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THIRTEEN YEARS OF PAIR PRODUCTION

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Thirteen Years of Pair Production

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Pair production from relativistic heavy-ion collisions is reviewed. Physical processes produced from two photons are discussed, and their calculation in terms of Feynman diagrams and classical perturbation theory are compared. The experiments at CERN energies are discussed and compared to Monte Carlo QED calculations. We also discuss a number of phenomena and their consequences: (i) the creation of electron pairs and their importance for the detection of dileptons at RHIC; (ii) the creation of electron pairs with capture and implications for accelerator design, and (iii) the creation of the vector bosons and the Higgs boson and consequences for electron-weak theories.

I. INTRODUCTION

This talk surveys a thirteen-year collaboration with Chris Bottcher on various aspects of strong field electrodynamics. Most of the work centers on the atomic physics associated with the peripheral collisions of ultrarelativistic heavy atoms. The earliest, beginning in about 1979, dealt with the spontaneous emission of positrons from nuclear quasimolecules and touched briefly on the formation of axions as a possible explanation of the anomalous peaks in the spectrum. This work stimulated the extensive studies of particle production from coherent fields that laid the foundations for investigations of nuclear form factors, structure functions, and production mechanisms for the Higgs and other exotic particles. Chris conjectured that the strong fields that are present in these collisions would give rise to nonperturbative effects. Thus, during this time, Chris also worked to develop basis-spline collocation methods for solving dynamical relativistic fermions in super strong fields. This was perhaps one of the best of times for Chris; on these problems alone, he co-authored fifty articles with more than twenty different collaborators.

II. QED EFFECTS IN STRONG FIELDS

In the early 1980's several experimental searches were under way to try and isolate the spontaneous production of positrons from superheavy nuclear molecules. It was generally accepted that electron-positron pairs should be excited from the vacuum in low-energy collisions of heavy ions whenever the combined charge is > 173. These processes had been extensively studied by

Greiner and co-workers for more than 15 years (1). In an independent electron approximation the dominant positron production mechanism is by particle-hole excitations out of the vacuum induced by the time-dependent electric field of the ions. While this level of approximation neglects many-body effects (including the Pauli principle), it suffices to establish the general features of the positron spectrum. This is completely determined from the solutions of the time-dependent Dirac equation for the motion of a single electron

$$[H_f + H_p(t)]\Psi(t) = i\partial_t \Psi(t), \tag{1}$$

where H_p denotes the the field from the projectile, $H_p(t) = \vec{\alpha} \cdot \vec{A}_p(t) - A_p^0(t)$, and where H_f denotes the furry Hamiltonian

$$H_f = -i\vec{\alpha} \cdot \vec{\nabla} + \beta - A_t^0. \tag{2}$$

These equations are given in a frame fixed in the target atom for purposes of exposition. In our work (2), these equations were solved on a basis of finite elements using a Galerkin approach. The spectrum of emitted positrons was studied as a function of the nuclear sticking time and is shown in Fig. 2 of Ref. (2). One interesting point to note is that although many theoretical studies of this effect have been made, it has not yet been observed. A closely related issue concerned the interpretation of the positron peaks in terms of more fundamental quantities. In Ref. (3) we studied the possibility that a light-mass axion served as a precursor to the peak formation. However, this phenomenon played a secondary role to Chris's major interest – namely solving exactly, in a numerical sense, the general problem of particle production from relativistic fields.

III. COHERENT PARTICLE PRODUCTION

By about 1985, all of the concepts for a large-scale numerical solution of the heavy-ion collision problem were in place. At this time, Chris presented 1-D model simulations of relativistic collisions in terms of basis splines at the Gaithersburg Conference (4). A number of important features of the spline method were discussed, including the solution to the fermion-doubling problem, flux conservation theorems, and projection methods for final-state probabilities. The 1-D models proved to be very fruitful and pair production spectra and distributions were calculated and discussed at a number of major workshops (5,6). These results were highly controversial and Chris greatly enjoyed explicating them in great detail. A direct consequence of the workshop interactions was the development of a hybrid method for studying the collision problem. The method was developed by Chris and was called the Feynman-Monte Carlo approach. This method used massively parallel supercomputers to evaluate the multi-dimensional integrals that enter into a perturbation expansion of the time-dependent field solution (7,8). This approach has proved to be of great benefit in studying complex experimental results, including the design of detectors for searching for the quark-gluon plasma at ultrarelativistic colliders. The development of the formalism for this paper characterizes Chris's approach to science more than any other.

We treat the production of electron pairs using a semiclassical formalism, wherein the electron states evolve in the presence of classical electromagnetic potentials. The source currents for the potentials are the Lorentz-boosted charge distributions of the two heavy ions. For the present purposes of exposition, it is most convenient to consider the symmetric collision of two heavy nuclei in the center of momentum (CM) frame. In the case of structureless-point charge nuclei, these potentials are the retarded Liénard-Wiechert interactions. The realistic calculation of electron pair production requires a detailed knowledge of the charge currents of nucleons within each nucleus, and also the charge structure functions of the individual nucleons. Though a fully dynamical understanding of such currents in terms of a fundamental theory like QCD is presently not possible, models for incorporating these effects into the calculations are well known (9), and they will be treated in future work.

We develop the equations governing pair production in the spirit of Schwinger's space-time picture of electrodynamics (10), with appropriate modifications to allow for the classical motion of the heavy ions. The two important differences are the localization of the fields along an impact parameter transverse to the motion of the ions, and the large number of virtual photons in the fields. The semiclassical coupling of electrons to the electromagnetic field is given by the Lagrangian density (11)

$$\mathcal{L}_{int}(x) = -\overline{\Psi}(x)\gamma_\mu \Psi(x) A^\mu(x) \quad , \tag{3}$$

which separately conserves electron number (7), and only depends on the field variables via the classical four-potential A^μ. Given the Lagrangian (Eq. (3)), the equations of motion for state vectors in the Schrödinger picture follow from three basic assumptions:

- We construct a semiclassical action in terms of a time-dependent *many-electron* state $\Phi(t)$

  $$\mathcal{S} = \int d^4x < \Phi(t) \mid : \mathcal{L}_0(x) + \mathcal{L}_{int}(x) : \mid \Phi(t) > \quad , \tag{4}$$

  where the normal ordering is with respect to an unspecified reference state, and where $\mathcal{L}_0$ is the usual noninteracting fermion Lagrangian

  $$\mathcal{L}_0(x) = \overline{\Psi}(x)[\gamma_\mu i\partial^\mu - m]\Psi(x) \quad . \tag{5}$$

  In Eq. (4), the dynamical coordinates which are varied to make the action stationary are the parameters labeling the state vector $\Phi(t)$, and not the field operators.

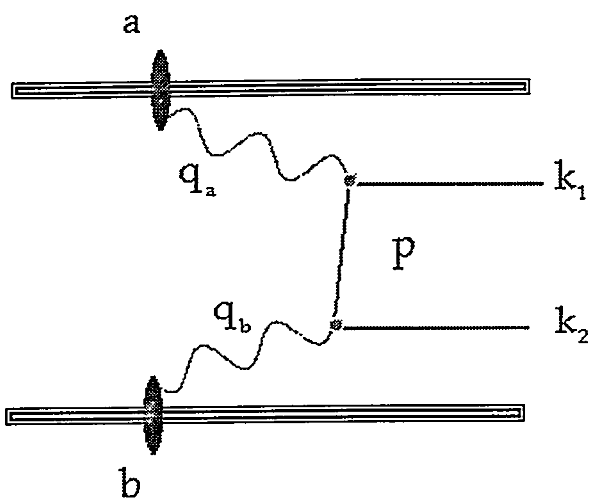

FIG. 1. The lowest-order Feynman diagrams for producing lepton pairs from the electromagnetic fields of the colliding ions.

- We assume that the initial-state vector corresponds to a single Slater determinant, $\mid O >$

$$\lim_{t \to -\infty} \mid \Phi(t) > \to \mid O > \quad . \tag{6}$$

Since we shall only consider pair production out of the vacuum, we choose $\mid O >$ as the vacuum state, which we also identify as the reference state for the ordering in Eq. (4).

- We assume the dynamics governing the time evolution of the states in Eq. (4) is unitary; that is

$$\mid \Phi(t) >= K(t, -\infty) \mid O >, \tag{7}$$

where $KK^\dagger = K^\dagger K = 1$. There are several important consequences of these assumptions. Equations (6) and (7) guarantee that the state Φ is at all times a representation of a single Slater determinant, and equations of motion can be cast into the form

$$H(x)K(t,t') = i\partial_t K(t,t'), \tag{8}$$

where,

$$\begin{aligned} H(x) &= H_0(x) + V(x), \\ H_0(x) &= -i\vec{\alpha} \cdot \vec{\nabla} + \gamma_0 m, \\ V(x) &= -\vec{\alpha} \cdot \vec{A}(x) + A_0(x). \end{aligned} \tag{9}$$

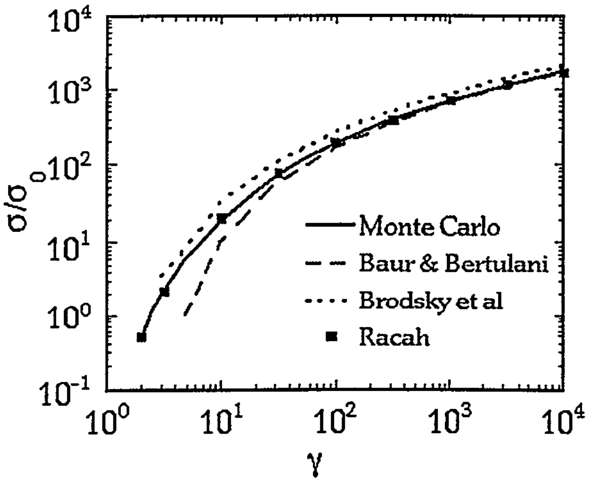

FIG. 2. Dependence of the pair cross section with energy. The ratio of the cross section to the reduced cross section, σ_0, is plotted vs. γ. Solid line: exact numerical result; dashed line: equivalent photon approximation; long-dashed line: modified Weizsäcker-Williams method; square boxes: Weizsäcker-Williams method of Racah; (see text).

With the above-noted assumptions, all orders of processes can be obtained from the solutions to Eq. (8). The lowest-order process is given in Fig. 1 in terms of Feynman diagrams. In Fig. 2 we show the variation of the total cross section with energy for colliding beams of heavy ions. The beam kinetic energy per nucleon in units where the nucleon mass is one is $\gamma - 1$. The cross section for producing an electron pair from ions of charges (Z_a, Z_b) is expressed in terms of the reduced cross section σ_0

$$\sigma_0 = \lambda^2 Z_a^2 Z_b^2 \alpha^4, \tag{10}$$

where $\lambda = \hbar/mc$ is the reduced Compton wavelength of the electron. For $Au + Au$, $\sigma_0 = 1.49Kb$. This figure compares the exact results with two currently quoted approximations. The dotted curve is the *equivalent photon approximation* of Ref. (12), which is essentially a modern version of the Weizsäcker-Williams method. The dot-dashed curve is calculated from the formulae of Ref. (13), which attempt to refine the Weizsäcker-Williams result. It is correct for large γ, but appears to break down by as much as a factor of ten for $\gamma \leq 20$.

The above work was extended to other classes of particles (15–17) and effects from microstructure in the nuclear currents (18). Generally, electron pair production is not strongly affected by the details of the nuclear currents. However, this is strongly a function of the lepton mass. We note that muon and tauon pair production is extremely sensitive to these form factors, as

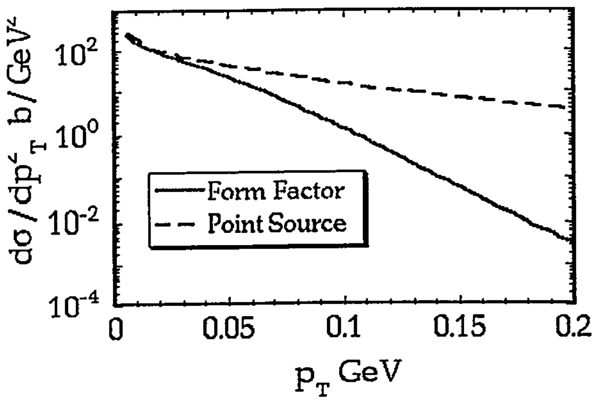

FIG. 3. Transverse momentum distribution of dimuons produced from colliding beams of Au nuclei at an energy per nucleon of 100 GeV. The solid curve is the calculated result including the proton and Au form factors; the dashed curve is the result from point charge nuclei.

shown in Table 1. Note that the results for the electrons are equivalent to the same results using point nuclear currents. The effects of the nuclear form factors on the heavy lepton production can be understood in greater depth by examining their effects on the pair distributions in rapidity, transverse momentum, and invariant mass. We show in Fig. 3 the transverse momentum distribution for muons calculated with nuclear form factors and for point nuclei. Again, we are considering colliding beams of Au nuclei at 100 GeV per nucleon. The solid curve gives the results which include the nuclear form factors, and the dashed curve gives the results for point nuclear currents. We note that the two curves are approximately the same for values of P_T near zero. The Compton wavelength of the μ is about 0.1 GeV/c; at this value of P_T, the nuclear form factors have reduced the cross section by an order of magnitude, and hence the resulting distribution is concentrated at small values of P_T.

As indicated earlier, the theoretical study of the direct production of electron pairs in high-energy collisions has been extensively investigated. Results of different calculations vary substantially in quantitative detail, including

TABLE 1. Results for the coherent muon and tauon pair production for 100 GeV per nucleon colliding beams of Au nuclei.

	μ (mb)	τ (mb)
Point Source	770	2.7
Form Factor	200	2.8×10^{-3}

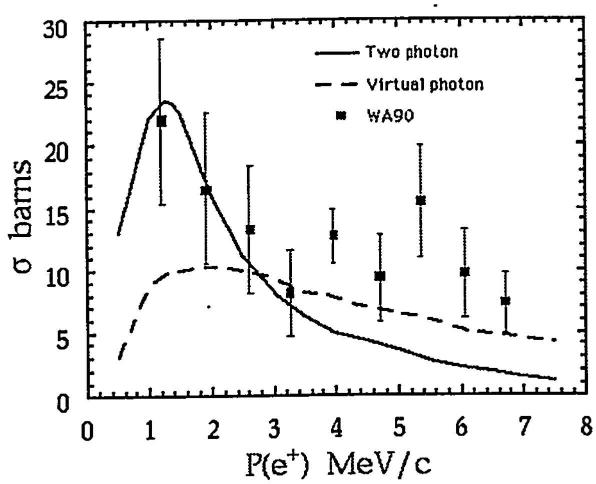

FIG. 4. Measured positron momentum distribution for the collision of S+Au at a bombarding energy per nucleon of 160 GeV. The solid curve is the result of a Monte Carlo calculation, and the dashed curve that of an equivalent photon calculation.

single- and multiple-pair formation rates. However, few experiments have been reported that can test the differences between the various calculations (19–21). It is at the highest bombarding energies that the differences among the various theories are most apparent. Figure 4 shows the positron spectrum obtained by Refs. (19,21) in the collision of S+Au at a bombarding energy per nucleon of 160 GeV. The experimental spectrum is compared to two different calculations; the solid line is the result of calculations by Bottcher *et. al* from Ref. (19). The total experimental yield for this production of pairs is 85 ± 22 barns compared to the calculated result of 98 barns. Even though the cross sections are in agreement, details of the momentum distribution need to be further understood.

IV. SEARCH FOR THE HIGGS

The search for the Higgs boson and the new physics responsible for electroweak symmetry breaking was a major interest of Chris' during the last three years of his life. Its confirmation or denial by experiments will clarify the ambiguities in the present electroweak theory, and experiments will naturally focus on exploring new physics above the TeV energy scale. However, if the mass of the Higgs lies between the mass of the Z_0 and twice the W mass, it will be difficult to separate its decay products from the background signals. Consequently, extensive interest has been aroused in relativistic heavy-ion colliders as a possible way to form an intermediate mass Higgs via a two-photon mechanism. This process has two important features: (1) it is coherent in the electromagnetic field of the nuclei and, under appropriate conditions, is enhanced by the fourth power of the nuclear charge, Z^4, over the corresponding mechanism at pp colliders, and (2) the two-photon production occurs at distances of several nuclear radii and can be separated from the usual backgrounds which obscure the formation and decay of the Higgs. The details of designing a clean experimental veto for these events is of paramount importance, since typical heavy-ion cross sections are of the order of 10 barns, whereas the Higgs cross sections are about 100 pb, so that the experimental acceptance is $1 : 10^{11}$. Furthermore, it is essential to have a clean experiment since heavy-ion luminosities are significantly smaller than those for protons. For example, integrated luminosities at the LHC for Pb beams are no more than 1 pb^{-1}, yielding estimates of 100 Higgs per year. LHC experiments might detect about 25 Higgs events per year. Heavy-ion experiments could be designed for the SSC. Since the charge-to-mass ratio of heavy nuclei is about 0.4, the expected beam energy per nucleon would be 8 TeV. The additional energy results in an increased Higgs cross section about a factor of seven larger than for the LHC. The proposed ELOISATRON (23) might accelerate heavy ions to a beam energy per nucleon of 40 TeV. This could yield as many as 4000 Higgs events per year.

Our method of treating the production is semiclassical. The coherent production S-matrix is obtained from the excitations of the quantum fields using perturbation theory (2). For the lowest-order processes, the production cross sections are obtained in terms of Feynman integrals in momentum space, which are integrated numerically using Monte Carlo methods. It can be proved that the expressions obtained from the classical field approximation are exactly the same as the full Feynman integration whenever the four-momentum transfer of the nuclei is small compared to their respective momenta (22). These conditions are well satisfied for the collisions studied. The Higgs boson is a neutral particle and does not directly couple to electromagnetic fields. However, the coherent production of the Higgs via two virtual photons can occur through the production and subsequent fusion of charged pairs. The lowest-order Feynman diagram for this process is shown in Fig. 5. In this figure, the intermediate loops can be formed from any charged particles, quarks, leptons, and bosons. The contribution of these loops is usually treated as an effective three-body vertex. The Higgs production cross section is obtained from the leading order term in the S-matrix, which is first order in the Hamiltonian. For two-photon pair production of massive charged particles, both of the virtual photons, q_a, q_b, are space-like and "off-shell." Since the mass of the Higgs boson is unknown, we must make some assumption regarding it. The result would then set limits on the uncertainty of the Higgs boson mass if it still remains unobserved. The production cross section of the Higgs boson is given in Fig. 6 as a function of the Higgs mass, for $m_t = 100$ GeV. Here we are assuming a symmetric collision of beams of uranium at beam energies per nucleon of 3.5, 8, and 40 TeV, corresponding to possible experiments at the

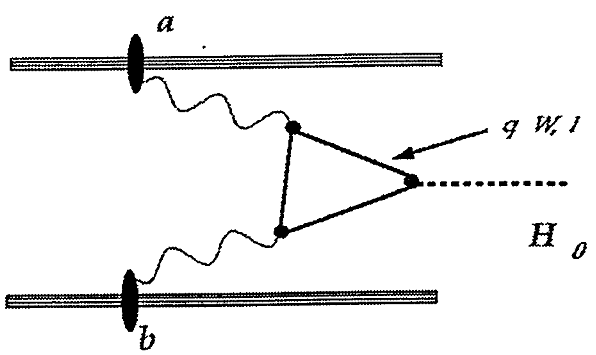

FIG. 5. The lowest-order Feynman diagrams for Higgs production from the electromagnetic field. The loop sums over all charged fermions and bosons.

LHC, the SSC, and the ELOISATRON.

These calculations yield, for Higgs masses in the range 100–200 GeV, cross sections of 10–1000 pb. For the very massive Higgs, the production is extremely sensitive to the beam energy. We note that the general features of the cross section dependence with energy are similar to those for lepton and W-pair production, all increasing as $(\ln\gamma)^3$ for large values of γ. The rise in the cross section at about 160 GeV is due to the coupling to the W bosons.

V. PAIR PRODUCTION AND CAPTURE

In this section, we focus on one important aspect of peripheral collisions of heavy ions at relativistic energies by calculating the cross section for electron capture in the two-photon limit using Monte Carlo integration over intermediate and final momenta. The present study spans an energy range that includes the present AGS experiments and the future RHIC program.

The new physics associated with strong and pulsed electromagnetic fields from relativistic heavy ions has important practical implications for detector design and machine performance at RHIC. The largest nuclear depletion mechanism of the RHIC beam is estimated to be the electron capture process following pair production during beam crossing.

The electron capture mechanism is shown schematically in the two-photon diagrams of Fig. 1. We use time-dependent perturbation theory to expand the S-matrix in orders of the electromagnetic interactions of the two colliding ions a, b. The leading contributions to pair production arise from the two Feynman diagrams, in which each ion interacts exactly once (7). When the direct and crossed diagrams are correctly included, the resulting amplitude is Lorentz covariant and gauge invariant (24). Capture is then described in a sudden (or impulse) approximation by convoluting the electron line in Fig. 1 into

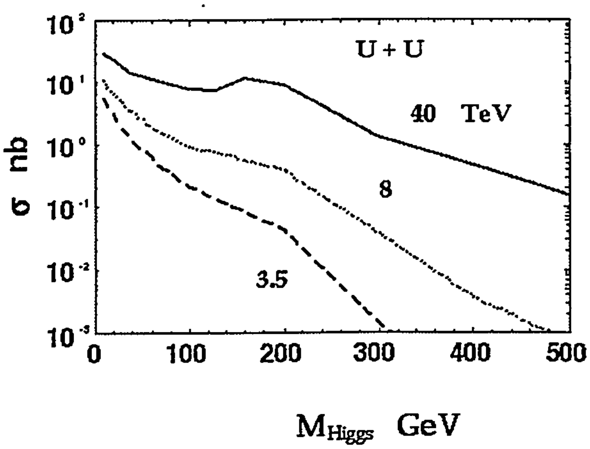

FIG. 6. The cross section for Higgs production as a function of the Higgs mass for uranium collisions at beam energies per nucleon of 3.5, 8, and 40 TeV. The top quark mass is assumed to be 100 GeV.

the momentum wavefunction of the final bound state. This *ansatz* can also be derived by summing the ladder diagrams for repeated interactions of the electron and the capturing nucleus. From the perspective of RHIC accelerator performance, the top RHIC energy per nucleon for Au is 100 GeV. At this energy, the capture cross section is 72b per beam. For comparison, the total pair production is about 32 Kb, or 440 times larger.

In Fig. 7, the capture cross sections are shown for a sample of symmetric heavy-ion collisions used in the design of RHIC. These are full Monte Carlo calculations. To convey the variation with Z, all the results here are scaled according to $\lambda^2 = 1.49Kb$. It appears that the cross sections are very small for ions lighter than Cu, and indeed very few heavy ions would be lost to this mechanism for $A \sim 100$. The result for Au is in harmony with a ten-hour beam lifetime at RHIC. An equally encouraging aspect of Fig. 7 is the saturation at large values of Z; the electron capture cross section for U+U is only twice that for Au+Au. It has recently been determined that U may be injected into RHIC (25).

VI. SUMMARY

In the previous sections I have tried to give you a view of the work that Chris Bottcher and I carried out on pair production. Others at this conference will have discussed at great length physics specific to their collaboration with Chris. Most of his twenty collaborators working in this area of physics are present at this conference. All of the participants here have expressed their

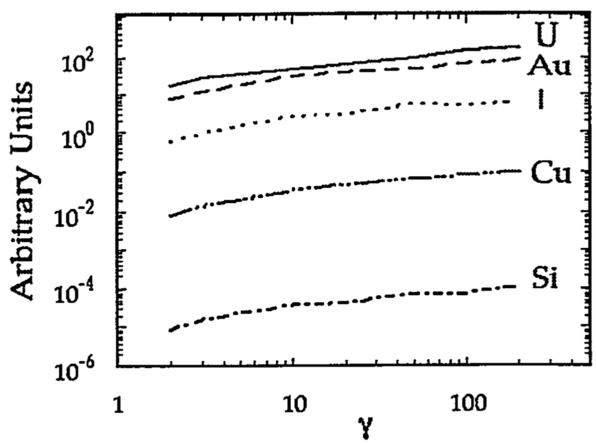

FIG. 7. Capture cross sections for symmetric A_Z+A_Z collisions scaled with respect to $\lambda^2 = 1.49$ Kb. The curves correspond, as labeled, to the ions A(Z) = Si(14), Cu(29), I(53), Au(79), U(92).

deep sense of loss. As time goes by, I know that I shall think of Chris and of the wonderful spirit with which he approached science.

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