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Detection of Gross Instabilities in a Toroidal Pinch

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NOTICE

DETECTION OF GROSS INSTABILITIES IN A TOROIDAL PINCH

by

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ABSTRACT

Magnetic pickup windings for detecting the mode structure of the gross instabilities in a toroidal pinch are discussed.

I. INTRODUCTION

The primary question concerning the stability of the reversed field pinch is the stability of the gross modes, which move the plasma bodily to the chamber walls. Based on the sharp-boundary model, these modes have $m = 1$ and $k = n/R \cong 1/a$, where R is the major radius of the torus and a is the plasma radius. For R/a fairly small, only a few n values are unstable. It is naturally of interest to determine experimentally the n-value spectrum of $m = 1$ modes to see if, in fact, $n \cong R/a$ instabilities are limiting the plasma lifetime. In this report, pickup windings that could be used for this measurement are described.

II. GEOMETRY OF THE WINDINGS

The plasma is assumed to be surrounded by vacuum, with the outer plasma boundary at $r = a$, and a conducting wall at $r = b$. The major radius of the torus is R, with $R \gg a$. The pickup winding lies on the surface $r = b - \Delta$.

The individual wires making up the winding have a helical structure with $\theta = n'z/(m'R)$.

The induced voltage from all wires connected in series is given by

$$V = \sum_i \pm \int E \cdot d\ell_i \quad , \tag{1}$$

where the $\pm$ shows the relative phasing of the ith wire, and $d\underset{\sim}{\ell} = (\underset{\sim}{e}_z + n'r/(m'R)\underset{\sim}{e}_\theta)\, dz$. The electric field can be represented as $E = \nabla\phi - \partial A/\partial t$. The gradient term does not contribute to the loop voltage. Because the magnetic field perturbation can be assumed to depend sunusoidally on $m\theta - nz/R$, a vector potential for the vacuum magnetic field, $B = \nabla\chi$, is given by

$$A_r = \frac{r}{m^2}\frac{\partial^2\chi}{\partial\theta\partial z}$$

$$A_\theta = 0$$

$$A_z = -\frac{r}{m^2}\frac{\partial^2\chi}{\partial\theta\partial r} \quad . \tag{2}$$

The voltage is therefore

$$V = \sum_i \pm \int \frac{r}{m^2}\frac{\partial^3\chi}{\partial\theta\partial r\partial t}\, dz \tag{3}$$

which can be written as

$$V = \int \frac{1}{m^2}\frac{\partial^2 B_r}{\partial\theta\partial t}\, \rho(u)\, r d\theta\, dz \quad , \tag{4}$$

with

$$\rho(u) = \sum \pm m' \, \delta(u-u_i) \equiv \frac{\partial N}{\partial \theta} \quad , \tag{5}$$

where $u = m'\theta - n'z/R$ and u_i denotes the angular location at $z = 0$ of the ith wire. Integrating Eq. (4) by parts and using the function $N(u)$ defined by Eq. (5), the final expression is obtained.

$$V = \int \frac{\partial B_r}{\partial t} N(u) \, r d\theta \, dz \quad . \tag{6}$$

The functions $\rho(u)$ and $N(u)$ for two cases are illustrated in Fig. 1. For a mode with $B_r \sim \sin(m\theta - nz/R)$, Eq. (6) gives

$$V \sim \int \sin \left[\frac{mu}{m'} - \frac{(m'n-mn')z}{m'R}\right] N(u) \, du \, dz$$

so that if $N(u) \sim \sin(u)$, the winding detects only the $m = m'$, $n = n'$ mode.

III. EQUATION FOR THE DETECTED VOLTAGE

The magnetic field perturbation in the vacuum region can be represented as $B = \nabla\chi$ with $\chi = [CI_m(kr) + DK_m(kr)] \sin(m\theta - kz)$, where $k = n/R$. The coefficients C and D are determined by the boundary conditions $B \cdot n = 0$ at the plasma and conducting wall surfaces,

$$CI_m'(ka) + DK_m'(ka) = -\frac{\xi_r}{ka}(mB_\theta - kaB_z) \quad ,$$

$$CI_m'(kb) + DK_m'(kb) = 0 \quad . \tag{7}$$

Here ξ_r is the $\cos(m\theta - kz)$ fourier component of the plasma surface deformation, and B_θ and B_z are the equilibrium fields at the plasma surface. The induced voltage from Eq. (6) depends on $\partial B_r/\partial t(b-\Delta)$. A first-order expansion in Δ gives

$$\frac{\partial B_r}{\partial t}(b-\Delta) = -k^2 \Delta \left(1 + \frac{m^2}{(kb)^2}\right) [\dot{C} I_m(kb) + \dot{D} K_m(kb)] \sin[m\theta - nz/R] \quad . \tag{8}$$

Defining a quantity of induced voltage as

$$V_0 = \frac{2\pi^2 R [1 + m^2/(kb)^2]}{a [I_m'(kb) K_m'(ka) - I_m'(ka) K_m'(kb)]} \Delta \dot{\xi}_r (mB_\theta - kaB_z) \quad , \tag{9}$$

the final result for the voltage induced in an m′, n′ winding is as follows.

$$V = \sum_{m,n} V_0(m,n) \int \sin\left[\frac{mu}{m'} - \frac{(m'n - mn')z}{m'R}\right] N(u) \frac{du\, dz}{2\pi^2 R m'} \quad . \tag{10}$$

In practical units $\Delta \dot{\xi}_r B_\theta$ is a voltage in volts equal to 10 Δ(cm) $\dot{\xi}_r$(cm/μs) B_θ(kG).

IV. DISCUSSION FOR m′ = 1

For a double wire m′ = 1 winding, N(u) is as shown in Fig 1b. This winding rotated by 90° detects $B_r \sim \cos(\theta - n'z/R)$. From Eq. (10) such a winding would pick up a signal from all modes with m odd and n = mn′, as follows.

$$V = \sum_{m\ \mathrm{odd}} \frac{2}{\pi m} V_0(m, mn') \quad . \tag{11}$$

Note that the pickup from higher m-value modes is reduced by 1/m. However, if the plasma is not unstable to m > 3 modes (which is probably true for the reversed field pinch), the plasma perturbation is uniquely determined. In this case the measured (time-integrated) voltages on the various n′ windings show unambiguously which n values are present and with what mode amplitudes. Figure 2 shows the dependence of the detected voltage on n.

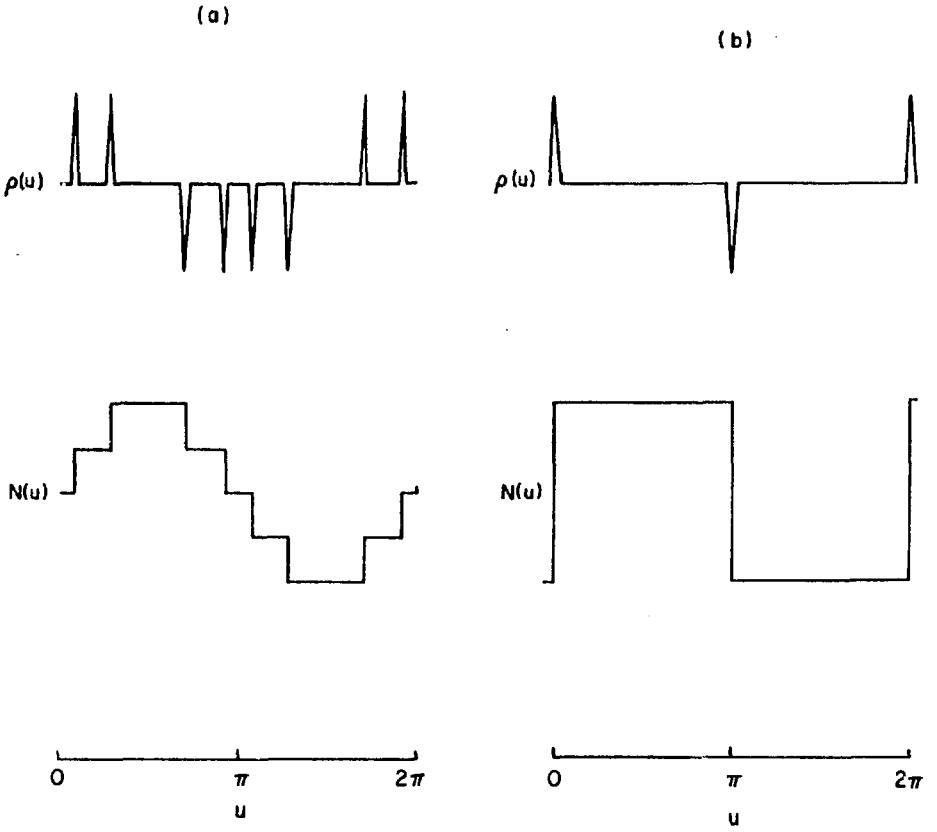

Fig. 1.
The functions ρ(u) and N(u) for two cases: (a) an 8-wire m = 1 winding, and (b) a 2-wire m = 1 winding.

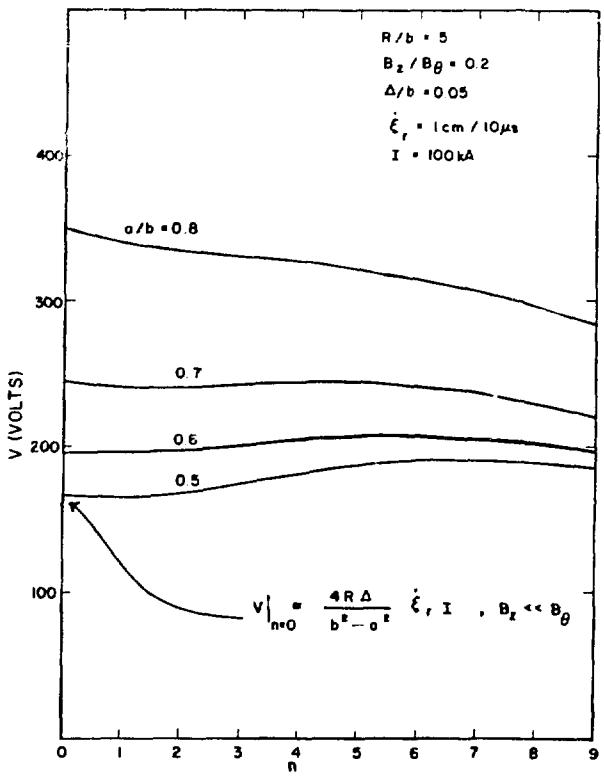

Fig. 2.
Induced voltage vs toroidal mode number n of the plasma m = 1 instability for a double wire m′ = 1 winding.