ_c^f is the composite strength contributed by the short fibers, *i.e.* the direct short-fiber strengthening.

3. DISCUSSION

The new direct short-fiber strengthening theory developed in this paper uses maximum load criterion for composite failure, which is straight forward and easier to use than the previous theory by Zhu *et al* [12]. It also takes into account the probability variation of fibers being intercepted by a sample cross-section at different inclination angles in the calculation of the total load. The previous theory by Zhu *et al* [12] predicted the direct short-fiber strengthening as

$$\sigma_c^f * = \frac{V_f \sigma_f (1+v^2)}{8((1+v)} \left(1 - \frac{l_c}{2l}\right) \tag{27}$$

The present theory predicts a much higher direct short-fiber strengthening than Eq. 27. For example, taking the Poisson's ratio of the composite as $\nu = 0.33$, the direct short-fiber strengthening predicted by the present theory (Eq. 26) is 60% higher than that by Eq. 27. It should be noted that the present theory does not consider the non-uniform deformation between fibers and the matrix and it uses an average fiber effective fiber length for simplicity. This makes the high direct fiber strengthening predicted by the present theory still conservative.

Taking the Poisson's ratio of the composite as $\nu = 0.33$ and substituting it into Eq. 26 yields

$$\sigma_c^f \approx \frac{V_f \sigma_f}{6} \left(1 - \frac{l_c}{2l}\right) \tag{28}$$

Eq. 28 is close to Friend's [1, 10] empirical equation for randomly-oriented short-fiber reinforced metal matrix composites:

$$\sigma_c^f = \frac{V_f \sigma_f}{5} \left(1 - \frac{l_c}{2l}\right) \tag{29}$$

This explains why Friend's empirical equation can describe the strengths of some aluminum alloy matrix composites. This also indirectly proves the validity of the present theory. The difference between Eq. 28 and 29 is probably due to the other strengthening mechanisms such as short-fiber dispersion hardening [12], thermally induced matrix work hardening and residual thermal microstress [13-17], which are empirically incorporated into Eq. 29.

4. CONCLUSIONS

The theory developed in this paper for calculating the direct fiber strengthening in randomly-oriented short-fiber composites overcomes some deficiencies of previous theories. It adopts a new maximum load composite failure criterion and takes into account the fiber orientation effect on the probability of a fiber being cut by a specimen cross-section. Basic material parameters such as the Poisson's ratio, fiber strength and critical load transfer length were included in the calculation of the direct fiber strengthening. The direct contribution of short fibers toward the composite strength predicted by the present theory is much larger than that predicted by previous theories, which helps to explain the high strength of some composites. The information provided by the present theory is useful for composite strength assessment and design.

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Figure captions

Fig. 1. Definition of off-axis angle θ

Fig. 2. Composite sample and its cross-section

Fig. 3. $g(\theta_0)$ as a function of θ_0 for different ν values. $g(\theta_0)$ is at its maximum at $\theta_0 = 0$.

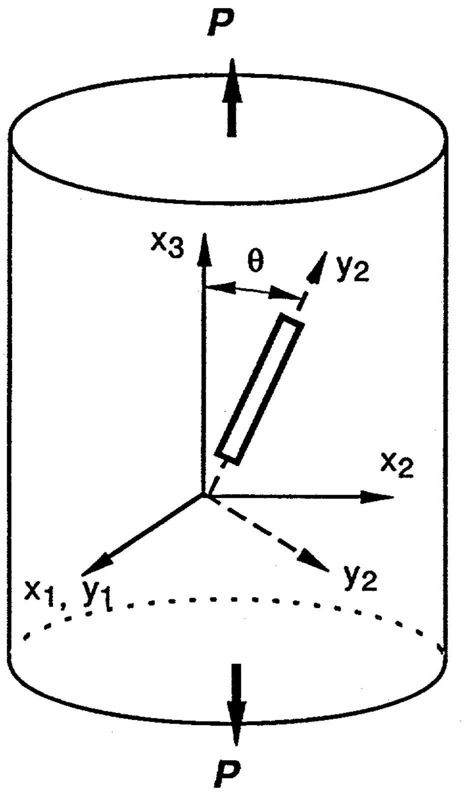

Fig. 1 Y. T. Zhu & W. R. Blumenthal

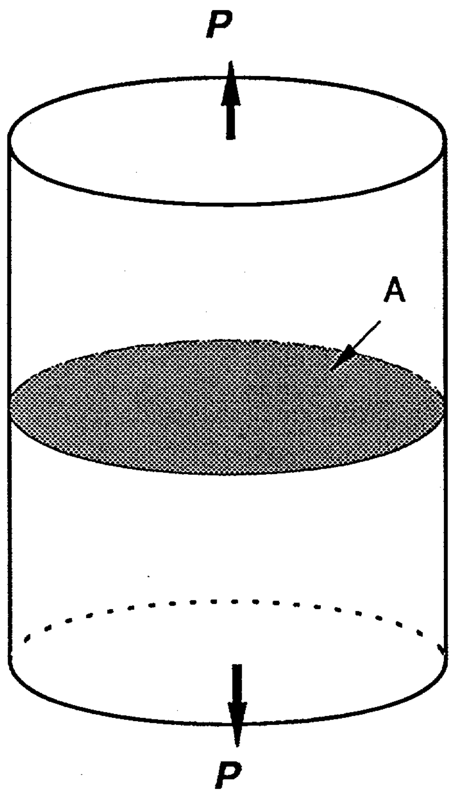

Fig. 2. Y. T. Zhu & W. R. Blumenthal

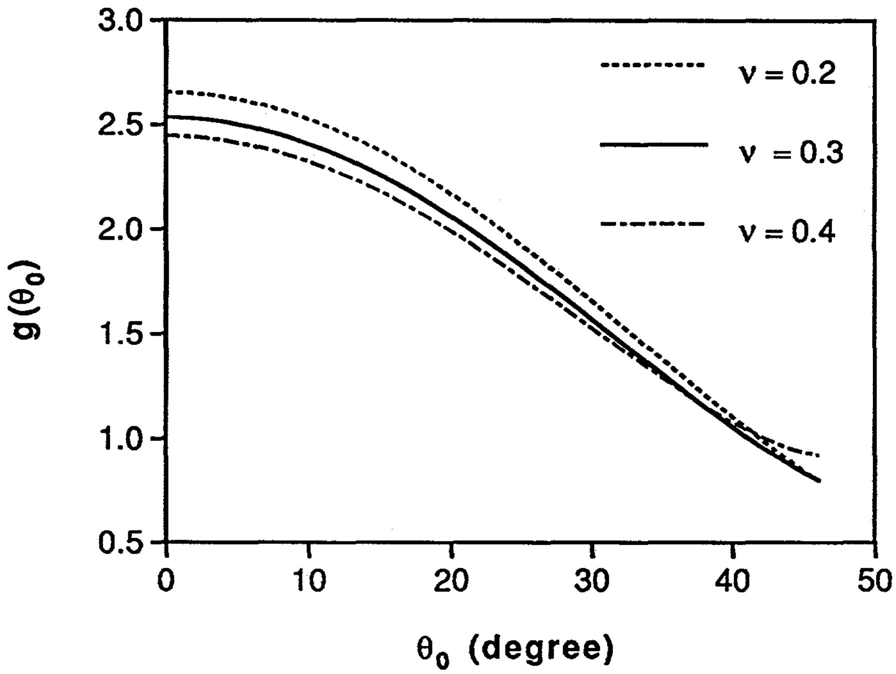

Fig. 3. Y. T. Zhu & W. R. Blumenthal