National Conference on Synchrotron
Radiation Instrumentation, 6/4-6/79

BNL-26199

CONF-790651--1

An Undulator for the 700 MeV VUV-Ring of the National Synchrotron Light Source*

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ABSTRACT

The magnet parameters and the spectrum of the emitted radiation are considered for an undulator to be used in the VUV-Ring of the NSLS.

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* Research supported by the U.S. Department of Energy.

We consider an undulator (also called a coherent wiggler) for use in the 700 MeV VUV-Ring of the National Synchrotron Light Source[1]. The magnet discussed is a transverse wiggler, i.e. in its midplane the magnetic field is in the vertical direction, and to a good approximation

$$B_y = B_o \sin(2\pi z/\lambda_o), \tag{1}$$

where λ_o is the wiggler wavelength. The peak field B_o decreases rapidly as one increases the ratio G/λ_o, where G is the full gap of the undulator. For example in a case with infinite permeability iron one can show that $B_o \sim 1/\sinh(\pi G/\lambda_o)$.

In order to reduce the cost of the magnet and to assure early operation of it in the storage ring, it is desirable that the undulator lie completely outside of the vacuum. This implies that the gap should be large enough to pass a vacuum chamber through it, allowing sufficient vertical aperture to obtain a long electron beam lifetime. Once the gap is fixed, reducing λ_o requires more current to achieve a given field strength B_o. As discussed by Kincaid[2], the peak field B_o and the undulator wavelength λ_o should be chosen such that the parameter,

$$K = \frac{e\lambda_o B_o}{2\pi mc^2} \cong \frac{\lambda_o(\mathrm{cm})B_o(\mathrm{KG})}{10}, \tag{2}$$

is near unity. We suggest that a reasonable choice of dimensions is

$\lambda_o = 5$ cm,

$G = 3$ cm.

Calculations using the two-dimensional magnet program POISSON indicate that $B_o = 4$KG is easily achievable, corresponding to a range of operation

$K \leq 2$.

One could obtain the necessary current density using water cooled copper coils, but the power dissipation is prohibitive in this case, so we consider a superconducting device.

The spectrum of the emitted radiation has been calculated by Alferov, Bashmakov, and Bessonov[3]. In this note we apply their results to the undulator described above with $\lambda_o = 5$ cm, and the number of wiggler periods taken to be

$$N = 50.$$

In the forward direction the spectrum is composed of the odd harmonics

$$\lambda_k = \frac{\lambda_o}{2k\gamma^2}\left(1+\frac{K^2}{2}\right), \quad (k = 1, 3, 5, \ldots) \tag{3}$$

where $\gamma = E/mc^2 = 1370$ for electron energy $E = 700$ MeV. The even harmonics do appear in non-forward directions. In the forward direction, the width of the spectral peaks is

$$\frac{\Delta\lambda}{\lambda_k} \approx \frac{1}{Nk}, \tag{4}$$

however, if one accepts radiation in a cone of half-angle θ about the forward direction, the spectral line is broadened,

$$\frac{\Delta\lambda}{\lambda_k} \approx \frac{\gamma^2\theta^2}{1+\frac{K^2}{2}} . \tag{5}$$

In order to maintain $\Delta\lambda/\lambda_k \approx N^{-1}$, it is necessary to restrict

$$\gamma\theta \leq \frac{1}{\sqrt{N}} . \tag{6}$$

The angular spread of the electron beam should satisfy (6), implying that the amplitude functions β_x and β_y describing the focussing properties of the storage ring must not be too small at the wiggler location. In the VUV-Ring we have:

Emittance	$E_x = 9 \times 10^{-8}$ m-rad	$E_y = 9 \times 10^{-10}$ m-rad
Amplitude Function	$\beta_x = 11.6$ m	$\beta_y = 5.5$ m
Beam Size	$\sigma_x = 1.0$ mm	$\sigma_y = 0.07$ mm
Angular Spread	$\sigma_x' = 0.09$ mrad	$\sigma_y' = 0.01$ mrad

Since $(\gamma/N)^{-1}$ = 0.10 mrad, we see that (6) is satisfied. If desired, the horizontal angular spread can be further reduced by increasing the coupling in the storage ring.

For the k-th harmonic (k odd) the central brightness of the undulator is given by[3]:

$$\text{photons/sec, 1\% bandwidth, } (0.1\text{mrad})^2\text{, Amp} = 4.56 \times 10^6 N^2 \gamma^2 F_k(K), \quad (7)$$

where the function $F_k(K)$ is explicitly defined in the Appendix and is plotted in Fig. 1. We can compare this result with the brightness of a bending magnet source,

$$\text{photons/sec, 1\% bandwidth, } (0.1\text{mrad})^2\text{, Amp} = 3.46\times10^5 \gamma^2 \left(\frac{\omega}{\omega_c}\right)^2 K_{\frac{2}{3}}^2\left(\frac{\omega}{2\omega_c}\right),$$

where $\omega_c = \frac{3}{2}(c/\rho)\gamma^3$ and ρ is the magnet bending radius. The brightness of the undulator exceeds that of the bending magnet by about four orders of magnitude. A wavelength range of 60Å - 400Å can be covered by the device by varying the magnetic field and using the first, third, and fifth harmonics. Of course it is necessary to eliminate the unwanted harmonics using appropriate optical elements for a given experiment. It follows from Eq. (7) that one can obtain on the order of 10^{16} photons/sec, Amp, 1% bandwidth from the undulator within a forward cone of half-angle 0.1 mrad. We note that for the 12 KG bending magnets of the VUV-Ring, it is necessary to accept 50 mrad of arc to obtain 10^{16} photons/sec, Amp, 1% bandwidth.

The energy radiated per unit solid angle, per unit frequency interval is[3] (Gaussian units)

$$\frac{dI(\omega)}{d\Omega} = \frac{e^2\gamma^2N^2}{c} \sum_{k=1}^{\infty} G_k(K,\gamma\theta,\phi)\, H_k(K,\gamma\theta,\omega), \quad (8)$$

where we choose spherical coordinates with the z-axis along the wiggler axis and the y-axis in the vertical direction parallel to the magnetic field. Then $\theta = 0$ corresponds to radiation emitted in the forward direction, and

$\phi = 0$ is the xz-plane of the electron motion.

$$H_k(K,\gamma\theta,\omega) = \frac{\sin^2\left[N\pi\left(\frac{\omega}{\omega_1} - k\right)\right]}{\pi^2 N^2\left(\frac{\omega}{\omega_1} - k\right)^2} , \tag{9}$$

where $\omega_1 = 2\gamma^2\omega_o/\left(1 + \gamma^2\theta^2 + K^2/2\right)$ and $\omega_o = 2\pi c/\lambda_o$. In the limit $N \to \infty$,

$$H_k(K,\gamma\theta,\omega) \to \frac{\omega_1}{N} \quad \delta(\omega - k\omega_1).$$

The function $G_k(K,\gamma\theta,\phi)$ is defined in the Appendix. To convert to mks units make the replacement $e^2/c \to e^2c/10^7$, and to obtain the number of photons/sec multiply (8) by $I/(e\hbar\omega)$, where I is the electron beam current in Amp and $\hbar$ is Planck's constant divided by 2π.

The frequency spectrum[2]

$$I(\omega) = \int d\Omega \frac{dI(\omega)}{d\Omega} \tag{11}$$

in units of photons/sec, Amp, 1% bandwidth is plotted in Fig. 2, for K = 0.4, K = 1, and K = 2, where the delta function approximation of Eq. (10) has been used. Significant peaks in the frequency spectrum remain after integration over all angles, and to appreciate the order of magnitude of the results recall that for a 12 KG bending magnet it is necessary to accept 50 mrad of arc to obtain 10^{16} photons/sec, Amp, 1% bandwidth.

In order to get a better idea of the broadening of the spectral line as one increases the angle θ of acceptance, we plot in Fig. 3, for K = 1,

$$I_\theta(\omega) = \int_0^\theta d\theta' \sin\theta' \int_0^{2\pi} d\phi' \frac{dI(\omega)}{d\Omega'} , \tag{12}$$

in the region of the first harmonic, when $\gamma\theta = 0.1$ and $\gamma\theta = 0.4$, using $H_k(K,\gamma\theta,\omega)$ as given in (9).

References

(1) For a more detailed discussion see, S. Krinsky, BNL25698.

(2) B.M. Kincaid, J. App. Phys. 48, 2684 (1977).

(3) D. F. Alferov, Yu. A. Bashmakov, and E. G. Bessonov, Zh. Tekh. Fiz. 43, 2126(1973), Translation: Sov. Phys. Tech. Phys. 18, 1336 (1974).

Appendix

$$G_k(K,\alpha,\phi) = \frac{4k^2}{A^2}\left\{\left[S_1\alpha\cos\phi - \left(S_1 + \frac{2}{k}S_2\right)\frac{A}{2\alpha\cos\phi}\right]^2 + S_1^2\alpha^2\sin^2\phi\right\},$$

where

$$S_1 = \sum_{n=-\infty}^{\infty} J_n\left(k\xi_z\right) J_{2n+k}\left(k\xi_x\right),$$

$$S_2 = \sum_{n=-\infty}^{\infty} nJ_n\left(k\xi_z\right) J_{2n+k}\left(k\xi_x\right),$$

$$\xi_z = \frac{K^2}{4A}, \quad \xi_x = \frac{2K\alpha\cos\phi}{A}, \quad A = 1 + \alpha^2 + \frac{K^2}{2},$$

and $J_n(u)$ is the Bessel function of order n. The central brightness is

$$\left.\frac{dI(\omega)}{d\Omega}\right|_{\substack{\theta=0\\ \omega=k\omega_1}} = \frac{N^2e^2\gamma^2}{c} F_k(K),$$

where

$$F_k(K) = \frac{K^2k^2}{\left(1+\frac{K^2}{2}\right)^2}\left[J_{\frac{k+1}{2}}\left(\frac{kK^2}{4+2K^2}\right) - J_{\frac{k-1}{2}}\left(\frac{kK^2}{4+2K^2}\right)\right]^2,$$

for k odd, and vanishes for k even.

Figure Captions

1. The function $F_k(K)$ is plotted for k = 1,3, and 5. The wavelengths correspond to $\lambda_k = \frac{\lambda_o}{2k\gamma^2}\left(1 + \frac{K^2}{2}\right)$, with λ_o = 5 cm and γ = 1370. For N = 50, the brightness in photons/sec, $(0.1\text{mrad})^2$, Amp, 1% bandwidth equals $2.1 \times 10^{16}\ F_k(K)$.
2. The integrated spectrum in units of photons/sec, Amp, 1% bandwidth.
3. Spectrum of radiation (for K = 1) in forward cone of half-angle $\theta = 0.1/\gamma$ and $\theta = 0.4/\gamma$ in the wavelength region of the first harmonic.

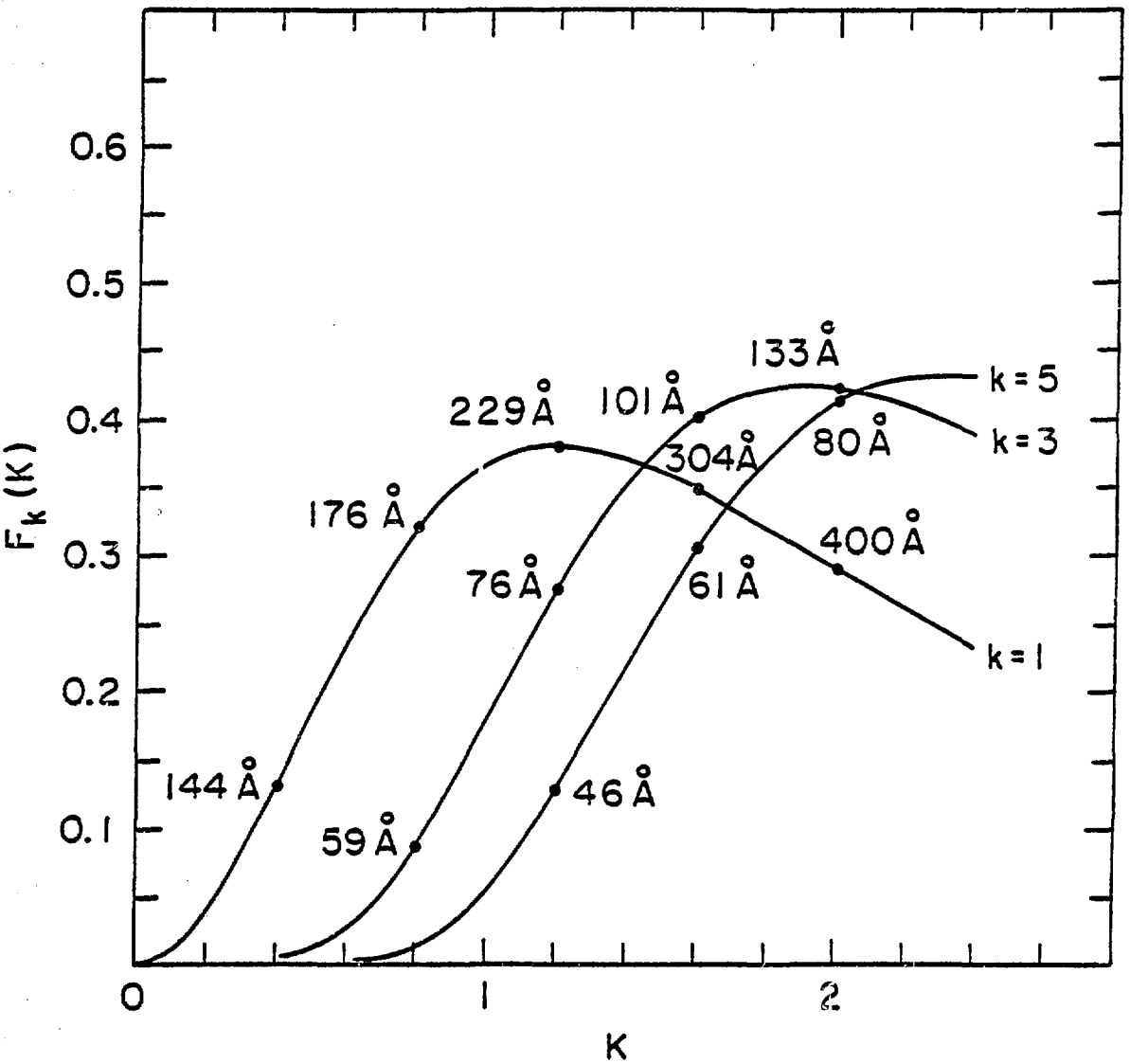
0.6
0.5
0.4
0.3
0.2
0.1
$F_k(K)$
0
1
2
K
k = 5
k = 3
k = 1
133 Å
229 Å
101 Å
80 Å
304 Å
176 Å
400 Å
76 Å
61 Å
144 Å
46 Å
59 Å

Figure 1.

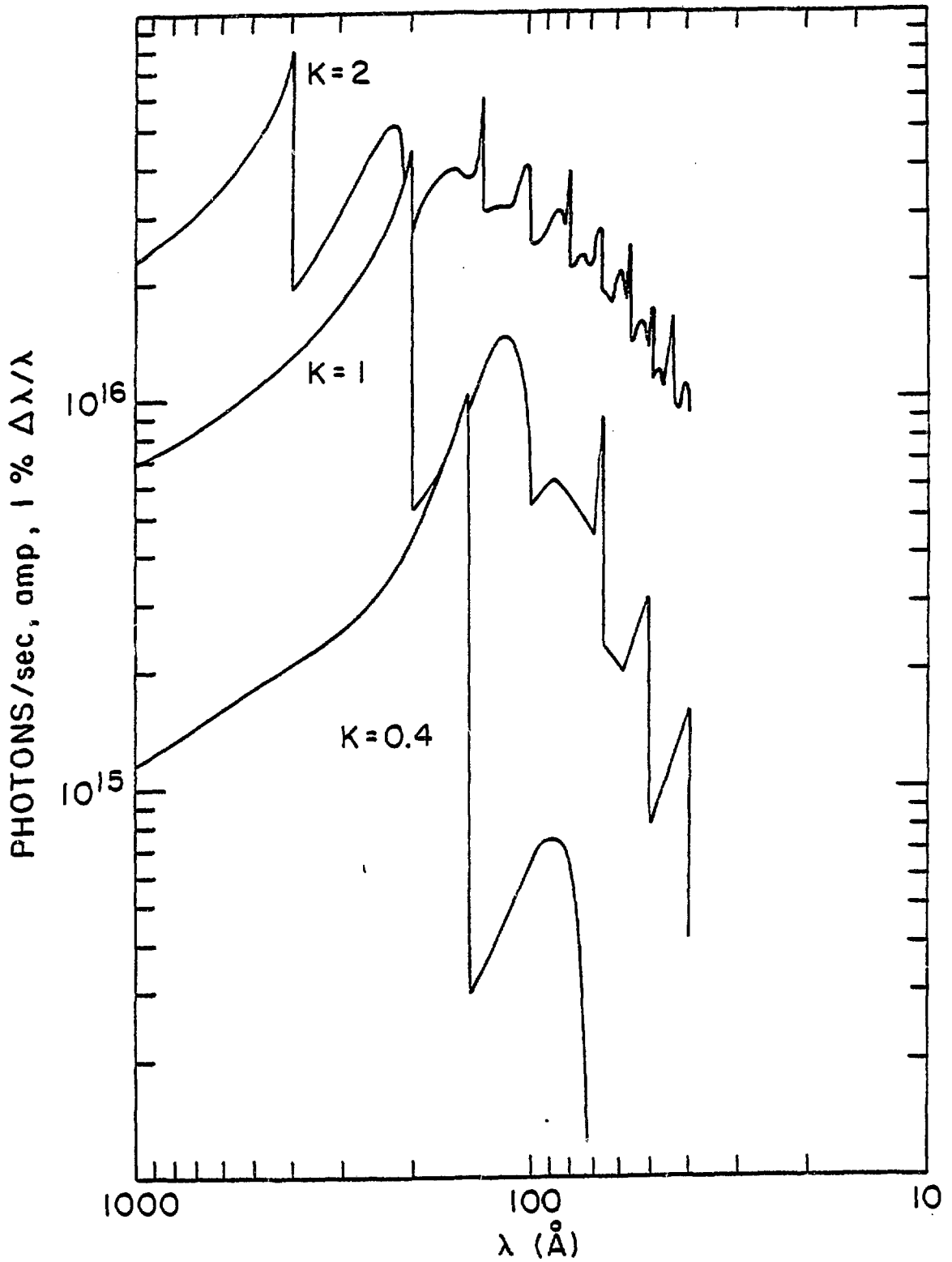

Figure 2.

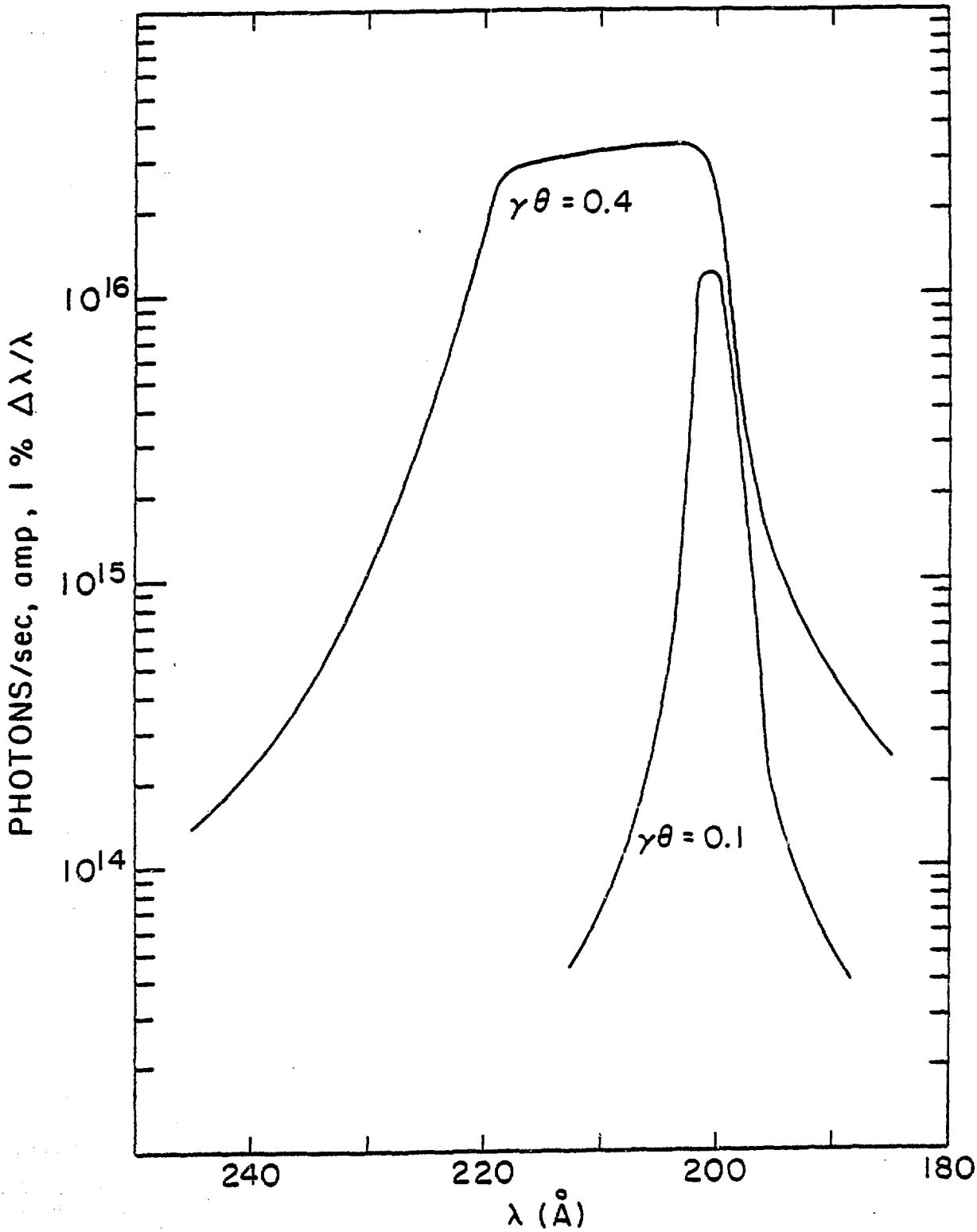
PHOTONS/sec, amp, 1 % Δλ/λ
10^16
10^15
10^14
γθ = 0.4
γθ = 0.1
240
220
200
180
λ (Å)

Figure 3.