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MEASUREMENT OF MAGNETIC FLUCTUATIONS ON ZT-40(M) *

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Abstract

The mathematical basis for experimental measurement of magnetic fluctuations in a Reversed Field Pinch is reviewed. A quasi-static drift model is introduced as the framework for analysis of the five-fixed-probe technique. The exprapolation of edge-measured $\tilde{B}_r$ fluctuations into the plasma is discussed. Correlations between magnetic and other fluctuations expected from a quasi-static model are derived and transport-relevant correlations are discussed. Data from ZT-40(M) are presented.

I. INTRODUCTION

Magnetic fluctuations and the associated field line stochasticity provide the key to understanding many of the special features of plasma behavior in the Reversed Field Pinch and other magnetic confinement devices. It seems very important, then, to begin a concerted wide-spread program of experimental measurement of magnetic fluctuations, and indeed, such a program is underway at several laboratories. Of particular interest are fluctuation quantities of direct relevance to transport. These are principally the magnetic field diffusivity $D_m(r)$ and the Chirikov (island overlap) parameter $S(r)$. Hutchinson *et. al.*[1] have discussed estimates of S in the edge region obtained from external edge measurements of $\tilde{B}_\theta$ and $\tilde{B}_\phi$. Measurements of $D_m(r)$ and $S(r)$ have been reported by Miller *et. al.*[2] using direct edge measurement of $\tilde{B}_r$. These measurements used a simplified five-probe technique to determine the mode spectrum, rather than mapping out the entire spatial autocorrelation function. This technique, based on the method of Beall *et. al.*[3] will be discussed in more detail in this paper. Magnetic fluctuation measurements are being carried out also on the REPUTE[4] and MST[5] Reversed Field Pinch experiments.

The ZT-40(M) measurements utilized the five probes shown in Fig. 1., each measuring three components of **B**. The probes were protected from plasma damage by Boron Nitride shields surrounding the ceramic vacuum jackets, allowing the centers of the coils to be inserted 1 cm into the plasma for shortened-time 60 kA discharges. Figure 2 shows the withdrawn and fully inserted positions of the probes relative to the bellows vacuum chamber.

Each probe had three pickup coils bifilar wound on a 11 mm long by 3 mm diameter coil form, with center tap grounded to a 0.0005-in-thick stainless steel electrostatic shield. The coils had an NA of about 10 cm^2. Twisted pair cable and differential-input preamplifiers were used. Signal differencing was used to minimize bit error, with 1024 level analog-to-digital conversion, at a sampling rate of 1 MHz.

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II. MODE SPECTRUM AND RELATED QUANTITIES

In this section some mathematical background is presented. A numerical example, an analytical fit to the mode spectrum measurements of Ref. 2, is also discussed. A toroidally-curved, cylindrical coordinate system will be used, with coordinates r, θ, and ϕ, where r is the minor radius, θ is the poloidal angle, and ϕ is the toroidal angle. The major radius R of the torus is assumed to be large compared to the minor radius r, so that the geometry is approximately cylindrical. The fundamental magnetic field fluctuation quantities will be taken as the mode amplitudes $b_{mn}(r,t)$ in the Fourier decomposition of $B_r(r,\theta,\phi,t)$,

$$B_r = \sum_{m,n} b_{m,n} \exp[i(m\theta + n\phi)]. \tag{1}$$

Since B_r is real, we have the relation

$$b_{m,n} = b^*_{-m,-n} \tag{2}$$

between amplitudes with positive and negative mode numbers.

From $\nabla \cdot \mathbf{B} = 0$, the surface integral of B_r over a cylindrical surface is zero,

$$0 = \int B_r \frac{d\theta}{2\pi}\frac{d\phi}{2\pi} = b_{0,0}\,, \tag{3}$$

which implies that the Fourier amplitude $b_{0,0}$ is zero.

Several different averages will be used subsequently: 1) time averages over some time interval for a particular shot at fixed spatial position, 2) shot-to-shot averages at fixed times and spatial positions, and 3) spatial averages over θ and ϕ at fixed times and shot numbers. Three simplifying assumptions allowing these different averages to be related to one another will be made: 1) that the fluctuations are stationary in time so that average quantities are time translation invariant, 2) that shots are statistically identical so that averages are independent of shot number, and 3) that there is approximate cylindrical symmetry, so that averages are cylindrically invariant. With these assumptions, all of the different averages are equal. Experimentally, it is usually most convenient to use time and shot-to-shot averaging. The time average $\overline{Q}$ is defined by

$$\overline{Q} \equiv \int_0^T \frac{dt}{T} Q(r,\theta,\phi,t), \tag{5}$$

and $\overline{Q}$ is assumed independent of θ and ϕ. This is a useful approximation but not an exact relation. In a real toroidal plasma there is not exact cylindrical symmetry and $\overline{Q}$ does depend somewhat on θ and ϕ.[1] Theoretically, it is sometimes advantageous to use cylindrical averaging over θ and ϕ. Thus we define the cylindrical average $\langle Q\rangle$ of $Q(r,\theta,\phi,t)$ as

$$\langle Q\rangle \equiv \int_0^{2\pi} \frac{d\theta}{2\pi} \int_0^{2\pi} \frac{d\phi}{2\pi} Q(r,\theta,\phi,t). \tag{4}$$

By assumption $\langle Q\rangle$ is independent of time. If $\overline{Q}$ is independent of θ and ϕ and $\langle Q\rangle$ is independent of time, then $\overline{Q} = \langle Q\rangle$, since $\langle\overline{Q}\rangle = \overline{\langle Q\rangle}$, and $\overline{Q} = \langle\overline{Q}\rangle = \overline{\langle Q\rangle} = \langle Q\rangle$.

An important quantity is the spatial autocorrelation function, defined by

$$C(\Delta\theta, \Delta\phi) \equiv \frac{\langle B_r(1)B_r(2)\rangle}{\sqrt{\langle B_r^2(1)\rangle\langle B_r^2(2)\rangle}} = \frac{\langle B_r(1)B_r(2)\rangle}{\langle B_r^2\rangle}, \tag{6}$$

where the arguments 1 and 2 refer to points with different θ's and ϕ's. In the surface average, the separations $\Delta\theta = \theta_1 - \theta_2$ and $\Delta\phi = \phi_1 - \phi_2$ are kept constant. Substituting Eq. (1) into Eq. (6), we obtain

$$C(\Delta\theta, \Delta\phi) = \frac{1}{\langle B_r^2\rangle}\sum_{m,n}|B_{m,n}(r,t)|^2 \exp[i(m\Delta\theta + n\Delta\phi)]. \tag{7}$$

Equation (7) shows how the mode spectrum $|b_{m,n}|^2$ determines the spatial autocorrelation is function. Fourier transforming, the inverse relation is

$$|b_{m,n}|^2 = \langle B_r^2\rangle \int C(\Delta\theta, \Delta\phi)\exp[-i(m\Delta\theta + n\Delta\phi)]\,\frac{d\Delta\theta}{2\pi}\frac{d\Delta\phi}{2\pi}. \tag{8}$$

Equation (8) is the basis of the primary experimental method for determining the mode spectrum. For example, a cross array of probes spanning θ and ϕ separately, as shown in Fig. 3, allows determination of $C(\Delta\theta, \Delta\phi)$ and hence $|b_{m,n}|^2$. Any $\Delta\theta$, $\Delta\phi$ combination can be obtained from measured points on the cross array.

In the case where the magnetic field is stochastic, field lines execute a random walk. The average radial displacement squared, in traveling a distance l along the mean field, is given by

$$\langle(\Delta r)^2\rangle = \int_0^l dl_1 \int_0^l dl_2 \,\langle\frac{dr}{dl_1}\frac{dr}{dl_2}\rangle. \tag{9}$$

But

$$\frac{dr}{dl} = \frac{B_r}{B} \tag{10}$$

so that

$$\langle(\Delta r)^2\rangle = \int_0^l dl_1 \int_0^l dl_2 \,\frac{1}{B^2}\langle B_r(1)B_r(2)\rangle. \tag{11}$$

The average in the integrand of Eq. (11) is just the spatial autocorrelation function $C(\theta_1 - \theta_2, \phi_1 - \phi_2)$ times $\langle B_r^2\rangle$, which is dependent only on the difference of the θ's and ϕ's. Thus, we write

$$\langle(\Delta r)^2\rangle = \frac{\langle B_r^2\rangle}{B^2}\int_0^l dl_2 \int_{-l_2}^{l-l_2} d(\Delta l)C(\Delta\theta, \Delta\phi), \tag{12}$$

where $\Delta l \equiv l_1 - l_2$. Along an unperturbed field line,

$$\begin{aligned} \Delta\theta &= \frac{B_\theta}{B}\frac{\Delta l}{r} \\ \Delta\phi &= \frac{B_\phi}{B}\frac{\Delta l}{R}. \end{aligned} \tag{13}$$

Substituting Eq. (7) for C into Eq. (12), we obtain, for $l \to \infty$,

$$\langle(\Delta r)^2\rangle = \frac{l}{B}\frac{2\pi R}{B|B_\phi|}\sum_{m,n}|b_{m,n}|^2\delta(n+m/q), \tag{14}$$

where the safety factor q is given by $q = rB_\phi/(RB_\theta)$.

The magnetic field diffusivity D_m is defined by the relation

$$\langle(\Delta r)^2\rangle = 2D_m l. \tag{15}$$

The factor of 2 results form the primary definition of D_m as the constant in the Folker-Plank equation (for simplicity in slab geometry)

$$\frac{\partial f_m}{\partial l} = D_m\frac{\partial^2 f_m}{\partial x^2}, \tag{16}$$

where f_m is the field line distribution function.[7,8] A solution of Eq. (15) is

$$f_m(x-x_i,l) = \frac{1}{\sqrt{4\pi D_m l}}\exp\left[-\frac{(x-x_i)^2}{4D_m l}\right], \tag{17}$$

where x_i is the initial location of the field line. This gives

$$\langle(\Delta x)^2\rangle = \int(\Delta x)^2 f_m\, d(\Delta x) = 2D_m l, \tag{18}$$

where $\Delta x \equiv x - x_i$. Equation (18) has the factor of 2 as in Eq. (15).

Comparing Eqs. (14) and (15) we see that

$$D_m = \frac{\pi R}{B|B_\phi|}\sum_{m,n}|b_{m,n}|^2\delta(n+m/q). \tag{19}$$

For $m = 0$, the only contribution comes when $n + m/q = n = 0$. But $b_{0,0} = 0$ from Eq. (3) so, using Eq. (2),

$$D_m = \frac{2\pi R}{B|B_\phi|}\sum_{\substack{n\\ m>0}}|b_{m,n}|^2\delta(n+m/q), \tag{20}$$

that is, D_m can be written as a sum over positive m values.

The along-field autocorrelation length is defined by the relation

$$D_m \equiv \frac{\langle B_r^2\rangle}{B^2}l_c. \tag{21}$$

In terms of the spatial autocorrelation function,

$$l_c = \frac{1}{2}\int_{-\infty}^{\infty} d(\Delta l)\, C(\Delta\theta,\Delta\phi). \tag{22}$$

Reference 2 used the following analytic form to fit the mode spectrum measured on ZT-40(M):

$$|b_{m,n}|^2 = \langle (B_r)^2 \rangle \frac{\sigma}{\sqrt{2\pi}} \{ \epsilon \delta_{m,0} \exp(-\frac{n^2\sigma^2}{2}) + (1-\epsilon)\delta_{m,1} \exp[-\frac{(n+N)^2\sigma^2}{2}] \}, \tag{23}$$

with $\sigma = 0.1$, $\epsilon = 0.1$, and $N = -15$. The case $m = n = 0$ is special as already discussed and $b_{0,0} = 0$. Equation (2) implies the symmetry operation $m \to -m$, $n \to -n$, so the $m = 1$ term should actually be separated into two terms, one with $m \to -m$ and $n \to -n$, each term with a factor of 1/2.

The spatial autocorrelation function is obtained from the mode spectrum using Eq. (7). We will approximate the summation over discrete n values as an integral over continuous values. Using Eq. (23) in Eq. (7) and doing the integral over n we obtain

$$C(\Delta\theta, \Delta\phi) = \epsilon \{ \exp[-\frac{(\Delta\phi)^2}{2\sigma^2}] - \sqrt{2\pi}\sigma \} + (1-\epsilon)\cos(\Delta\theta + N\Delta\phi)\exp[-\frac{(\Delta\phi)^2}{2\sigma^2}]. \tag{24}$$

The term multiplied by ϵ is Eq. (24) results from the $m = 0$ term in Eq. (23). Note that because $b_{0,0} = 0$, this term gives 0 when integrated over $\Delta\phi$. Thus it does not contribute to D_m or l_c. Using Eqs. (22) and (24), we obtain for l_c,

$$l_c = R\frac{B}{|B_\phi|}\sigma\sqrt{2\pi}(1-\epsilon)\exp[-\frac{\sigma^2}{2}(N - 1/q)^2]. \tag{25}$$

At the reversal surface, where $1/q \to \infty$, l_c vanishes, counterintuitive to the motion that $m = 1$ leads to $l_c \approx r_0$. The transistion from magnetic surfaces to stochasticity is thought to occur when the Chirikov parameter S exceeds 1, where S is the ratio of virtual island width to spacing between islands. Theoretical studies[9] have mostly been of the case of two islands, so the Reversed Field Pinch case with many islands is more uncertain, nevertheless, $S \geq 1$ throughout most of the plasma would seem to reasonably indicate stochasticity. In terms of the safety factor $q(r)$, the virtual island width is given by

$$w = 4\sqrt{|\frac{b_{m,n}Rq}{B_\phi n(dq/dr)}|}, \tag{26}$$

where $b_{m,n}$ combines positive and negative m as in Eq. (23). The distance between $m = 1$ islands is given by

$$\Delta r = |\frac{q}{n(dq/dr)}|. \tag{27}$$

Thus,

$$S = 4|\frac{b_{1,n}}{B_\phi}R\frac{n(dq/dr)}{q}|^{1/2}. \tag{28}$$

Near the $q = 0$ surface (the reversal surface), $m = 0$ islands with different n values interact with each other and with $m = 1$ islands. But $m = 0$ is a special case, already indicated by the fact that $m = 0$ does not contribute to D_m. The situation near the reversal surface is far from clear.

An important quantitative measure of chaos is the Kolmogorov-Lyapunov scale length, which is the length over which a tube of flux changes its shape, as shown in Fig. 4. Reference 10 gives a formula for L_K, derived using a slab approximation. After some manipulation, the result of Ref. 10 can be stated as follows:

$$\frac{L_K}{r_0} = 2.75(\frac{L_s}{r_0})^{2/3}(\frac{D_m}{r_0})^{-1/3}, \tag{29}$$

where L_s is the magnetic shear scale length. Reference 11 suggests the following formula for L_s in cylindrical geometry,

$$L_s = \frac{Bq}{B_\theta(dq/dr)}. \tag{30}$$

For the case where there are only $m = 1$ and $m = 0$ fluctuations, as in the analytic fit given by Eq. (23), a one-to-one relationship exists between the $m = 1$ mode spectrum absolute amplitude and the quantities D_m, S, and L_K. That is, one quantity is determined by any of the others.

III. QUASI-STATIC DRIFT MODEL

In this section a mathematical model, the quasi-static drift model, is introduced as a framework for analysis of the five-fingered probe technique. The quasi-static drift model is defined as follows. Consider a fluctuating quantity $Y(\theta, \phi, t)$. In the quasi-static drift model Y is assumed to be of the form

$$Y = \sum_{m,n} y_{m,n}(t) \exp\{i[(m\theta + n\phi) + \omega_{m,n}(t)t]\}, \tag{31}$$

where the time variation of $y_{m,n}(t)$ and $\omega_{m,n}(t)$ is small. Neglecting this time dependence, the fluctuations consist of static spatial modes, with mode numbers m and n, drifting with constant velocities $v_\theta = -r_0\omega_{m,n}/m$ and $v_\phi = -R_0\omega_{m,n}/n$. Because Y in Eq. (31) is real, we have

$$y^*_{-m,-n} = y_{m,n}$$
$$\omega_{-m,-n} = -\omega_{m,n},$$

and Eq. (31) can be written as

$$\begin{aligned} Y = &\sum_{\substack{m>0 \\ n}} 2|y_{m,n}| \cos(m\theta + n\phi + \omega_{m,n}t + \alpha_{m,n}) \\ &+ \sum_{n>0} 2|y_{0,n}| \cos(n\phi + \omega_{0,n}t + \alpha_{0,n}), \end{aligned} \tag{32}$$

where

$$y_{m,n} = |y_{m,n}| \exp(i\alpha_{m,n}).$$

Equation (32) shows that the fluctuations are drifting waves with amplitudes $y_{m,n}$ and phase angles $\alpha_{m,n}$.

One motivation for this model is the physical picture of rotating magnetic islands, where the source of the magnetic perturbation is the magnetic island structure. In the Reversed Field Pinch, the source of the low frequency magnetic fluctuations, while probably not islands, could be some concentration of current on the rational surface resonant for a particular mode, having integrity over enough time to be describable as a drift-like motion.

If we define the Fourier transform Y_ω as

$$Y_\omega = \int_{-T/2}^{T/2} Y(t) e^{-i\omega t}\, dt, \tag{33}$$

then, from Eq. (31),

$$Y_\omega = \sum_{m,n} y_{m,n} \exp[i(m\theta + n\phi)] 2\pi \delta_T(\omega_{m,n} - \omega), \tag{34}$$

where

$$\delta_T(x) \equiv \frac{\sin(xT/2)}{\pi x}. \tag{35}$$

The power spectrum $|Y_\omega|^2$ is given by

$$|Y_\omega|^2 = \sum_{\substack{m,n\\ m',n'}} y_{m,n} y^*_{m',n'} 2\pi\delta_T(\omega_{m,n} - \omega) 2\pi\delta_T(\omega_{m',n'} - \omega) \exp[i(m-m')\theta + (n-n')\phi]. \tag{36}$$

We now define the wide spacing approximation as

$$(\omega_{m,n} - \omega_{m',n'})T \geq 2\pi. \tag{37}$$

When the wide spacing approximation holds, the product of the δ_T's in Eq. (36) is as shown in Fig. 5, and it is a good approximation to make the replacement $\delta_T \to \lim_{T\to\infty} \delta_T = \delta$, where δ is the Dirac delta function. Using this approximation,

$$\delta_T(\omega_{m,n} - \omega)\delta_T(\omega_{m',n'} - \omega) \to \delta_T(\omega_{m,n} - \omega)\delta_T(\omega_{m',n'} - \omega_{m,n}). \tag{38}$$

The wide spacing approximation implies that $\delta_T(\omega_{m',n'} - \omega_{m,n}) \to \delta_{m,m'}\delta_{n,n'} T/(2\pi)$, where $\delta_{m,m'}$ is the discrete delta function, from Eq. (35).

Equation (36) will now be averaged a frequency interval $\Delta\omega$ large compared to the minimum nonzero spacing between mode frequencies $\omega_{m,n} - \omega_{m',n'}$. In the average over frequency,

$$\int \frac{d\omega}{2\pi}\, \delta_T(\omega_{m,n} - \omega) \to \left|\frac{\partial n_{\omega,m}}{\partial\omega}\right| \Delta\omega, \tag{39}$$

since, referring to Fig. 5, the integral in Eq. (39) is the count of the number of δ-function spikes in the interval $\Delta\omega$. With these approximations, the frequency-averaged power spectrum is given by

$$\int |Y_\omega|^2 \frac{d\omega}{2\pi} = T \sum_m |y_{m,n_{\omega,m}}|^2 \left|\frac{\partial n_{\omega,m}}{\partial\omega}\right| \Delta\omega. \tag{40}$$

Thus we see that the power in frequency interval $\Delta\omega$ is related to the spatial mode spectrum in mode number interval $|\partial n/\partial\omega|\Delta\omega$.

Using similar arguments we can demonstrate the following:

$$\overline{Y^2} = \int Y^2 \frac{dt}{T} = \sum_{m,n} |y_{m,n}|^2 = \langle Y^2 \rangle. \tag{41}$$

Also,

$$\overline{Y(\theta+\Delta\theta, \phi+\Delta\phi)Y(\theta,\phi)} = \sum_{m,n} |y_{m,n}|^2 \exp[i(m\theta + n\phi)]. \tag{42}$$

The relationship used to determine the spatial mode spectrum using fixed probes separated in the θ and ϕ directions is

$$\int Y_\omega(\theta+\Delta\theta, \phi+\Delta\phi) Y_\omega^*(\theta,\phi)\, \frac{d\omega}{2\pi} = T \sum_m |y_{m,n}|^2 \left|\frac{\partial n_{\omega,m}}{\partial\omega}\right| \Delta\omega\, e^{i(m\Delta\theta + n\phi)}, \tag{43}$$

which relates the cross power spectrum of spatially displaced signals to the mode spectrum and m and n of the modes. As in Eq. (39), the cross power spectrum is averaged over a frequency interval $\Delta\omega$, which is small yet contains many of the discrete frequencies $\omega_{m,n}$.

The simplest application of Eq. (43) is for the case when one m dominates. In that case the phase angle of the cross power spectrum varies with spatial separation like $\exp[i(m\Delta\theta + n\Delta\phi)]$ and measurements for one separation in θ and one separation in ϕ (four probes in all) suffice to determine m and n and the spatial mode spectrum. The mode spectrum is given by

$$|y_{m,n}|^2 = \frac{\int Y_\omega Y_\omega^* \frac{d\omega}{2\pi}}{T\left|\frac{\partial n_{\omega,m}}{\partial\omega}\right|\Delta\omega}, \tag{44}$$

with m and n determined by the phase angles for the separated measurements.

In actuality, the magnetic fluctuations in an Reversed Field Pinch consist mostly of $m = 1$, 0, and -1. Using probes separated across a minor diameter ($\Delta\theta = \pi$), one directly measures the relative amounts of $m = 1$ and $m = 0$ ($m = 0$ is found to be about 10% of $m = 0$). However the four-probe technique, using the dominate mode approximation, overestimates $m = 0$, as shown schematically in Fig. 6. This is because $m = 1$ and $m = -1$ together lead to a spuriously large inferred $m = 0$. With the five-probe method, the fifth probe is used to unambiguously determine the $m = 0$ component. The n-spectrum of $m = 0$ is assumed to be similar to that of $m = 1$.

Unlike the case for electrostatic fluctuations,[1] the drift velocities for magnetic fluctuations observed on ZT-40(M) are not constant as a function of frequency (or mode number). Data are shown in Fig. 7 for three cases, normal bank polarities, B_ϕ reversed, and B_θ and B_ϕ reversed. If $\alpha_\omega(\Delta\theta)$ and $\alpha_\omega(\Delta\phi)$ are the measured phase angles for the cross power spectrum at separations $\Delta\theta$ and $\Delta\phi$, then

$$\begin{aligned} v_\theta &= -\frac{\omega}{\alpha_\omega(\Delta\theta)} r_0 \Delta\theta \\ v_\phi &= -\frac{\omega}{\alpha_\omega(\Delta\phi)} R\Delta\phi. \end{aligned} \tag{45}$$

The reversals of B_θ and B_ϕ show that the drift velocity is of the form $\mathbf{v} \sim \hat{\mathbf{r}} \times \mathbf{B}$ (as for the diamagnetic drift). The coefficient, however, changes sign with frequency. Typical drift velocities are in the range $v_\theta, v_\phi \approx 2 - 5 \times 10^6$ cm/s.

The wide spacing approximation, Eq. (7), is most difficult to satisfy for modes having n numbers differing by 1. In this case, writing $\omega = -nv_\phi/R$,

$$\frac{\Delta\omega T}{2\pi} = \frac{v_\phi T}{2\pi R} \approx 2.4 > 1,$$

for $v_\phi = 3.5 \times 10^6$ cm/s, $T = 0.5$ ms, and $R = 114$ cm [as in ZT-40(M)], meaning that the wide spacing approximation is indeed valid for this case.

IV. EXPRAPOLATION OF $\tilde{B}_r$ INTO PLASMA

Using the present experimental technique, with stationary probes, the magnetic fluctuation measurements can be done without damaging the probes only in the edge region for limited duration, low-current discharges. The measurements clearly show changes of fluctuation level (and even steady vertical field level) as the probes are inserted into the plasma. This is attributed to the expected variation of $\tilde{B}_r(r)$ with r, which is particularly important near the edge, since B_r vanishes at the effective conducting wall (see Fig. 8). In the actual experiment, what plays the role of the conducting wall is a function of frequency. The liner bellows provides a conducting wall boundary condition for high frequency fluctuations, but for low frequencies, magnetic fields penetrate the liner and extend out to the thick aluminum shell ($\sim 0.1\, r_0$ outside the bellows in ZT-40M). The quantity $\Delta r = B_r/(\partial B_r/\partial r)$ was calculated using the relation $\nabla \cdot \mathbf{B} = 0$ to obtain $\partial B_r/\partial r$ from $\partial B_\theta/\partial\theta$ and $\partial B_\phi/\partial\phi$ measured with the four-fingered probe. Figure 9 shows Δr versus frequency for two probe insertions: 0 and 1 cm beyond the bellows. The curves are displaced by about 1 cm as expected. The variation of Δr with frequency is characteristic of the outer boundary condition and, to some extent, of the mode spectrum. For a single mode

$$\Delta r = \frac{\Delta r_{sl}}{F(\omega)} + \Delta r_{lp}, \tag{46}$$

where Δr_{sl} is the separation between shell and liner, Δr_{lp} the separation between liner and probe, and

$$F(\omega) = \frac{i\omega\Delta r_{sl} f\delta}{\eta} \frac{m^2 + k^2a^2}{f^2m^2 + k^2a^2} + 1, \tag{47}$$

with δ the liner thickness, η the liner resistivity, $k = n/R$, and f =(length of streched out bellows)/(actual compressed bellows length).[12] Also shown in Fig. 9 is a fit using Eq. (46).

To calculate $S(r)$, $D(r)$, $L_K(r)$, and $l_c(r)$ using formulas given in Sec. II, the fluctuation mode amplitudes b_{mn} must be extrapolated into the plasma. This was done using the quasi-static equation for $b_{mn}(r)$,[13] assuming model equilibrium current profiles consistent with the experimental constraints. Figure 10 shows examples of the radial dependencies obtained for dominant modes. Generally, $b_{mn}(r)$ does not satisfy the two required boundary conditions, at the outer conducting wall and at the origin, so the two solutions obtained by integrating out from the origin and in from the wall join with some discontinuity in derivative. Since B_r is continuous, the two solutions themselves must come together continuously. The point where the two solutions were brought together was the singular surface, if one existed for the particular mode, or at $r_0/2$ if there was no singular surface. The sign of Δ', the discontinuity of the radial derivative of B_r divided by B_r, indicates the mode's resistive stability (assuming ideal stability): $\Delta' > 0$ unstable, $\Delta' < 0$ stable.

To extrapolate B_r into the plasma we need the shape of the eigenfunction. This is not strongly influenced by details of the equilibrium profile, even though the sign of Δ' *is* dependent on details of the equilibrium profile. The radial eigenfunctions calculated assuming two different equilibrium current profiles consistent with the experimental constraints ($\Theta = 1.5$, $F = -.1$) are shown in Fig. 10 for $m = 1$, $n = -15$, and $m = 0$, $n \to 0$. Figure 11 shows a plot of Δ' as a function of n for $m = 1$ modes assuming the two profiles. There is an unstable region from about $n = -10$ to $n = -20$. One profile is actually ideally unstable near $n = -10$. The other profile has an unstable region near $n = 100$. Details of the current profile thus affect the stability results. However, instability in the range $n \simeq -10$ to -20 was a feature that occured for all of the several profiles studied, consistent with the maximum of the measured fluctuation spectrum.

The outer conducting wall boundary location for the $b_{mn}(r)$ calculation was determined using the value of Δr from Fig. 9 for the mean fluctuation frequency of 40 kHz. Reference 2 gives plots of $D_m(r)$, $S(r)$, and $L_K(r)$ obtained from the model mode spectrum given by Eq. (23) exprapolated into the plasma.

Ideally, the mode structure calculation would use a theory that included pressure driven modes and related Δ' of the modes to the measured fluctuation amplitudes. Parameters of the $j_{\|}/B$ and $p(r)$ profiles would be varied to fit calculated to measured fluctuation amplitudes. The simplified calculation done here involves: 1) neglecting pressure-driven modes entirely, 2) assuming a $j_{\|}/B$ profile, 3) neglecting inertial effects (quasi-static approximation), and 4) using a marginal stability approximation, where the modes actually present are assumed to be marginally stable, that is, to have $\Delta' \approx 0$. These approximations seem valid for the largest part of the measured magnetic fluctuations in ZT-40(M), as for example that part fit by Eq. (23). For the edge-resonant, pressure-driven modes that may determine Reversed Field Pinch confinement[14], however, a more accurate measurement technique to determine the small, very high n mode amplitudes, and a better modeling theory are needed.

V. CORRELATIONS BETWEEN MAGNETIC AND OTHER FLUCTUATIONS

Certain correlations are clearly seen in the data. In this section a quasi-static model that explains the largest part of these correlations will be discussed.

The quasi-static model is defined by the following equations:

$$\frac{\partial n}{\partial t} + \nabla\cdot(n\mathbf{v}) = 0, \tag{48}$$

$$\nabla \cdot \mathbf{v} = 0, \tag{49}$$

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = \eta \mathbf{j}, \tag{50}$$

$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} - \nabla\Phi, \tag{51}$$

$$\mathbf{B} = \nabla \times \mathbf{A}, \tag{52}$$

$$\mathbf{j} \times \mathbf{B} = \nabla p, \tag{53}$$

$$\mathbf{j} = \nabla \times \mathbf{B}, \tag{54}$$

Fluctuating quantities will be denoted by a superscript $\tilde{}$ and are spatially resolved into Fourier modes as in Eq. (1). The subscript m, n will be dropped for notational simplicity, as well as not showing explicit dependence on r and t. The radial derivative $\partial/\partial r$ is denoted by $'$. A time derivative is denoted by $\dot{}$.

Reference 13 shows how Eqs. (52) - (54) imply the following:

$$\tilde{B}_r \equiv b, \tag{55}$$

$$\tilde{B}_\theta = \frac{ir}{m^2 + k^2r^2}[\frac{m}{r}(rb)' - k\mu rb], \tag{56}$$

$$\tilde{B}_z = \frac{ir}{m^2 + k^2r^2}[k(rb)' - m\mu b], \tag{58}$$

where $\mu \equiv j_\parallel/B$. Also,

$$\tilde{j}_r = \mu\tilde{B}_r, \tag{57}$$

$$\tilde{j}_\theta = \mu\tilde{B}_\theta + \tilde{\mu}B_\theta, \tag{59}$$

$$\tilde{j}_z = \mu\tilde{B}_z + \tilde{\mu}B_z, \tag{60}$$

with

$$\tilde{\mu} = \frac{irb\mu'}{mB_\theta + krB_z}.$$

A vector potential for the magnetic perturbations given by Eqs.(55) - (57) is[13]

$$\tilde{A}_r = \frac{r}{m^2 + k^2r^2}[\frac{m}{kr}(rb)' + \mu rb], \tag{61}$$

$$\tilde{A}_\theta = \frac{ib}{k}, \tag{62}$$

$$\tilde{A}_z = 0. \tag{63}$$

Substituting Eq. (51) into Ohm's law, Eq. (50), and dotting with $\mathbf{B}$, we get, to first order neglecting resistivity,

$$\tilde{\Phi} = -\frac{i\dot{b}\frac{B_\theta}{k} - \Phi' b}{i(\frac{m}{r}B_\theta + kB_z)}. \tag{64}$$

The radial velocity obtained by crossing Ohm's law, Eq. (50), with $\mathbf{B}$ is given by

$$\tilde{v}_r = -\frac{iB_z}{kB^2}\dot{b} + \frac{i(krB_\theta - mB_z)}{rB^2}\tilde{\Phi}. \tag{65}$$

Using Eqs. (48) and (49), the density fluctuation $\tilde{n}$ is given by

$$\tilde{n} = -\frac{\tilde{v}_r n'}{i(\omega + \mathbf{k} \cdot \mathbf{v})}, \tag{66}$$

assuming time dependence $e^{i\omega t}$ and with

$$k_\theta = \frac{m}{r}$$
$$k_z \equiv k,$$

where $\mathbf{v} = -\Phi' \hat{\mathbf{r}} \times \mathbf{B}/B^2$ is the rotation velocity caused by the radial electric field $E_r = -\Phi'$.

Figures 12 and 13 show some experimental data on correlations from ZT-40(M). Figure 12 shows the correlation in phase angle between $\tilde{B}_r$ and $\tilde{n}$ as measured by Langmuir probe ion saturation current. Figure 13 shows the correlation in phase angle between $\tilde{B}_r$ and $\tilde{j}_\parallel$ of hot electrons as measured by an electrostatic energy analyzer.

Transport of particles results from the correlation

$$\Gamma_r = \langle \tilde{n} \tilde{v}_r \rangle. \tag{67}$$

From Eq. (66), the quasi-static model, which accounts for most of the fluctuations gives zero transport.

Transport of kinetic electrons along stochastic field lines obeys Eq. (67) except that

$$v_r = v_\parallel \frac{B_r}{B},$$

and we have a kinetic electron radial flux

$$\langle \Gamma_r^{kin} \rangle = -\frac{1}{eB} \langle \tilde{B}_r \tilde{j}_\parallel \rangle, \tag{68}$$

where $j_\parallel = -\int e f v_\parallel \, dv_\parallel$ with f the electron distribution function. Equation (68) allows direct experimental measurement of stochastic transport using $\tilde{j}_\parallel$ measured by an electrostatic analyzer. A quasilinear calculation shows that

$$\langle \Gamma_r^{kin} \rangle = \int |v_\parallel| B D_m \frac{\partial}{\partial r} \left(\frac{\langle f \rangle}{B} \right) dv_\parallel,$$

where D_m is given by Eq. (20).

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VI. Summary

The mathematical basis for the experimental measurement of the magnetic fluctuation mode spectrum has been reviewed. A Gaussian fit of the mode spectrum, given in Ref.

2, has been used to calculate quantities of interest and the subleties involving the role of $m = 0$ discussed.

The quasi-static drift model and the wide spacing approximation were introduced as the framework for analysis of the five-probe technique. The drift velocities of magnetic fluctuations are not constant, but depend on mode number, unlike the case for electrostatic fluctuations. This goes along with the nonlocal nature of magnetic fluctuations.

In order to derive quantities of interest like magnetic diffusivity, the edge-measured fluctuations must be extrapolated into the plasma. The quasi-static model is used for this and seems adequate for the current driven modes responsible for most of the magnetic fluctuations.

Finally, the correlations between magnetic and other fluctuations expected from the quasi-static model are derived. Mostly, this is what is actually seen in the data, except that the important transport-related correlations are not present in the model. The transport-related correlations can be directly measured experimentally, but because they are not the leading order effect the measurements must be done carefully.

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