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Task A

Table of Contents

I. Introduction

II. Progress report of L. N. Chang

III. Progress report of C. H. Tze

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I. Introduction

This report summarizes and highlights the work of the theory task in high energy physics supported by *DOE* during 1989-1990.

In 1989-1990, the theory group consisted of one professor, L. N. Chang, one associate professor, C. H. Tze, and one research associate physicist, Waichi Ogura. One graduate student, Chopin Soo, working for the doctoral degree under Chang's supervision, was supported by the *DOE*. In addition, two other students, Zoltan Trocsanyi, working with Chang, and Marek Grabowski working with Tze, also contributed to the research, but they were not supported by the existing contract.

During the course of the year, we benefited greatly from interactions with Robert E. Marshak, University Distinguished Professor Emeritus, members of the *Institute for High Energy Physics*, R. Workman of the intermediate energy physics group, and T. K. Lee and R. K. P. Zia of the condensed matter group.

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II. Research progress report of Lay Nam Chang

During the past year, my research has continued to center on gauge theories involving chiral fermions. Specifically, I have been studying generic properties of anomalous higher point Green's functions for axial currents, and applying these to various high and low energy processes. These properties have also been used to study the topological phase of quantum gravity in four dimensions. In addition, I have continued to examine the temperature dependence of chiral symmetry breaking in QCD in four dimensions. The main result that was obtained this past year has been the demonstration of how chiral symmetry breaking persists to all temperatures, and how this result can be understood within the lattice framework. Some of the details of these works have been described in the report submitted last year. In what follows, I will elaborate on the results obtained in the past year.

Higher point anomalous amplitudes

Anomalies in chiral fermion theories imply that certain amplitudes involving divergences of axial currents are not vanishing, even though the underlying Lagrangian is invariant under the corresponding transformations. It may be shown that as a consequence, these amplitudes describe the behavior of certain phases, which originate from topological structures in field configuration space[1,2,4]. As a result, the actual values of these amplitudes can be computed explicitly, either by making use of an underlying cohomology sequence, or by directly calculating the associated Feynman diagram. In those cases where the currents represent symmetries which are spontaneously broken, the interactions of the associated Nambu-Goldstone particles represented by these amplitudes are completely determined.

The form of these interactions for some lower point functions has already been discussed in the literature[5]. For example, the three point amplitudes give the usual Adler-Bardeen-Bell-Jackiw results, but only a partial form of the results for the four point functions is discussed in the literature. The interactions described by the higher point functions are new. In the past year, I have been studying the phenomenological implications of these terms, especially in connection with the proton spin crises[9]. Of late, I have also been considering the effects of these interactions on neutral pion photo-productionn at threshold, where there appears to be a discrepancy between the classic low energy predictions and experiment[13].

An alternative way of describing these phases is obtained by ascribing to commutators of current densities certain *extension* terms. For example, it is known that the usual triangular anomaly results can be re-expressed in terms of an extension on the charge-density algebra. The higher point current amplitudes place further constrains on these extensions[3], and the related operator product expansions. One of these expansions is relevant to polarized deep inelastic lepton nucleon scattering[9]. In the literature, this expansion is fixed by studying the Delbrück box diagram[10,11]. The higher point current amplitudes alluded to above provides for a different approach to this problem, which should take into account some of the non-perturbative effects discussed in the literature[12]. I have been working on this question during the past year with a graduate student, Zoltan Trocsanyi[7]. As a by-product of this work, a computer code was created, which can be used to calculate higher order loop diagrams in the back-ground field method[8].

The extended commutator relations following from the higher point functions can also be used directly to constrain low energy processes. It has been known for some time now, that observed photo-production of neutral pions at threshold disagrees substantially with classical current algebra predictions[13,14]. The reaction receives no contributions from the usual current algebra. Threshold photo-production of charged pions, on the other hand, does get contributions from this algebra, and is described rather well by these low energy theorems. The extensions referred to above, however, do modify the neutral elements of

the algebra, which enter into neutral pion photo-production, and work is in progress to check if in fact they can account for this discrepancy.

These results can be applied also to quantum mechanical systems. The existence of the topological phases presents obstructions to a proper classical limit for the system[15].

Topological phase of quantum gravity

The identification of the proper canonical variables to use in quantum gravity has been a long standing problem. The best choice of varables appears to be those introduced by Ashtekar[16], in terms of which the the appropriate constraint equations take a simple form. These variables can be interpreted as a set of $SU(2)$ gauge potentials, A^a. The classical constraints then take the form:

$$\mathcal{D}_i \sigma^{i,a} = 0 \tag{1}$$

$$\sigma^{ia} F_{ija} = 0 \tag{2}$$

$$\epsilon_{abc} \sigma^{ia} \sigma^{jb} (F^c_{ij} + \frac{\sqrt{2}}{3} \Lambda \epsilon_{ijk} \sigma^{kc}) = 0 \tag{3}$$

$$\tag{4}$$

Here F_{ij} denote the field strength of the vector potential, the indicies i, a refer to the three directions in ordinary and isospin spaces respectively, σ the conjugate of A, and $\mathcal{D}$ denotes the covariant derivative. In the above, a possible cosmological term Λ has also be included. In the absence of matter, and apart from boundary terms, the Hamiltonian is simply a linear combination of these constraints.

In going over to the quantum theory, there is the usual ambiguity with respect to operator ordering. In the past year, a graduate student, Chopin Soo, and I have been studying this problem, and have come up with a possible consistent prescription for this ordering. Within

this ansatz, the ground state wave-functional depends on the Ashtekar vector potential only through the Chern-Simons form. The first two constraints are automatically satisfied, since they express nothing more than invariance under small $SU(2)$ gauge transformations, and three dimensional diffeomorphisms of ordinary space. The last constraint is simply the Ashtekar-de Witt-Wheeler equation[16]. What we showed is that relative to this class of functionals, this equation is identical to the condition that the wave functional annihilates a $BRST$ charge enforcing topological invariance. By this invariance, we mean the invariance introduced by Witten in his construction of topological field theories[18]. Hence the wave-functional we are looking at describes quantum gravity in the unbroken topological phase, where there are no gravitons. In this phase, one may treat the cosmological constant as the coupling constant. The wave-functional so constructed has the additional property that it automatically satisfies the Ashtekar-Renteln ansatz, which is known to satisfy the constraints classically[17]. Various other implications of this construction are now being worked out[19].

Chiral symmetry breaking at finite temperatures

Over the last few years, I have been involved with a study of spontaneous chiral symmetry breaking in *QCD* with massless quarks[22]. The technique we used is essentially that of the renormalization group (*RG*). We first set up the quark two-point function with massive fermions, consistent with the requirements of *RG*. We then study the chiral flip part of this amplitude which is proportional to to m_r, the renormalized mass. This is dependent upon certain *RG* invariant parameters, which are essentially the integration constants of the *RG* equations. We then extract the *RG* invariant part of the chiral flip amplitude, and isolate its dependence on these invariant parameters. To study chiral symmetry breaking (*XSB*), we let these parameters go to the critical values such that $m_r \to 0$, and inquire if the chiral flip amplitude survives this limit. The result of our analysis is that for an asymptotically free theory such as *QCD*, *XSB* indeed does occur, and, in addition, we are able to calculate the value of the chiral symmetry order parameter $\langle\bar{\psi}(x)\psi(x)\rangle$ in terms of the invariant cut-off Λ. The actual value we obtain turns out to be in agreement with those obtained from current algebra analysis of low energy pion-nucleon scattering[22]. It is also consistent with estimates based upon lattice considerations[21]. Furthermore, we are able to show that the order parameter, regarded as a function of momentum, goes to zero for values of momentum higher than a critical value. The order parameter exhibits the phenomenon of *bifurcation*[20].

During the past year, we examined the question of the temperature dependence of the order parameter. The idea is to repeat the calculation at finite temperatures, and ask if the XSB solution exhibits any phase transitions. The surprising result for the RG invariant mass, defined as the zero of the inverse two-point fermion function, is $\mathcal{M}^2 = (2\pi^2/3)(T^2)(\ln T^2/\Lambda_{QCD})$, for high temperatures. Notice that it is independent of three-momentum, and is different from a conventional plasmon excitation in that respect. A straightforward computation of the order parameter, using only the minimal renormalization necessary to get rid of ultra-violet divergences, gives zero[23]. An examination of the equations shows that the calculation is valid only for temperatures above a critical value, so that one is tempted to conclude that there is a phase transition. Indeed, the value of this critical value agrees with those obtained in lattice studies[21]. So chiral symmetry is apparently restored, despite the appearance of a mass for the fermion. However, a careful analysis shows that the Dirac equation for this particle is non-local. It is therefore unclear what is meant by chiral transformations. A further transformation is necessary in order that the effective Dirac equation be brought into local form. This is accomplished by a Cini-Touschek transformation[24]. With respect to the field expressed in this basis, the order parameter is directly proportional to the RG invariant mass, and so persists to arbitrarily high temperatures. Therefore, chiral symmetry is not restored at high temperatures. Lattice calculations generally do not perform this final transformation, and so give the apparent restoration of

chiral symmetry at high temperatures. The reason is that chiral transformations depend upon the wave functions used in expanding the fermi field, and are therefore sensitive to the choice of the zero of energy. For finite temperatures, this choice acquires a dependence on this temperature. The Cini-Touschek transformation is necessary to put in this dependence on the wave-functions in the expansion of the fermi field. Relative to this basis, where chiral transformations are well-defined, XSB persists to arbitrarily high temperatures[23]. A better comparison with lattice considerations, therefore, is to examine if the calculation there for the same two point function will yield an RG invariant mass with the same value as the one obtained here. Such a comparison is now being sought for.

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III. **Progress report on the research of C.H. Tze**

During the past year my research work continues to be on the physics and mathematics of nonlinear phenomena in particle physics. It focuses on very specific problems in four areas relating to a nonperturbative group theoretic approach to strong interactions, various global aspects of quantum field theory and to possible extensions of the Standard Model and their relevance at SSC energies.These four areas are:

1) D≥4 dimensional solitons, mainly topological ones. Search for explicit examples of higher dimensional fermi-bose transmutation via the representation theory of D≥3 infinite dimensional Kac-Moody group. Skyrmions in effective lagrangians of QCD, of the standard electroweak model and its possible extensions. Their applications to SSC physics.

2) Self-duality, conformal structures and covariant quantization of field theories . Further extension and applications of hypercomplex analysis to four dimensional conformal field theories. Applications of special geometries associated with division, Jordan and related algebras. Non-associative, exceptional algebraic and group theoretical aspects of superstrings and supermembranes

3) Topological field theories in D≥3 dimensions : Aharonov-Bohm-Berry phase phenomena in field theories, anyonic properties of relativistic membranes and the possibility of exotic spins and statistics in higher dimensions. Various Applications to Kaluza-Klein scenarios and to

condensed matter physics. Their geometric quantization via the Kirillov-Kostant theory of group orbits.

4) Integrable systems in D≥ 2 dimensions : search for a) explicit realizations of Yang-Baxter-Zamolodchikov liked systems in topological quantum field theories.b) physical realizations of quantum groups

The battery of mathematical tools to be used is varied. They include index theorems, homology theory, theories of harmonic mappings, of holomorphic function over a quaternionic variable, twistors techniques and the Penrose Radon transform, the representation theory of D≥ 2 current and diffeomorphism algebras, theories of links and knots, of Hopf and affine Jordan algebras, differential geometry of exceptional manifolds.

Before going into the specific problems to be studied, I should briefly retrace some past results which are natural stepping stones for the above topics. Since the details of this research were already described in last year progress report, I will mostly elaborate on further results and the status of ongoing work of these topics.

Section 1 recalls the relevance of the Skyrmion approach to strong interactions, mainly to the Tev physics of the standard model's Higgs sector, then state the proposed problem of research

Section 2 reports on ongoing work on higher dimensional analogs of

D=2 conformal field theories, specifically the study of relations between general covariance, infinite symmetries, self-duality and hypercomplex analyticity in self-dual Yang-Mills theory and in the broader context of cohomoloigal field theories.

Section 3 discusses some outstanding issues concerning nontrivial Bohm-Aharonov-Berry phases of anyonic membranes. I report on the progress achieved in the geometric quantization of D≥3 topological gauge theories by the Kirillov-Kostant method of group orbits.

I. Skyrmions as representations of current algebras

QCD is the undisputed gauge theory of strong interactions. Yet for lack proper technical tools only its perturbative short distance ultraviolet properties have been succesfully confronted with experiements. Indeed, asides from the advances in numerical simulations on lattice gauge theories and the insights they have provided, analytic nonperturbative techniques are nonexistent. This frustating situation is in sharp contrasts to the abundance and ever greater accuracy of experimental data on hadrons.

However at a more phenomenological and semi-quantitative level, a tractable analytic method of attack of the strong interactions problem has emerged under the name of Skyrmion physics [1]. This solitonic approach pioneered by T.R. Skyrme [2] was revived in 1979 by N.K. Pak and myself [3] in the context of effective chiral lagrangians and current

algebra . In recent years, after the pioneering works of Balanchandran [1], of Witten[4] and their schools, much research activity has taken place in this field, specially among nuclear physicists. A semi-quantitative unified field theoretical picture has emerged for hadrons and their interactions [5].

Asides from strong QCD interactions at Gev energies, there could well be detectable new strong forces at work elsewhere. Consider the standard SU(3)xSU(2)xU(1) model of strong and electroweak interactions While meeting every experimental test, it is nevertheless structurally incomplete . The reason rests in the arbitrary mass of the Higgs particle; $M_H = [\, 8\pi \ \alpha_H \sqrt{2}\, G_F]^{1/2}$; α_H the quartic scalar coupling being unknown. The same arbitrariness extends to fermion masses. Consequently should the physical Higgs mass exceed 1 Tev (i.e. $\alpha_H \sim 1$), a very likely experimental possibility, perturbative analysis will fail as the Higgs sector becomes strongly coupled. The three components of the Higgs doublet , making up W_L, the longitudinal components of the W 's and the Z , will interact strongly among themselves, with the quarks and the leptons. These interactions will thus reveal an additional strong "Higgs force", determined within the standard model, and apparently not accounted by a gauge principle. It has been clear since Veltman's analysis [6] that the Tev physics at this new regime closely parallels the chiral dynamics of the SU(2) sigma model of low energy hadronic interactions at Gev energies. As was the case for hadrons, one is led to expect from the strongly coupled Higgs system a "Yukawa sector" potentially as rich if not richer in structure as the quark or lepton systems.

Indeed in a paper with J. Gipson [7], we realized that the effective 1-loop finite gauged sigma model [8] used to simulate heavy Higgs effects has exactly all global and dynamical features of the Skyrme's action and hence should admit Skyrmions. Due to the topology of the field space, these solitons could be fermions upon canonical quantization [3,4]. We carried over to the strongly coupled Higgs regime all the physical implications of the Skyrme model : we predicted a rich calculable isobaric-liked spectrum of fermionic resonances and estimated the bounds for the lowest soliton mass to be $2.5 \leq M_{Soliton} \leq 4.0$ Tev .These bounds depends only logarithmically on the Higgs mass. This soliton scheme is in fact a "minimal extension" of the standard model as it makes no specific assumption whatsoever about the compositeness of the Higgs particle(s). These "weak skyrmion" couple to leptons which induce on them with such exotic attributes as a leptonic charge L equal to their baryonic charge B. In the past few years the above approach to the Higgs sector has been investigated by several groups worldwide [9]

In 1987 , I began with Dr. P.Q. Hung of UVA to further apply the skyrmion approach to the electroweak sector. The first outcome of this project was a paper entitled " Can a fourth lepton family exist without its quark counterpart ? " [10] (#31 in Vita). It put forth the novel idea of accommodating via Skyrme's fermionization mechanism,new leptons within the Standard Model;, without the customary need of accompanying new quarks and without having to cope with the specifics of the possible compositeness of the Higgs bosons.

To carry further the above ideas behond their qualitative basis, a key technical ingredient has been sorely missing in all applications of the Skyrme idea. We need a systematic mathematical quantum approach to describe one and several Skyrmions. We need a wide ranging formulation and determination of the precise nature and extent of the D≥2 fermi-bose correspondance behond the semi-classical analysis available thus far. At a minimum we seek a 1- soliton operator analogous to Mandelstam 's for the quantum soliton in the Sine-Gordon model [11]. The latter intertwining operator emerges naturally from the representation theory of D= 2 Kac-Moody algebra. The latter D≥3 counterpart , e.g. the mapping group { S^3" G } with its noncentral Abelian Q-number extension has been studied in recent years by Rajeev and Mickelsson [11,12,14]. Lately these authors have finally found some <u>unitary</u> representations [13]. The status of this problem was surveyed in my talk this august at the Voralberg Symposium on Symmetries V [15]. In the past months and in the year ahead I propose to investigate, with my student , Marek Grabovski, the possible physical relevance of these interesting representations in the context of QCD and the strongly coupled HIggs sector. This investigation will be done as part of our continuing search [14] for the explicit algebraic construction of a 1 or multi-Skyrmion creation operator in the representation theory of D≥3 current (and diffeomorphism) algebras[15].

2. D≥ 4 Critical Phenomena: Self-Duality, Infinite Dimensional Symmetries and Hypercomplex Analyticity.

Mathematical tools tailored made for the study of field theories in critical (e.g. D= 3, 4, 6 and 10) spacetime dimensions have shown their efficacity in connection with supergravity, superstring and supermembrane theories [16]. That two dimensional conformal field theories are elegantly solved by complex analys is widely appreciated. Yet little known is the existence of hypercomplex analysis of a quaternion [17,18], octonion [19] . In several papers with Gürsey (see Vita) , the power and relevance of the latter function theories have been demonstrated in the case of four dimensional self-dual Yang Mills theory, sigma models, and in higher dimensional strings and membrane theories [18,20,21,22]. They closely related to Penrose's twistor program [23] for solving nonlinear classical and quantum field theories.

In particular in Ref[21] we showed that a conformally Kahlerian structure of the euclidean Schwarzschild Ricci flat geometry leads to its form invariance under the quaternionic analytic Fueter transformations The latter form a remarkable infinite subgroup of the general covaraince group. In some recent works [24] further significant progress has been made on grafting an essential self-(antiself-dual) structure on this infinite dimensional the Füeter group. The latter now has as a subgroup, the D=4 conformal group . Well combined with the construction of self and antiself dual quaternionic functions, this renewed approach has provided a striking D=4 analog of infinite dimensional D=2 Virasoro conformal group. In recent months I have been picking up this subject anew . My goal has been a) to elaborate further on these new results of Gürsey et al.[25] seeking their connections to the representation theory of D=4

diffeomorphism groups, b) to explore possible applications in explicit D=4 critical field theories and by merging them into a larger structure with higher dimensional Kac-Moody groups discussed in Section 1.

Indeed as an overview of the whole subject of division algebras and function theories over them has been gained in the writing of a comprehensive (500 pages) book with F. Gürsey of Yale . The book entitled " On the Role of Division, Joradn and Related Algebras in Particle Physics" will be published by end of summer of 1991 by World Scientific. Except for some minor retouchings and small additions and the inserion of references the book is essentially finished. A table of content is enclosed.

3. D≥3 Topological Field Theories: Anyonic Membranes and Division Algebras, geometric quantization by the method of orbits

This study of this topic triggered by a paper of Polyakov [26] had preoccupied me for the two previous years and resulted in four publications [27-30]. The object has been to understand the possible physical role of anomalies and physical relevance of added topological action to the standard actions on the spin and statistics of exrended objects. We managed to show on primarily topological grounds that certain membrane solitons in D≥3 spacetime dimensions resulting from remarkable Hopf mappings are endowed with œ-spin and statistics. Yet the definitive proof should come from actually quantizing the membranes themselves and show that the generalized Polyakov's phase factor

corresponds indeed to a spin factor. The difficulties of doing so are at least the same as those of quantizing supermembranes [31]: the nonlinear character of the constraints, the possible lack of quantum consistency due to anomalies. As with all work on membrane quantization, our progress has been slow but our work is continuing.

Recently the method of orbit of Kirillov and Kostant [32] has been successfully extended to the geometric quantization of the Hamiltonian action of an infinite group of symmetries such as the Virasoro group [33-34] . Two dimensional topological quantum field theories such as the Wess-Zumino-Witten model have been constructed on the co-adjoint orbits of infinite dimensional Lie groups. As a first step toward a geometric path integral quantization of $D \geq 3$ topological field theories via the orbit method, Marek Grabowski and I have succeeded [35] in reformulating several classical gauge theories on the coadjoint orbits of the semi-direct product of the the gauge group and the Weyl group. Our construction is illustrated in the acse of the D-dimensional Yang-Mills theories, the d=3 Chern-Simons and D-dimensional topological field theories [36]. A copy of our preprint is enclosed. Work on the supersymmetric extension of our scheme, the connection to supercoherent states and on the application to the quantization of topological theories is in progress.

4. Novel Non-Perturbative Approaches

4a Background

Much of my research goes under the general heading of non-perturbative methods. Though the years I have applied them to dual resonances models, to theories of quark confinement, to the topological classification and dynamics of monopoles, vortices, instantons and solitons . In last decade or so, powerful algebraic methods have been developed and tailored to solve whole families of integrable systems, classical and quantum. They are the method of the (quantum) inverse transform, of the generalized Bethe ansatz, " vertex operators" or representation theory of Kac-Moody, and more recently the representation theory of Hopf algebras [32] and quantum groups.

Always keen on tackling the more difficult cases of higher dimensional integrable systems, I have particularly been seeking further explicit realizations of $D \geq 3$ counterparts of the Yang-Baxter- equations [37,38], called simplex equations [39] and connecting them to higher dimensional topological quantum field theory such as the Donalson theory in four dimensions. Some preliminary work has been initiated in 3 spacetime dimensions in the form of a generalized Bethe ansatz in collaboration with Dr. Peter Orland, now at Baruch College of the City University of New York. Difficulties encountered so far are connected to our current inability to solve for delta function interactions in two spatial dimensions. These issues of higher dimensional integrability are intimately connected to the representation theory $D \geq 3$ infinite

generalized Kac-Moody and Virasoro groups outlined in Section 2 [15] and to the quaternionic analyticity approach to D≥2 analogs of D=2 conformal field theories.

Presentations and Seminars in 1989-1990

Finally. I wish to report on my professional travels this past year. In October 1989, I lectured on my work on Berry phases of antisymmetric gauge fields at the department of physics of the University of Cincinnati, Cincinnati, Ohio. In May 1990, I paid a one week visit to the Institute for Advanced Study, Princeton, N.J. at the invitation of Prof. Edward Witten. During the last week in June and the first week of July 1990, I visited and gave a seminar at two french laboratories at the Ecole Normale Superieure in Paris, as a guest of Prof. B. Julia and at the Ecole Polytechnique, Palaisseau, as a guest of Prof. Tran T. Truong. During the first week of August, at the invitation of Prof. Biedenharn, I attended and was principal speaker at the Symposium on " Symmetries in Science V: Algebraic Systems: their Representations , Realizations and Physical Applications" in Vorarlberg, Austria. My talk entitled " New Phases in D≥2 Current and Diffeomorphism Algebras in Particle Physics" is enclosed. During the second week in August, as a guest of Profs. H.B. Nielsen and P. Olesen, I visited and lectured at the Niels Bohr Institute,the University of Copenhagen, Denmark.

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