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Full Wave Computation of Electromagnetic Wave Excitation, Propagation, and Absorption at the Ion Cyclotron Frequency in Fusion Experiments*

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High-power electromagnetic waves at frequencies ranging from a few megahertz to a few hundred gigahertz serve many important functions in modern fusion experiments. Probably the most important application is plasma heating. Ignition of a fusion reactor will require a plasma to be heated until the average particle energy is ~10 keV (temperature > 10^8 K). This is routinely accomplished in existing large devices. Waves at the ion cyclotron frequency (typically f = 30 to 100 MHz) are very important for fusion devices because of low cost/unit power compared to other frequency regimes and because of their ability to directly heat fusile ions. These waves are also useful for modifying the velocity distribution for improved stability and to drive currents which affect plasma equilibrium. Study of this frequency range is, however, greatly complicated by long wavelengths compared to device size, nonsymmetric device geometry, and the tendency of the waves to linearly transform to shorter wavelength modes. Geometrical optics is generally inapplicable. Thus, codes have been developed to solve the vector wave equation in toroidal geometry for hot plasmas having anisotropic, spatially nonuniform, dispersive constitutive relations.

In this paper we describe the code ORION [1–4] developed at Oak Ridge National Laboratroy and present illustrative applications to a range of fusion experiments. Specific applications of the code include detailed modeling of the antennas used to launch the waves, calculation of wave propagation throughout the plasma, and modeling of the absorption of the waves by the plasma.

1. Mathematical Formulation

For purposes of this paper we restrict consideration to toroidally axisymmetric devices such as tokamaks (Fig. 1) [5,6]. These devices consist of a toroidal shell vacuum vessel, assumed to be perfectly conducting, filled with a hot magnetized plasma. The plasma is contained by a magnetic field $\mathbf{B}^0(\mathbf{x})$, which on the time scale of the wave motion, 1/f, is steady state. The field lines of this equilibrium magnetic field wrap helically around the device and form approximately elliptical, nested surfaces called flux surfaces. The plasma density and temperature are approximately constant on flux surfaces; therefore, it is sometimes convenient to introduce a radial like coordinate, ψ, which labels the flux

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surfaces. The magnetic field strength does not follow the flux surfaces but varies roughly as 1/R, where R is the distance from the axis of symmetry. Another convenient coordinate is the poloidal angle, θ, which is measured the short way around the torus (see Fig 1).

Conceptually the wave formulation is straightforward. The waves of interest have periods much shorter than any other time scale (e.g., heating time or plasma flow time) and the absorption processes are linear, so we may safely assume the solutions to be time harmonic. Thus, the wave equation is simply the $\nabla\times\nabla\times\mathbf{E}$ Maxwell's equation,

$$\nabla \times \nabla \times \mathbf{E} - \frac{\omega^2}{c^2}\, \sigma\cdot\mathbf{E} = i\omega \mathbf{J}_{\mathbf{ext}} \tag{1}$$

where σ is the plasma conductivity tensor operator and $\mathbf{J}_{\mathbf{ext}}$ is the external current due to the antenna. Boundary conditions are that the tangential components of $\mathbf{E}$ vanish on the conducting wall. A second version of the code, which is needed for certain applications as explained below, introduces the potentials $\mathbf{A}$ and Φ along with the Coulomb gauge $\nabla\cdot\mathbf{A} = 0$. Then the waves are described by

$$\nabla^2\mathbf{A} + \frac{\omega^2}{c^2}\, \sigma\cdot\mathbf{A} + \frac{i\omega}{c^2}\, \sigma\cdot\nabla\Phi = -\mathbf{J}_{\mathbf{ext}} \tag{2}$$

$$\nabla\cdot(\sigma\cdot\nabla\Phi) - i\omega\, \nabla\cdot(\sigma\cdot\mathbf{A}) = -\rho_{ext} \tag{3}$$

where ρ_{ext} is the external charge on the antenna. The boundary conditions are $\mathbf{A}_t = 0$ and $\Phi = 0$ on the conducting wall.

The field quantities and source terms can be expanded as a Fourier series in ϕ:

$$\mathbf{E}(r,\theta,\phi) = \sum_N \mathbf{E}_N(r,\theta)\, e^{iN\phi} \qquad \mathbf{J}_{ext}(r,\theta,\phi) = \sum_N \mathbf{J}_N(r,\theta)\, e^{iN\phi} \tag{4}$$

Since the plasma and boundary are uniform in the ϕ coordinate, the Fourier modes are uncoupled. The matrix to be inverted is two-dimensional (2-D), although the structure of the summed fields is three-dimensional (3-D). The 2-D equations obtained by substituting Eq. (4) into Eqs. (1)-(3) are solved by finite difference techniques.

2. Plasma Constitutive Relations

The primary challenge in this work is to correctly model the current distribution induced in the plasma by the wave fields. To place the complexity of plasma wave phenomena in perspective, recall that in a uniform volume of air there are two basic waves: acoustic waves, which are longitudinal, and light waves, which are transverse. These waves are non-dispersive ($v_{phase} = \omega/k = \text{const}$) and except in the most extreme cases are uncoupled. However, in a hot magnetized plasma an infinite number of distinct waves exist [7]. Polarizations range from purely longitudinal (electrostatic waves) to purely transverse (electromagnetic waves). Phase speed and wavenumber can vary appreciably over short distances within the plasma volume. Waves can be very strongly absorbed, with damping length $< \lambda$. And local matching of wavenumber and polarization can occur, resulting in conversion from one wave type to another.

The simplest model is the cold plasma approximation. In the older literature, particularly in the ionospheric propagation field, this is referred to as magneto-ionic theory [8]. If the average particle velocity, $v_{thermal}$, is much smaller than the wave phase velocity, $v_{phase} = \omega/k$, the particles experience an oscillatory acceleration which is proportional to the local electric field. The wave-particle interaction is then approximately local in space and harmonic in time. The plasma current is of the form

$$\mathbf{J}_{plasma}(\mathbf{x},t) = \sigma(\mathbf{x},\omega)\cdot\mathbf{E}(\mathbf{x},t) \tag{5}$$

where $\sigma(\mathbf{x},\omega)$ is a nondiagonal tensor that depends locally on the equilibrium magnetic field vector $\mathbf{B}^0(\mathbf{x})$ and the densities of the various plasma particle species $n_j^0(\mathbf{x})$. This tensor is singular, permitting resonant absorption, at surfaces where the local cyclotron frequency ($\Omega_j = e_j B^0/m_j c$) equals the wave frequency for some species j. There also exist other resonances (wavenumber $k \rightarrow \infty$) called hybrid resonances and cutoffs, or surfaces of reflection ($k \rightarrow 0$), within the plasma volume.

This simple model describes the plasma response reasonably well over most of the plasma. However, near resonances the wave phase velocity, ω/k, decreases and may approach the particle thermal speed. Also near cyclotron resonance, the Doppler-shifted wave frequency in the particle frame of reference, $\omega - kv$, may equal the local cyclotron frequency. Here v is the particle velocity parallel to the wave. In either case the particle experiences a slowly varying accelerating field and may have coherent interactions with the wave over extended regions of space and time. In such cases the conductivity is a nonlocal, integral tensor operator on the wave field:

$$J_{plasma}(x,t) = \int d^3x'dt'\ \sigma(x,x',t,t')\cdot E(x',t') \tag{6}$$

This introduces dissipation throughout the plasma volume (not just at singular surfaces) and introduces a number of new, short-wavelength, electrostatic modes called Bernstein waves. The mathematical problem in this form is equivalent to an infinite-order differential system. In the ORION code the integral operator is expanded to second order in the gyromotion of the plasma particles about the equilibrium magnetic field [9]. In this expansion the conductivity tensor remains local, but now depends on a local wavenumber **k**, which is determined from a local dispersion relation. In this formulation, conversion of the electromagnetic ion cyclotron modes to Bernstein modes is neglected but the energy lost from the incident ion cyclotron waves is adequately modeled.

3. Numerical Considerations

Two different situations occur in practice. Case (1): In most cases of interest the plasma densities in tokamaks are sufficiently high that the electron conductivity parallel to $\mathbf{B}^0$ is much greater than that perpendicular to $\mathbf{B}^0$. In that case the wave electric field parallel to $\mathbf{B}^0$, $\mathbf{E}_{||}$, is very much smaller than $\mathbf{E}_\perp$ and can be calculated perturbatively. The numerical approach in this case is to solve Eq. (1) for $\mathbf{E}_\perp$. Case (2): When low plasma densities are of interest, for example for modeling the outside edge near the antenna, the parallel conductivity may be much smaller so that $\mathbf{E}_{||}$ must be solved for directly. In this case the numerical approach is to solve Eqs. (2) and (3) for the potentials. This approach resolves mathematical difficulties associated with direct inversion of the $\nabla\times\nabla\times$ operator and properly includes electrostatic effects. Special care must be taken to ensure that the finite differencing scheme is consistent with the gauge condition $\nabla\cdot\mathbf{A} = 0$. The finite difference representation preserves the natural grouping of partial derivatives as they occur in the differential equation and in this sense is "conservative." In both cases the finite difference problem is solved by direct matrix inversion using the MA32 package [10] for sparse, unsymmetric systems. This method is based on Gaussian elimination using a modification of the frontal scheme of Hood [11]. It has the advantage that only a small part of the total matrix need be stored in memory at one time.

4. Results from the ORION Code

Figure 2 shows calculations for the poloidal cross section of the Alcator C-Mod tokamak, which is under construction at MIT. The height of the vacuum vessel is about 75 cm. The axis of symmetry is to the left of the cross sections shown, approximately 66 cm from the center. For these calculations, the plasma consisted of electrons with peak density $n_e = 4 \times 10^{14}$ cm^{-3}, deuterium ions with density $n_D = 0.96n_e$, and a minority hydrogen ion species with density $n_H = 0.04n_e$. The temperature of all species was T = 2 keV. The magnetic field strength at the center was $B_0 = 5.0$ T. The wave frequency was 80 MHz. The

antenna consists of a strap of width $L_s = 12.5$ cm located outside the plasma in the recessed area to right [see Fig. 2(a)]. The vertical dashed line is at the location of the hydrogen fundamental cyclotron resonance ($\Omega_H = \omega$). Another plasma resonance called the ion-ion hybrid resonance lies just to the left of the hydrogen fundamental. Figure 2(a) is a grey scale intensity plot of one component of the wave electric field, $\mathrm{Re}\{E_x\}$. The free space wavelength at 80 MHz is 375 cm. However, it is seen that the high effective refractive index N of the plasma to these waves reduces the actual wavelength in the high density, central region to $\lambda = c/\omega N \sim 6$ cm. In the outer regions where the density is low, the refractive index is also small. Since the finite extent of the antenna in the toroidal direction induces a minimum toroidal wavenumber, $k_T = 1/\lambda_T \sim 1/L_s >> \omega/c$, and the radial wavenumber k_R satisfies $k_R^2 = N^2\omega^2/c^2 - k_T^2 < 0$, the radial wavenumber is imaginary at the outside of the plasma. The waves are therefore evanescent and must "tunnel" through to the inner region. One can see the focusing toward the center of the propagating waves due to the radially decreasing refractive index and the decrease in wave amplitude due to absorption at the cyclotron and hybrid resonances.

The primary quantity of interest in these calculations is the spatial distribution of power absorption. This, as shown in Fig. 2(b), occurs at the hydrogen cyclotron resonance and the ion-ion hybrid resonance. A check on the accuracy of the computation can be made by comparing the total absorbed power obtained by integrating $\mathbf{E}^* \cdot \sigma \cdot \mathbf{E}$ over the plasma volume with the total radiated power obtained by integrating $\mathbf{E}^* \cdot \mathbf{J}_{ext}$ over the antenna. For the present calculations, these agree to within 8%. This calculation also gives a useful quantity, the antenna load resistance. To obtain these results the grid consisted of 101 radial mesh points and 50 poloidal points. A total of 50 toroidal Fourier harmonics were included in the summed field, requiring 80 min of CPU time on a Cray 2. It was possible to perform these calculations with full memory storage; 9 million words were required.

An important application of codes such as ORION is to the detailed modeling of fields near the antenna structures. This is both for the purpose of understanding the radiation characteristics of the antennas and for minimizing high-voltage breakdown and sputtering due to near fields. A common technique for improving antenna performance is to eliminate stray toroidal electric fields by placing a Faraday shield consisting of conducting bars in the toroidal direction in front of the antenna.

Figure 3 shows a sequence of calculations for the DIII-D tokamak at General Atomics that focuses on the details of the fields near the antenna. This is a somewhat larger device than Alcator C-Mod, with a 260-cm-high vacuum vessel and a radius from the symmetry axis to the center of the cross section of 167 cm. For these calculations the peak electron density was $n_e = 8 \times 10^{13}$ cm^{-3} and the plasma temperature was T = 3 keV. The magnetic field at the center was $B_0 = 1.7$ T. In Fig. 3(a) the solution for $\mathrm{Im}\{E_y\}$ was computed across the entire cross section using the version of ORION which solves Eqs. (2) and (3). In Fig. 3(b) the computational domain was reduced to that indicated by the rectangular box. Thus,

the density of grid points in this region was increased by a factor of about 3. Again, perfectly conducting boundary conditions were imposed at the edge of the domain, but reflections were eliminated in the portion of the boundary lying in the plasma by introducing an artificially large collisional absorption near the edge. Except near the artificial absorber the agreement between the solutions in Figs. 3(a) and 3(b) is good. Figure 3(c) shows the same case but with conducting metal rods inserted as a Faraday shield. One can see the field compressed between the shield blades and some perturbation of the radiated fields in the plasma region. These calculations have proved very useful in understanding the mechanism for increased plasma sputtering due to RF fields near antennas [4]. For this calculation the grid was an 85×85 rectangular mesh. Owing to the increased number of mesh points and increased number of equations per point ($A_{||}$ and Φ are now included) it was no longer possible to invert the matrix in full storage. A 100 million word disk file was required. Solution for each toroidal mode takes 16 min of Cray 2 time. The solutions shown in Fig. 3 represent a single mode.

5. Conclusion

We see that it has now become feasible to study wave phenomena in very complicated 2-D geometries and in nonuniform, dispersive, dissipative media. These calculations are of great practical importance for understanding the results of fusion experiments and for designing future devices. The assumption of a 2-D boundary is adequate for studying the penetration of the waves into the center of the plasma and for calculating the absorption profiles. It would be desirable to include the three-dimensionality of the antenna structures at the plasma edge. However, it does not appear computationaly feasible to include such a complicated medium as a hot plasma in a 3-D code at this time.

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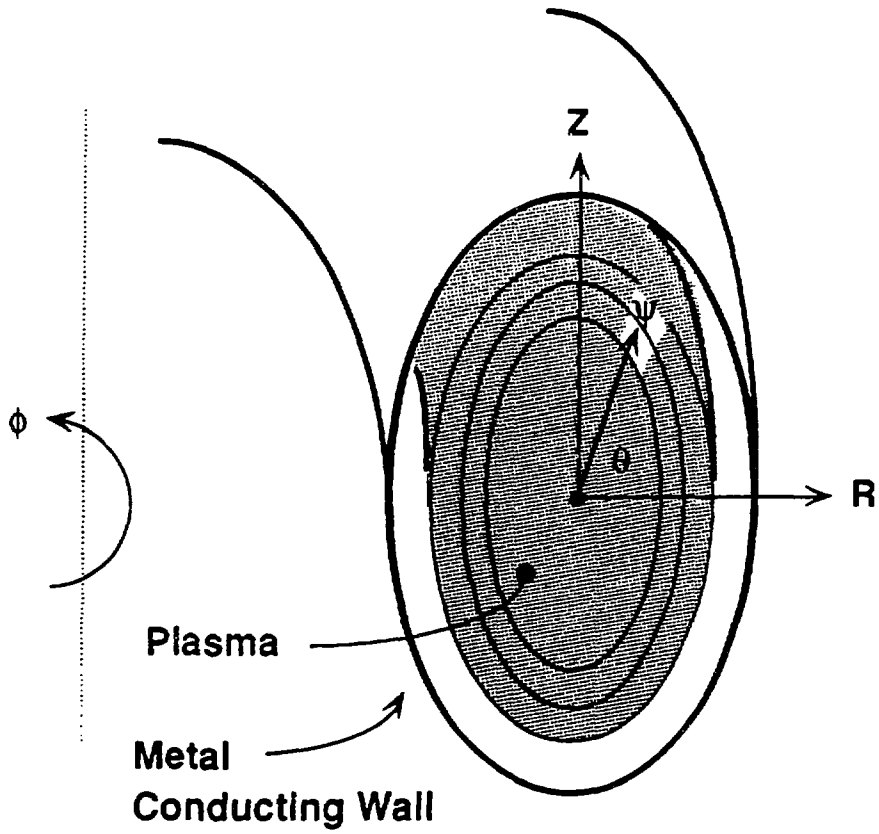

Fig. 1. Tokamak - toroidally symmetric geometry.

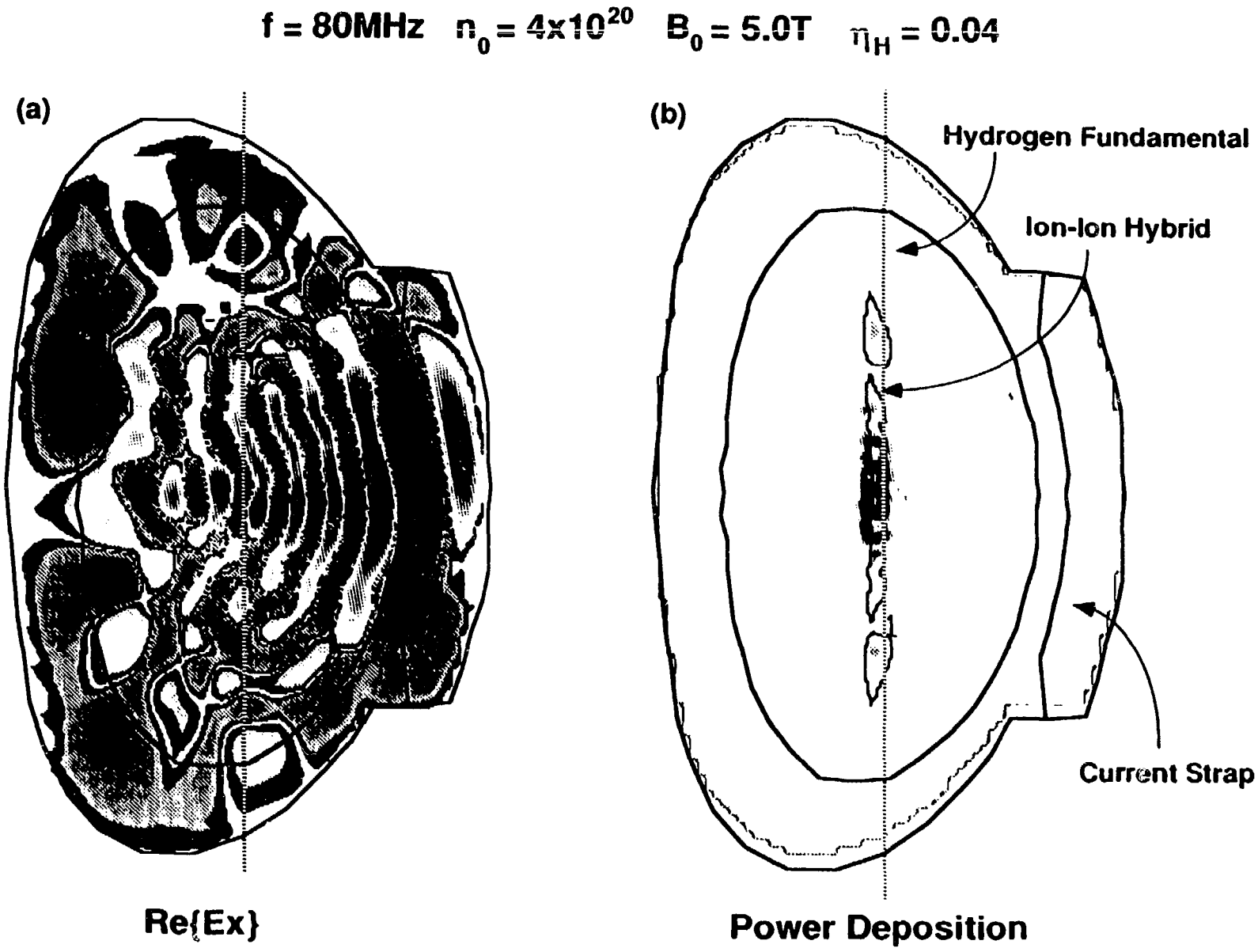

Fig. 2. Alcator C-mod D-H.

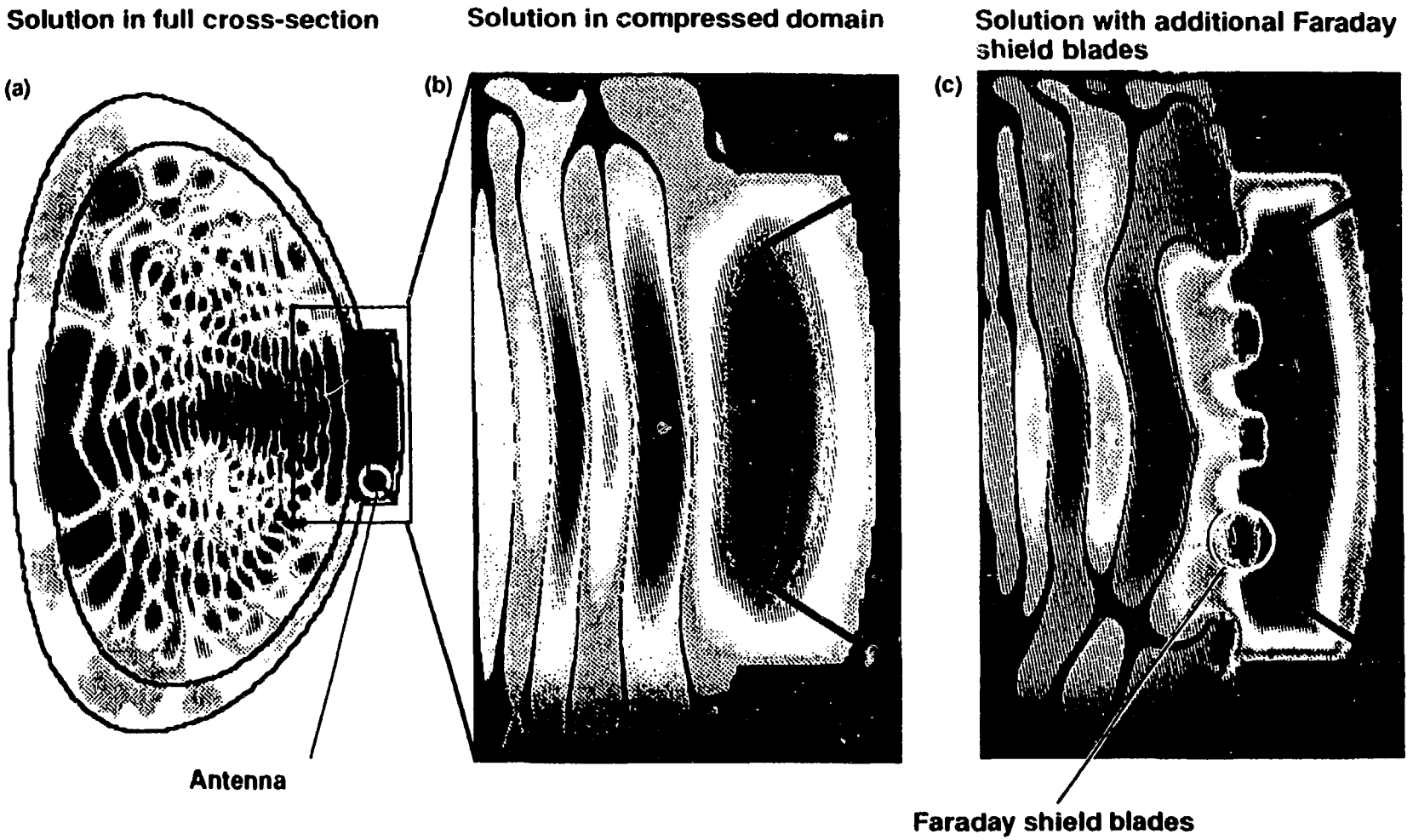

Fig. 3. Detailed modeling of antenna near fields in Tokamak geometry.