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Some Remarks on Antenna Response in a Reverberation Chamber

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Abstract - The simple formula, $\langle P_r \rangle = (E_0^2/\eta)(\lambda^2/8\pi)$, for the received power of an antenna with a matched load in an over-moded cavity actually holds for an antenna of any shape and size. This can be seen from the close connection between the correlation tensor of the cavity field at two different points and the imaginary part of the free-space dyadic Green's function.

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In a recent article [1], Hill has showed that the statistical average of the received power of a linear dipole with a matched load and a *special* form of current distribution is given by

$$\langle P_r \rangle = \frac{E_o^2}{\eta}\frac{\lambda^2}{8\pi} \tag{1}$$

First, we will show that (1) is valid for a linear antenna with *any* current distribution. To do this we notice that the correlation function $\rho_z(z_1,z_2)$ given by (18) in [1] can be expressed as

$$\rho_z(z_1,z_2) = \frac{3}{2k}(1+\frac{1}{k^2}\frac{\partial^2}{\partial z_1^2})\frac{\sin[k(z_1-z_2)]}{z_1-z_2} = \frac{6\pi}{k}\mathrm{Im}\,\Gamma_{zz} \tag{2}$$

where $\mathrm{Im}\,\Gamma_{zz}$ is the imaginary part of the zz-component of the free-space dyadic Green's function Γ [2],

$$\Gamma(\vec{r}_1,\vec{r}_2) = \frac{1}{4\pi}(\mathbf{u}+\frac{1}{k^2}\nabla_1\nabla_1)\frac{e^{ikr}}{r}, \tag{3}$$

along the z-axis, with $r = |\vec{r}_1 - \vec{r}_2|$, $\mathbf{u}$ = unit dyad. One can show that (2) is indeed true by simply carrying out the differentiation, as is done in [2]. Now the power radiated, P_{rad}, by a linear antenna with current distribution $I(z)$ is given by

$$\begin{aligned} P_{rad} &= \frac{\omega\mu}{2}\iint \vec{J}(\vec{r}_1)\bullet \mathrm{Im}\,\Gamma \bullet \vec{J}^*(\vec{r}_2)\, dV_1\, dV_2 \\ &= \frac{\omega\mu}{2}\iint I(z_1)[\mathrm{Im}\,\Gamma_{zz}(z_1-z_2)]I^*(z_2)\, dz_1\, dz_2 \\ &\equiv \frac{1}{2}R_d|I_o|^2 \end{aligned} \tag{4}$$

Hence the double integration in (22) of [1] has the value

$$\iint \rho_z(z_1,z_2)I(z_1)I^*(z_2)\,dz_1\,dz_2 = \frac{3\lambda^2}{2\pi\eta}R_d|I_o|^2 \tag{5}$$

where we have made use of (2) and (4). Substitution of (5) in (22) of [1] gives formula (1) for *any* current distribution on a linear antenna.

Next, we will show that formula (1) also holds for a matched antenna of any shape and size. To prove this we have to work out the statistical average of the correlation dyad of the electric field at two different points in the cavity, i.e., $\left\langle \vec{E}(\vec{r}_1)\vec{E}^*(\vec{r}_2)\right\rangle$. We define

$$R_{ij} = \frac{1}{2}\left[\left\langle E_i(\vec{r}_1)E_j^*(\vec{r}_2)\right\rangle + \left\langle E_i^*(\vec{r}_1)E_j(\vec{r}_2)\right\rangle\right] \qquad i,j = x,y,z \tag{6}$$

In view of (2) one should not be surprised to find that

$$R_{ij} = \frac{6\pi}{k}\operatorname{Im}\Gamma_{ij} \tag{7}$$

To verify (7) we first carry out all the necessary differentiations in (3) and obtain

$$\begin{aligned}\frac{4\pi}{k}\operatorname{Im}\Gamma_{ij} = &\left[3\frac{\sin u}{u^5} - 3\frac{\cos u}{u^4} - \frac{\sin u}{u^3}\right]u_i u_j \\ &+ \left[\frac{\sin u}{u} + \frac{\cos u}{u^2} - \frac{\sin u}{u^3}\right]\delta_{ij}\end{aligned} \tag{8}$$

where $u = kr$, $u_x = k(x_1 - x_2)$, $u_y = k(y_1 - y_2)$, etc., and δ_{ij} is the Kronecker delta. To work out the left-hand side of (7) we invoke what has been known in isotropic homogeneous turbulence, which enables one to write [3]

$$R_{ij} = -\frac{1}{2r} f' r_i r_j + (f + rf'/2)\delta_{ij} \tag{9}$$

where $r_x = x_1 - x_2$, etc. and $f' = df/dr$. The scalar function f can be identified with $\rho_z(z_1, z_2)$ in [1] and is often referred to as the longitudinal correlation function given by

$$f(r) = \frac{3}{k^2 r^2}\left(\frac{\sin kr}{kr} - \cos kr\right) = \frac{3}{u^2}\left(\frac{\sin u}{u} - \cos u\right) \tag{10}$$

Using (10) in (9) gives 3/2 times the right-hand side of (8), and thus (7) is verified. Hence,

$$\begin{aligned} \langle P_r \rangle &= \frac{E_o^2}{12 R_d |I_o|^2} \iint J_i(\vec{r}_1) R_{ij}(r) J_j^*(\vec{r}_2)\, dV_1\, dV_2 \\ &= \frac{E_o^2}{\eta} \frac{\lambda^2}{8\pi} \end{aligned} \tag{11}$$

where repeated indices are summed. Note that the trace of R_{ij} is

$$R_{xx} + R_{yy} + R_{zz} = 3\frac{\sin kr}{kr} \tag{12}$$

Thus far we have proved that formula (1) is valid for a matched receiving antenna of any shape and size. The underlying assumption for (1) to hold is that the cavity is highly over-moded in the sense that many overlapping modes are present within the bandwidth of one resonant mode. It would be interesting to show how the simple formula (1) would be modified for a receiving antenna in an under-moded cavity where the frequency is still quite high but operates in a range where the spectra of the resonant modes are well separated.

Before concluding two remarks are in order. It is worthwhile to mention that formula (1) can also be obtained by averaging the effective receiving area A_e over all angles of incidence and polarization of an incident plane wave in free space, i.e.,

$$\langle P_r \rangle = \frac{E_o^2}{\eta}\langle A_e \rangle = \frac{E_o^2}{\eta}\left\langle \frac{\lambda^2}{4\pi} Gpq \right\rangle = \frac{E_o^2}{\eta}\frac{\lambda^2}{8\pi} \tag{13}$$

since $\int G\, d\Omega / 4\pi = 1$, $\int p\, d\chi / 2\pi = 1/2$ and $q=1$ for a matched load, where G is the antenna's directivity gain and p the polarization mismatch factor [2]. The other point to be made is that there is a boundary layer near the cavity wall, which is about one wavelength thick (see Fig. 1 and [4]). It is not clear if formula (1) needs to be changed when part of the receiving antenna is immersed in the layer.

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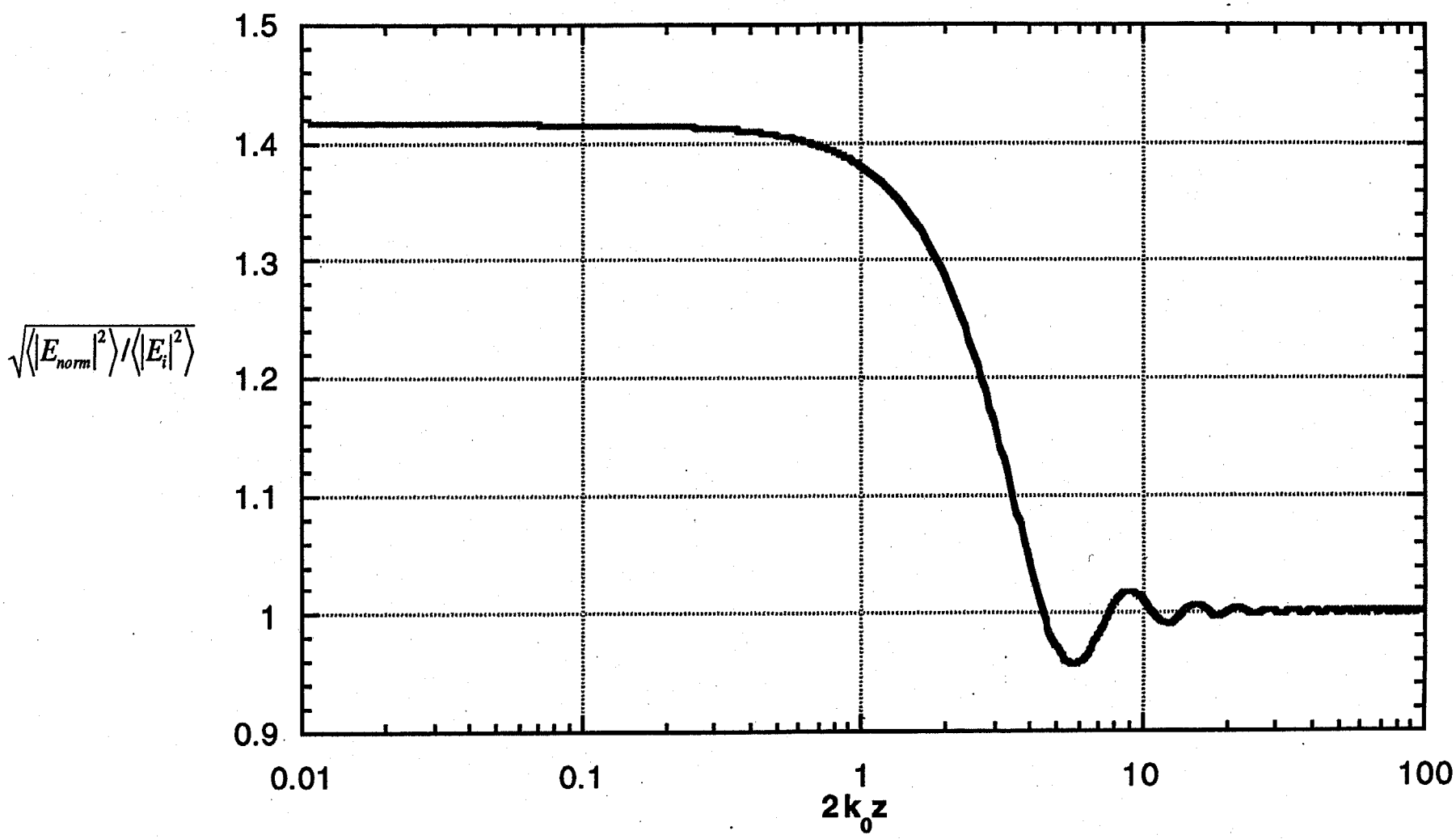

Figure 1. The transition region between cavity wall and isotropic homogeneous region.
$\langle |E_i|^2 \rangle = E_0^2/3, \;\; i = x, y, \text{ or } z$