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A MODEL OF ENERGY CONFINEMENT IN THE REVERSED FIELD PINCH

by

Guthrie Miller

ABSTRACT

Assuming Rusbridge's Tangled Discharge picture of the reversed field pinch, a simple parallel transport model describing energy confinement is obtained.

I. INTRODUCTION

Several of the observed properties of existing reversed field pinches are explained by assuming completely stochastic field lines. This model was termed the tangled discharge model by Rusbridge who first proposed it for Zeta in the 1960's (published accounts are more recent[1,2]). One of the most noteworthy features of reversed field pinches, the tendency for discharges to relax to the Taylor state with $j = \mu B$ and μ constant, is explained in a very simple and natural way in this picture. For plasma pressure and acceleration terms in the equation of motion small, it is generally true that $j = \mu B$; however, μ is not spatially constant. But from $\nabla \cdot j = 0$, it follows that $B \cdot \nabla \mu = 0$ or that μ is constant along a field line. Thus, if individual field lines wander in a random manner throughout the plasma, and in particular, in the limiting case where a single field line ergodically fills the entire plasma, μ will be constant. The origin of the stochasticity is presumably tearing mode activity, driven by parallel current, which causes islands to form, in conjunction with

the high density of rational surfaces in reversed field pinches, which readily allows the islands to overlap. Tearing mode stable configurations exist; however, resistive diffusion in a driven system leads to steady-state profiles with μ peaked on axis that are not stable. In the tangled discharge picture, resistive diffusion is modified by the nonexistence of flux surfaces through electrostatic effects, and stable steady-state configurations with flattened μ profiles are possible. The steady-state sustainment of reversal (with a constant applied electric field) comes about quite differently in the tangled discharge model than in dynamo effect theories,[3] which rely on $v \times B$ effects.

II. ENERGY CONFINEMENT MODEL

The energy confinement of a plasma with stochastic field lines can be estimated by making the simplifying assumption that cross field transport is completely negligible. The system is then characterized by a typical field line with length ℓ that begins and ends on the wall. To further simplify, slab geometry is assumed as shown in Fig. 1. Equilibrium in slab geometry demands

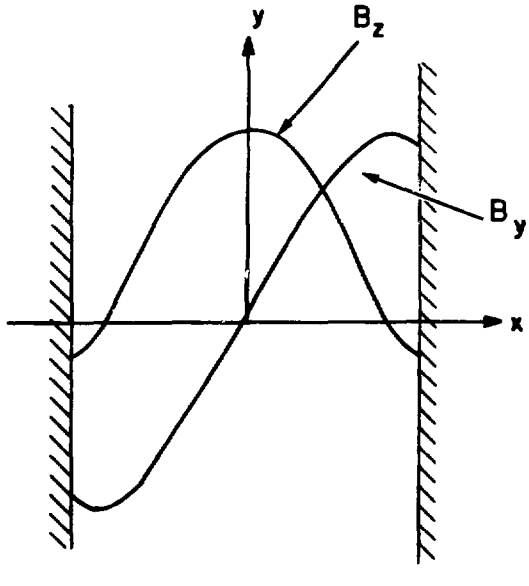


Fig. 1.
Cartesian coordinate system used for the slab model of the reversed field pinch.

that $|B|$ be spatially constant so that the constancy of μ along a field line implies constancy of j as well. The steady-state electron energy equation is

$$\frac{d}{dz} q + \eta j^2 = 0 \quad , \tag{1}$$

where z is a length coordinate along a field line, q is the parallel energy flux, and ηj^2 is the ohmic heating power per unit volume.

The energy flux q (in the −z direction) is given by

$$q = \kappa \frac{dT}{dz} + 1.5 \frac{j}{e} T \tag{2}$$

with the first term representing transport of energy by heat conduction and the second term transport of energy by convection caused by the electron flow implied by the current. From Braginskii,[4] $\kappa = \alpha T^{5/2}$ and $\eta = \beta/T^{3/2}$, where $\alpha\beta e^2$, with e the electron charge, is a dimensionless number given in Table I.

TABLE I

DEPENDENCE OF MODEL PARAMETERS ON Z

Z	$\alpha\beta e^2$	γ
1	1.61	1.4
2	2.16	1.04
3	2.44	0.92
4	2.62	0.86
∞	3.63	0.62

This system of two equations can be reduced to the following universal form:

$$\frac{dg}{d\zeta} = -\frac{1}{f^{3/2}} \quad ,$$

$$\frac{df}{d\zeta} = \gamma \frac{g-f}{f^{5/2}} \quad , \tag{3}$$

where f, g, and ζ are the scaled temperature, energy flux, and length:

$$f = \frac{T}{T_{max}} \quad ,$$

$$g = \frac{q}{1.5 \frac{j}{e} T_{max}} \quad ,$$

$$\zeta = \frac{z}{L} \quad ,$$

with

$$L = 1.5 \frac{T_{max}}{e\eta_{min}j} \quad .$$

The quantity γ is given by $\gamma = (1.5)^2/(\alpha\beta e^2)$. The numerical value of γ is of order 1 but somewhat dependent on the effective Z of the plasma, as shown in Table I.

Analytic solutions can be obtained in the two simplifying cases of 1) convection losses only, and 2) conduction losses only. The first case corresponds to the limit $\gamma \to \infty$ in Eq. (3) and has the solution

$$g = f$$

$$f = \left(1 - \frac{5}{2}\zeta\right)^{2/5} \quad .$$

In this case the maximum temperature point is at one end of the system and all the losses occur at this end, as would be expected for purely convective losses. The purely conductive solution, which in the limit $\gamma \to 0$ is the solution of Eq. (3), is given by

$$g = \left(\frac{1-f^2}{\gamma}\right)^{1/2}$$

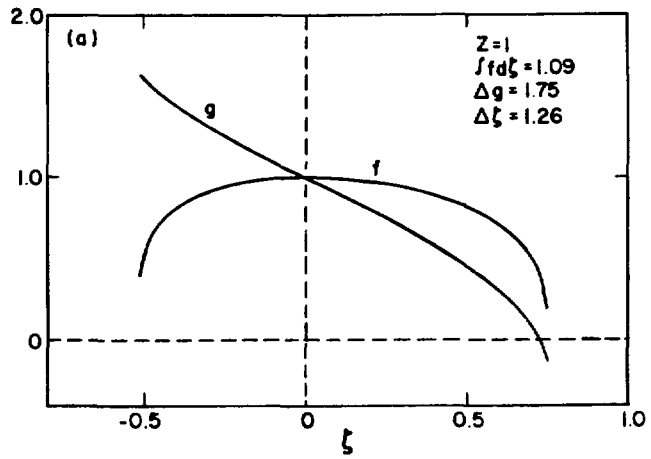
$$\zeta = \frac{1}{\gamma^{1/2}} \int_f^1 \frac{u^{5/2}}{(1-u^2)^{1/2}}\, du \quad .$$

In this case the maximum temperature point is in the center and the energy losses occur equally at the two ends of the system.

Numerical solutions of Eq. (3) for the two cases $Z = 1$ and $Z = 2$ are shown in Fig. 2. The numerical solution shares properties of both the convective and conductive limiting cases. The temperature distribution $f(\zeta)$ seems to lowest order to have a conductive rather than convective character. The energy flux $g(\zeta)$, on the other hand, seems more aptly described as convective.

III. DISCUSSION

As seen from the behavior of g in Fig. 2, the energy losses are not symmetric at the two ends of the system and almost all the energy is lost in the convective direction. This agrees with experimental observations of damage patterns at the wall in ZT-40M. Figure 3 is a photograph of a limiter used in ZT-40M, showing asymmetric damage. This observation would seem to imply that current carrying field lines from deep within the plasma intersect the limiter. If edge field lines on good flux surfaces intersected the limiter, the damage pattern would be symmetrical or perhaps come preferentially from the direction electrons on outer field lines are accelerated by the applied electric field, opposite from the observed direction.



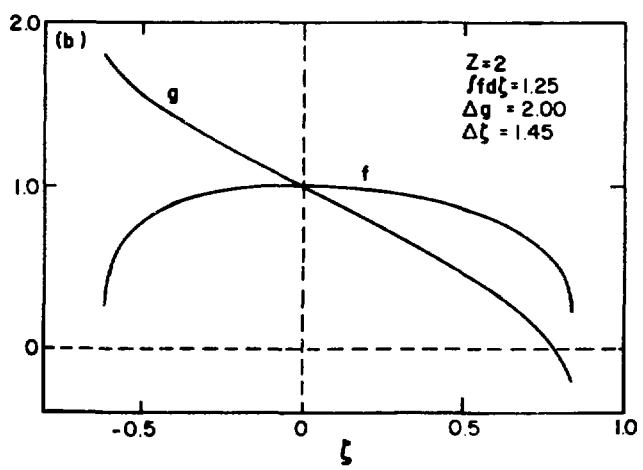


Fig. 2.
Universal solutions for temperature, f, and energy flux, g, as functions of length ζ. Part (a) shows the solution for Z = 1 and part (b) the solution for Z = 2.

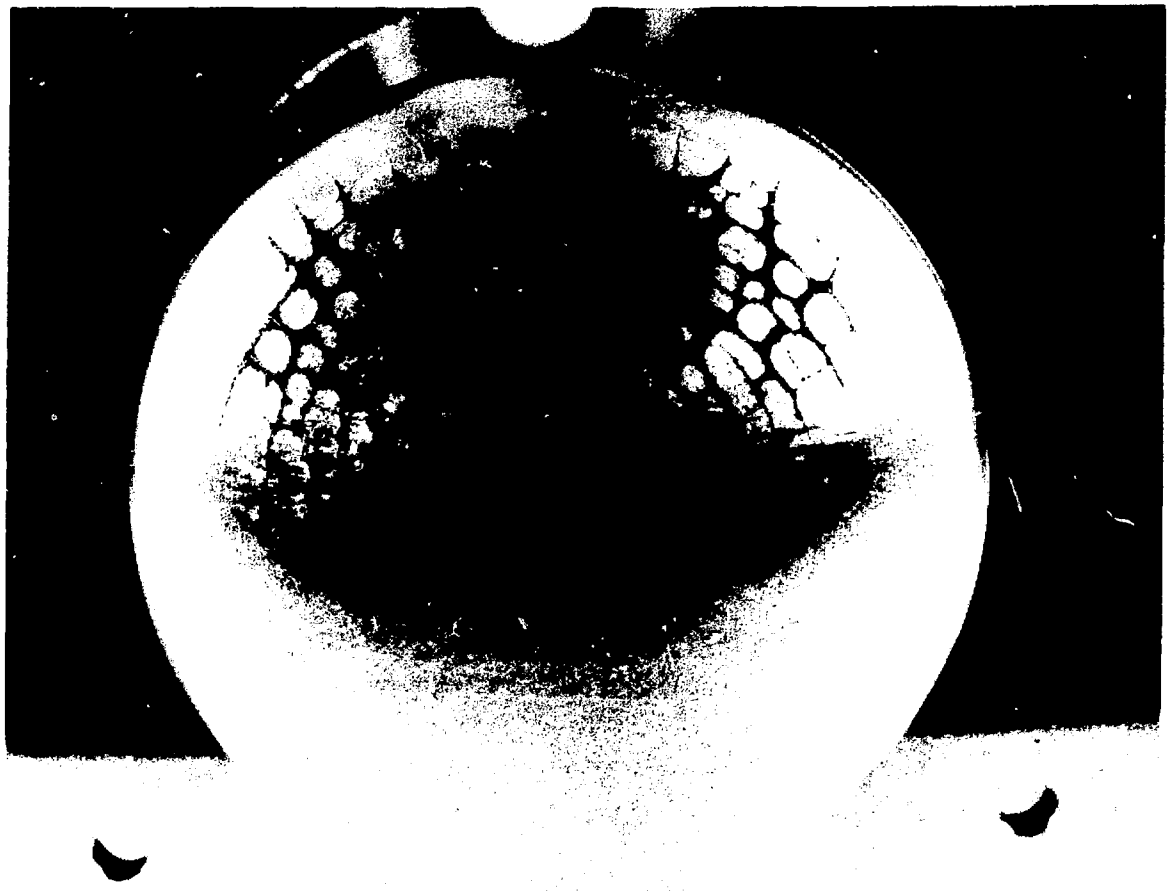


Fig. 3.
Limiter damage pattern. This mushroom-shaped limiter was placed on the outside of the torus. In the photograph, the toroidal current, flux, and electric field are directed from left to right. The reversed field at the limiter is directed from bottom right to upper left. The energy flux to the limiter is in the direction the electron drift would be, assuming the field lines intersecting the limiter have wandered deep within the plasma.

A consequence of this energy confinement model is the convective scaling of energy confinement time, $\tau = 1.5n\int Tdz/\Delta q$, where n is assumed constant and Δq is the total energy flux from both ends of the system,

$$\tau = 0.49 \frac{\ell e n}{j} \quad , Z = 1$$

$$\tau = 0.43 \frac{\ell e n}{j} \quad , Z = 2 \quad .$$

Another consequence of the model, obtained from the definition of the scaling length L, is that the maximum electron temperature scales like

$$T \sim (j\ell)^{2/5} \quad ,$$

where ℓ is the field line length. These same scaling relations are obtained in the two limiting cases of pure convection and pure conduction. Remarkably, the numerical factors in the two extreme cases are not greatly different from the values obtained by numerical integration of Eq. (3). The reason for this fact is the relationship between ohmic heating and thermal conductivity that causes $\alpha\beta e^2$ to be of order 1.

Experimental values of T and j in ZT-40M (300eV, 500A/cm^2) imply quite long lengths on the order of 1 km. Using a typical value of n (2×10^{13} cm^{-3}), the model predicts confinement times on the order of 0.5 ms, roughly in agreement with experimentally measured values.

Equation (3) is independent of density so density explicitly enters only in the determination of the total energy content of the system. However, density does affect the mean free path, which is implicitly assumed to be sufficiently small. Except for a Z dependent numerical factor of order 1, the length in units of the mean free path is equal to the inverse of the drift parameter (the electron drift velocity divided by the electron thermal velocity). This can be seen by expressing the scaling length in terms of the mean free path. As the drift parameter becomes large, the assumed Braginskii transport is no longer valid for two reasons: the conducted heat flux becomes

unreasonably large (exceeding the kinetic flux limit), and the system length becomes inadequate for thermalization of the electrons.

The condition that the drift parameter be near 1 is also the extreme runaway electron condition. While the drift parameter is large in ZT-40M, hard x-ray signals (energy > 50 keV) do not seem to imply a large number of runaways. This fact would be explained by finite length field lines. For example, assuming a 50-V loop voltage (0.07 V/cm) and 1-km field line length in ZT-40M, the maximum runaway electron energy would certainly be less than 7 keV and hard x rays would be entirely absent.

ACKNOWLEDGMENTS

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