CONF-870543--10

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DE89 010417

to be published in

Proceedings of the Second Workshop on Experiments and Detectors for the RHIC
Berkeley, California
May 25-29, 1987

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*Research sponsored by the U.S. Department of Energy under contract DE-AC05-84OR21400 with Martin Marietta Energy Systems, Inc.

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PULSED ELECTROMAGNETIC PRODUCTION OF LEPTONS*

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It seems likely that significant heavy lepton production and emission will occur from the electromagnetic fields which arise in relativistic heavy-ion collisions. This possibility was first suggested several years ago by Gould,[1] based on estimates using the Weizsäcker-Williams method. The technique can be derived from perturbation theory, as discussed by Soff[2] in the context of electron pair production in relativistic collisions of uranium. In such collisions, the near-zone electromagnetic field becomes very large, transverse, and very sharply pulsed.

In our studies, which are nonperturbative, we find large cross sections from this mechanism, much larger than from the Drell-Yan mechanism, which has been suggested as a possible means of diagnosing the transition from hadronic matter to a quark-gluon plasma in such collisions.[3] Positron production in slow, heavy-ion collisions has been the subject of intense experimental and theoretical investigation for over a decade.[4] Preliminary investigations using nonperturbative techniques have extended the theory to relativistic energies[5,6] and necessitate the accurate calculation of all of the particle-antiparticle states which are excited out of the vacuum in the presence of the strong transient fields. For each state, the time-dependent Dirac equation must be solved in three space dimensions, which is very difficult and expensive. In this paper, we address the production of lepton pairs out of the vacuum using these methods and a local field model. The formal details of this are given elsewhere.[7,8] In our current work, we shall apply the model and discuss in detail the production cross sections as a function of the transverse momentum, the invariant mass, and the rapidity.

For simplicity, we consider the symmetric collision of two bare, heavy nuclei of charge Z and mass A at relativistic velocities. We shall work in an inertial frame fixed in one of the nuclei, referred to as the target; in units

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of the nucleon mass, the kinetic energy of the projectile in this frame is γ-1. The collision is assumed to occur at a fixed impact parameter b, and we shall also assume complete hadronic transparency between the two nuclei, even when b is less than the interaction radius of the nuclei. This assumption will be discussed later. The action of the projectile nucleus creates an intense electromagnetic pulse which is treated classically, and which excites pairs out of the vacuum.[5-8]

The local equivalent field model, as employed here, is a variation of a model attributed to Schwinger, in which pairs are excited in a given region of space by the local electric field, supposedly uniformly over the region. This model has an extensive literature,[9] and we find that for arbitrary time dependence, there are nearly integrable solutions. Our results indicate that most of the production occurs near the Compton wavelength of the lepton. Thus, muons are produced partially inside, and tauons are produced entirely inside the nucleus.

The lepton pair production cross section predicted by the Weizsäcker-Williams method is given by

$$\sigma_W = 2\pi \int_{2m_\ell c^2}^{\gamma m_\ell c^2} d\omega \int_{\lambda\!\!\!^-}^{\infty} b\,db\; F(b,\omega) \left(\frac{S(b,\omega)}{b^2}\right) \tag{1}$$

where F is the flux of virtual photons and S is the cross section for photon-induced pair production. The integral requires a cutoff at small values of the impact parameter, usually taken as the Compton wavelength, $b = \lambda\!\!\!^-$. In the high-energy limit, σ_W scales with lepton mass, m_ℓ, with the charges, Z_1, Z_2, and with the bombarding energy per nucleon, γ-1,

$$\sigma_W \sim \left(\frac{Z_1 Z_2}{m_\ell}\right)^2 \ln^3(\gamma) \tag{2}$$

Equation (2) is incorrect in the high-energy limit, since it violates the Froissart limits for cross sections,[10]

$$\sigma < \sigma_0 \ln^2(\gamma)$$

The nonperturbative model has associated with it a dimensionless parameter, κ, which, in simple systems, behaves as the expansion parameter for pair production via time-dependent perturbation theory,[6]

$$\kappa = (\omega/m_\ell)\, E/E_0. \tag{3}$$

In (3) E_0 is the critical field for a lepton of mass m_ℓ,

$$E_0 = m_\ell{}^2/e,$$

and ω is the frequency of the interacting field of strength E. We have evaluated κ for collisions of U+U at the AGS and at RHIC, and we find that $\kappa \gg 1$ for muon production at the AGS and tauon production at RHIC. This suggests that perturbative methods of calculation are probably not applicable.

In the nonperturbative method, the effective Lagrangian coupling leptons of mass m_ℓ to an external classical electromagnetic field, A^μ, is

$$\mathscr{L}(x) = \overline{\psi}(x)\left[\gamma_\mu(i\partial^\mu - A^\mu) - m_\ell\right]\psi(x) \tag{4}$$

Note that this Lagrangian separately conserves electron, muon, and tauon number. For our purposes here, we assume that A^μ is completely specified by the classical motion of the heavy ions. For each species of lepton, a semiclassical action is defined by

$$\mathscr{S} = \int d^4x \; \langle 0(t) \,|\, :\mathscr{L}(x): \;|\, 0(t)\rangle \tag{5}$$

where $|\, 0(t)\rangle$ denotes the many lepton-antilepton state which evolves from the vacuum. The normal ordering is taken with respect to the vacuum. The parameters labeling this state constitute dynamical coordinates which are varied to make the action stationary. This procedure yields a set of time-dependent single-particle equations,[5-8]

$$\left[(\vec{\alpha}\cdot(\vec{P}-\vec{A}) + \beta m + A_0) - i\partial_t\right] |\, \psi_\lambda^{(s)}(t)\rangle = 0 \tag{6}$$

the label s = (+,-) denotes states which evolve from single lepton or single antilepton states, and λ denotes all of the other necessary quantum numbers. The solution to (6) for particular field configurations yields the inclusive number of negatively charged leptons as

$$N_s = \sum_{\lambda,\mu} \left|\, \langle \psi_\lambda^{(+)}(-\infty) \,|\, \psi_\mu^{(-)}(+\infty)\rangle \,\right|^2. \tag{7}$$

The summation is over indices which cover all available positive- and negative-energy single-particle states. The emission of pairs of leptons from the projectile and target nuclei is incoherent, in part due to the classical motion of the heavy ions, and in part due to the intrinsic time delay for the

emission. Thus, we work in a frame at rest in the target nucleus, and only consider the time-varying fields from the projectile. In this frame, the total inclusive singles cross section can be written as

$$\sigma_S = 2\pi \int_0^\infty bdb \; [2 \; N_S(b)], \tag{8}$$

where the states in (7) are restricted to those of the target atom, and where we shall only consider symmetric projectile and target combinations. The above equations are evaluated using a local equivalent field approximation which is discussed in Ref. 8. The results of calculations for colliding beams of U+U, (solid curves), and Kr+Kr, (dashed curves) are shown in Fig. 1 for e^-, μ^-, and τ^- total cross sections. The dot-dashed curve is the e-pair uranium-induced cross section evaluated using the Weizsäcker-Williams formula, (1), which is included for comparision purposes. There are several noteworthy features in Fig. 1. At low energies the e-pair cross section is approximately the same as the Weizsäcker-Williams result. However, at energies per nucleon of about 100 GeV, these differ by about a factor of 100. This difference is principally due to the unphysical $\ln^3(\gamma)$ energy scaling of the perturbative method.

In contrast, the results for muons and tauons are dominated by the finite size of the nucleus, for which reason the Weizsäcker-Williams formula as usually quoted is completely invalid. Note that the mu and tau cross sections increase with energy as $\ln(\gamma)$, and at 100 GeV are, respectively, 100 and 10 mb. The dashed curves display the cross section as a function of bombarding energy for collisions of Kr+Kr and show qualitatively similar features as those of U+U.

In Fig 2 the cross section is shown as a function of the lepton mass at a bombarding energy of 100 GeV per nucleon in U+U. This figure illustrates the effects of a number of assumptions: In (a) both the positive- and negative-energy continuum states of the target are assumed to be plane waves, in effect assuming a coherent field over the entire nucleus. This result is clearly unphysical and is included for illustration. In (b), Coulomb distortion factors are included and averaged over the nuclear volume, in effect assuming that the field is coherent over distances the size of nucleons. In (c), these factors are treated without any averaging, yielding a field which is approximately coherent over distances comparable to the impact parameter. The result (d) is for point nuclear charges with none of the above effects treated, and

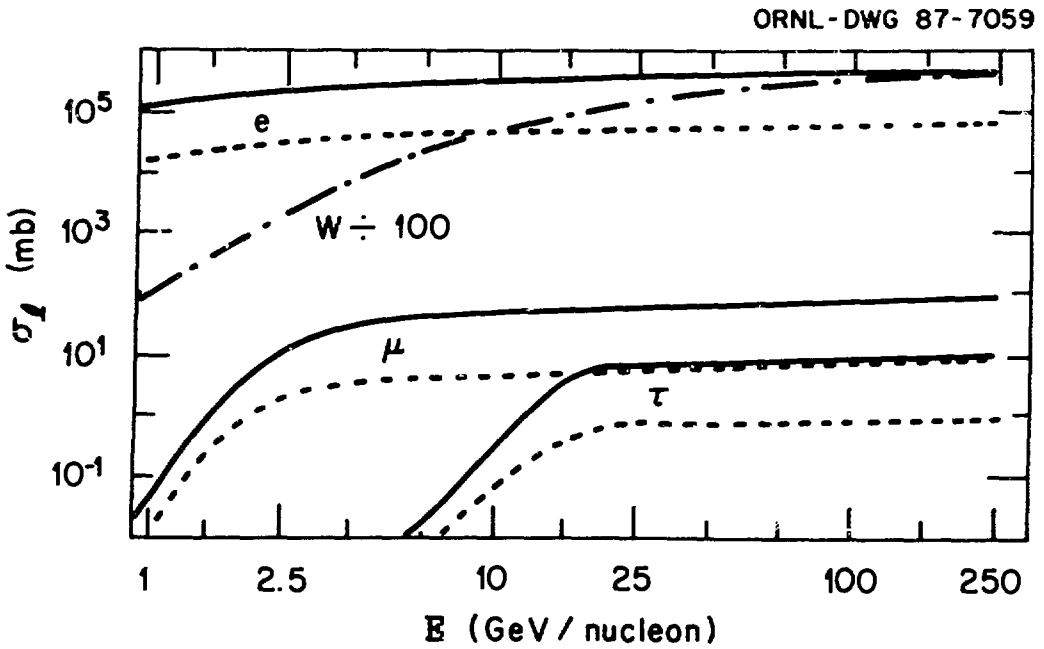

Fig. 1. Total inclusive singles cross sections vs beam energy in a collider computed for electrons, muons, and tauons. Full curves refer to the symmetric collision of uranium, and the dashed curves to the symmetric collisions of krypton. The curve labeled W is the production of electrons from a Weizsäcker-Williams model divided by one hundred.

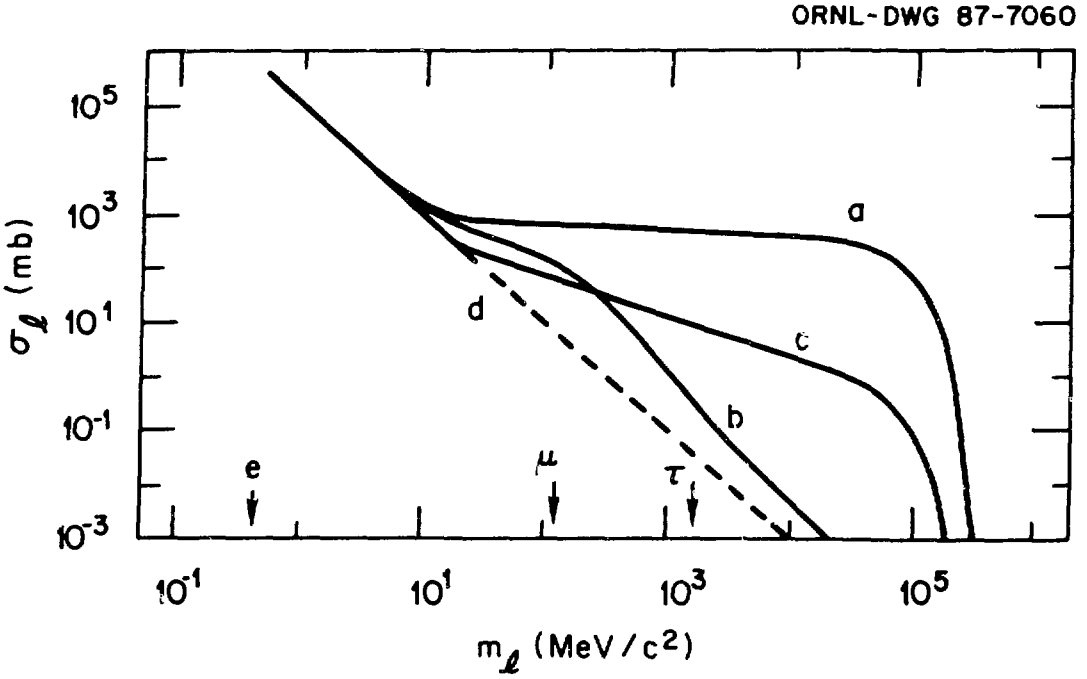

Fig. 2. Total cross section vs lepton mass for collisions of uranium at an energy per nucleon of 100 GeV. A range of assumptions yielding the curves labeled (a)-(d) are described in the text.

this case scales as m_ℓ^{-2}.

The differential cross sections, in terms of transverse momentum and rapidity, can be obtained from (7) and (8) by noting the relations

$$\sigma_s = \int_{-\infty}^{\infty} dY \int_0^{\infty} dP_\perp \frac{d\sigma}{dY dP_\perp} \tag{9}$$

with

$$Y = \frac{1}{2} \ln \left(\frac{P_0 + P_\parallel}{P_0 - P_\parallel} \right) \tag{10}$$

and where $P = (P_0, \vec{P})$ is the four vector associated with the positive-energy continuum state in (6). The vector $\vec{P}$ is decomposed into parts which are transverse, $\vec{P}_\perp$, and longitudinal, $P_\parallel$, to the beam direction. The transverse part is averaged over the azimuthal angle in order to simplify the computations in (9). The resulting e^- differential cross section is shown in Fig. 3 for the fixed target collision of Au+Au at an energy per nucleon of 4.2 GeV. This is an experiment that could be realized at the AGS. In Fig. 3 are given the contours of the inclusive cross section in (9), as a function of the transverse momentum, $P_\perp$, in units of $m_\ell c$, and of the rapidity. The cross section is given in units of $mb/m_\ell c$, and the contours are labeled by their exponent to the base 10. In this collision, the projectile rapidity is approximately 2.4, as indicated in the figure. Note the sharp side peaking at the projectile and target rapidity, reflecting the transverse character of the fields producing the pairs. Also, note that the distribution is a maximium for values of $P_\perp$ near the Compton momentum, $m_\ell c$, and has a broad distribution which decreases by three orders of magnitude as $P_\perp$ increases to about 20 $m_\ell c$.

The total inclusive pair cross section is another invariant which can be easily computed in our model. Since the time-evolved vacuum state is a Slater determinant, the multiplicity at fixed impact parameter for producing a lepton of momentum $\vec{k}$ and an antilepton of momentum $\vec{q}$ is

$$(2\pi)^6 \frac{dN_p}{d^3k\, d^3q} = \sum_{q^0<0} \left| \langle \psi_k^{(+)}(-\infty) \mid \psi_q^{(-)}(+\infty) \rangle \right|^2 \sum_{k^0>0} \left| \langle \psi_k^{(+)}(-\infty) \mid \psi_q^{(-)}(+\infty) \rangle \right|^2$$

$$+ \left| \sum_{q^0<0} \langle \psi_k^{(+)}(-\infty) \mid \psi_q^{(-)}(+\infty) \rangle \langle \psi_q^{(-)}(+\infty) \mid \psi_q^{(-)}(-\infty) \rangle \right|^2$$

where again we emphasize that the states $\psi^{(-)}$ are the negative-energy continuum states, and the states $\psi^{(+)}$ are the positive-energy continuum states,

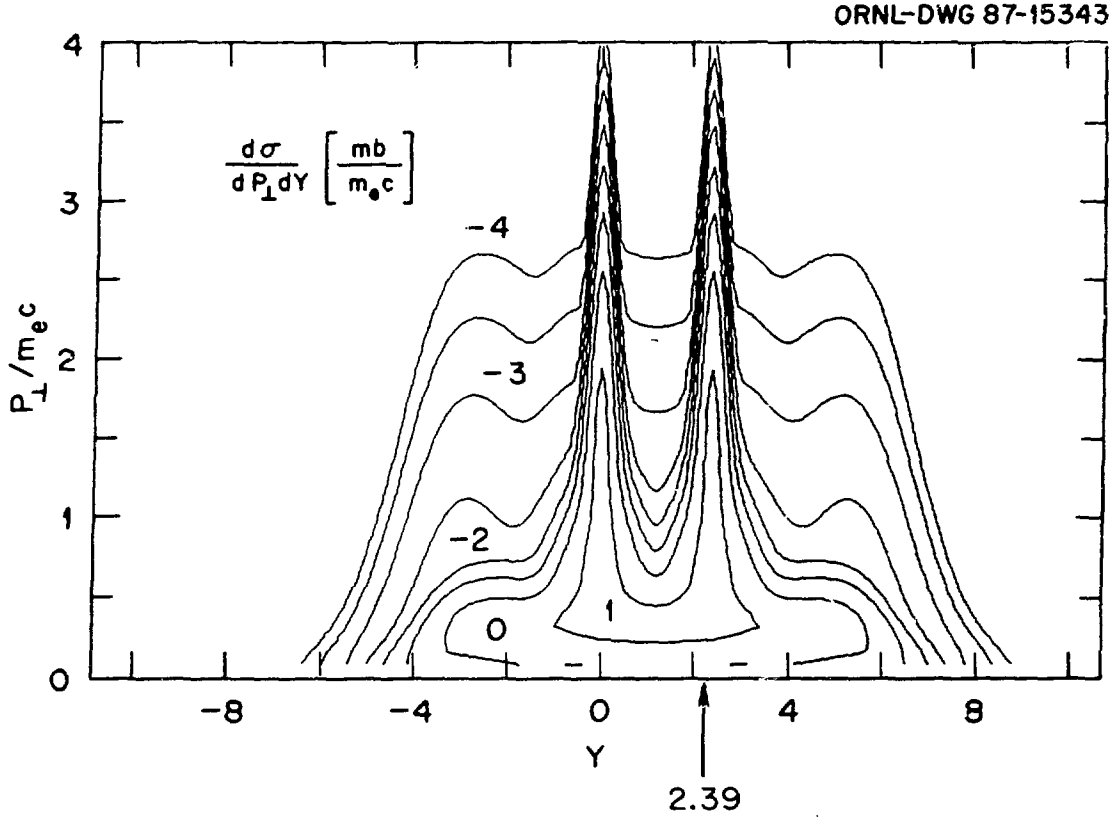

Fig. 3. Contours of the inclusive electron singles production cross section for the fixed target collision of Au+Au at an energy per nucleon of 4.2 GeV as a function of the electron transverse momentum and rapidity. The beam rapidity, 2.39, is indicated by the arrow. Log_{10} of the contour values are included as labels.

and the time labels either the initial states, $t = -\infty$, or the final states, $t = +\infty$. Detailed derivations are given in Refs. 6 and 8. Since $\vec{k}$ and $\vec{q}$ label single-particle momentum of the pair, it is straightforward to transform (11) and obtain the dependency on the total four momentum of the pair,

$$P = (k+q)$$

the invariant mass,

$$M^2 = [P_o{}^2 - \vec{P}^2]$$

and the total rapidity of the pair,

$$Y = \frac{1}{2} \ln \left(\frac{P_o + P_\parallel}{P_o - P_\parallel} \right)$$

Again we note that these results are obtained in Ref. 8 and will not be discussed here. The contours of the e-pair cross section as a function of the total transverse momentum of the pair, and of the total rapidity of the pair are shown in Fig. 4 for the Au+Au collision discussed in Fig. 3. Note that the pair distribution is much broader in comparison to the singles distribution shown in Fig. 3. This is, in part, due to the correlations arising from the second term in (11). The pair cross section as a function of the rapidity and the invariant mass for the same collision is given in Fig. 5. Here we clearly see that most of the cross section comes from invariant masses which can be as large as 2×10^3 lepton masses. This is also shown in Fig. 6, again for the same Au+Au collision, however, as a function of the transverse momentum and the invariant mass. In this collision, the total pair cross section is $\sigma_p \sim 116$ b.

Returning to the singles cross section in Fig. 3, the same features are apparent in Au+Au collisions at 100 GeV, as shown in Fig. 7. However, the transverse momentum and rapidity distribution is very broad in rapidity, reflecting the extreme violence of the collision. Note that the cross section still has a maximum for $P_\perp$ near the Compton momentum and decreases by about three orders of magnitude at about $P_\perp \sim 20\ m_\ell c$. In this particular case, the beam rapidities are ±5.4, and the total singles cross section yield is about 1260 b.

We have also studied these distributions for the production of muons and tauons, reaching the following conclusions:

i) The production of heavy leptons occurs mainly within the interior of

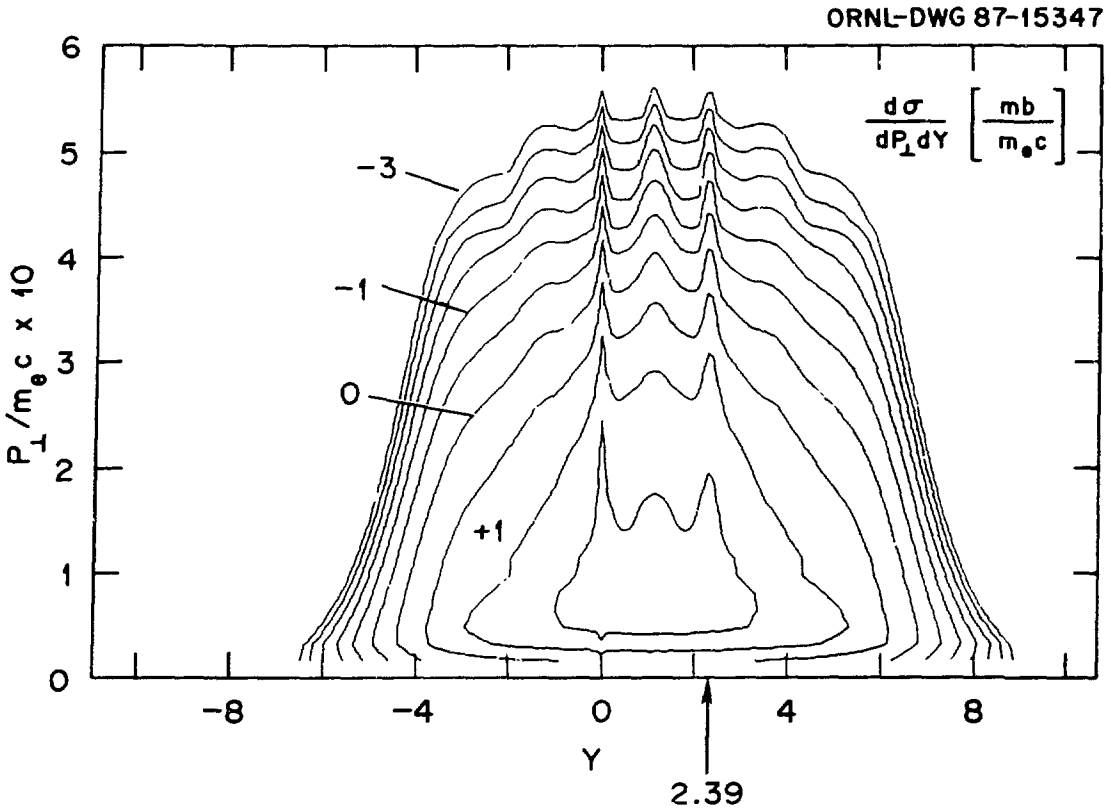

Fig. 4. Contours of the inclusive electron pair production cross section for the collision in Fig. 3 as a function of the total transverse momentum of the pair and the total rapidity. The beam rapidity, 2.39, is indicated by the arrow. Log_{10} of the contour values are included as labels.

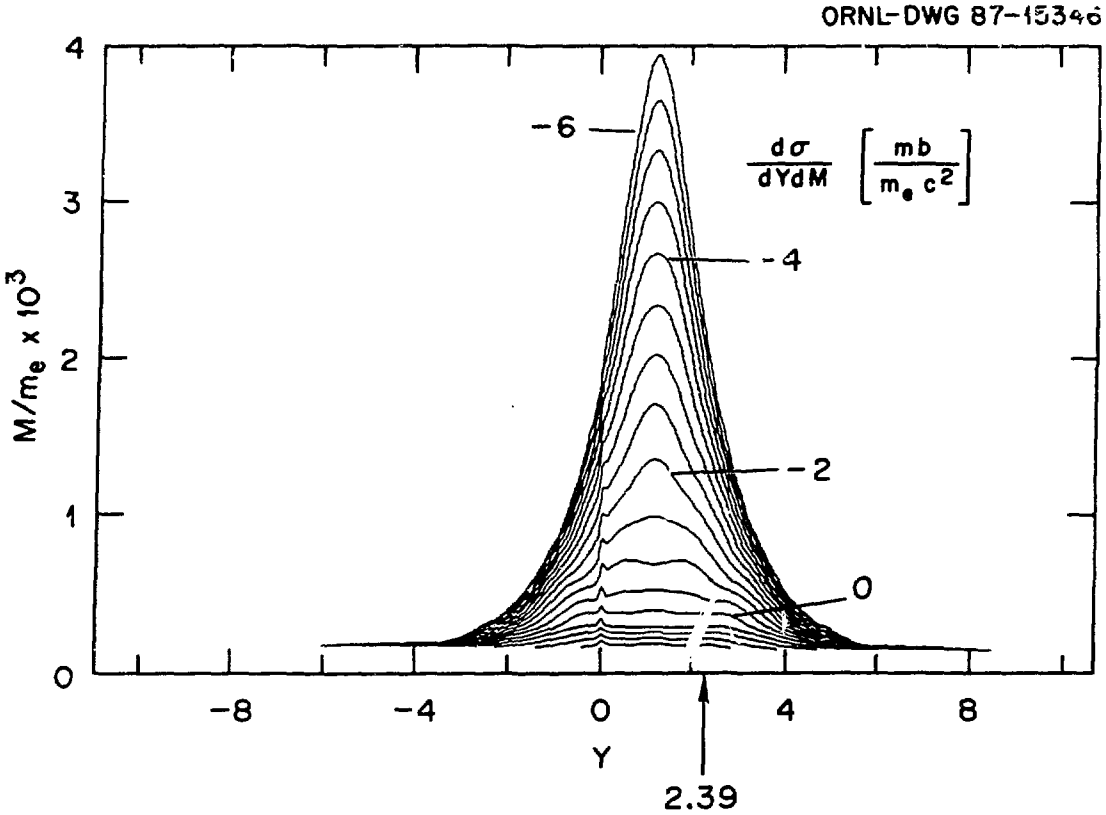

Fig. 5. Contours of the inclusive electron pair production cross section for the collision in Fig. 3 as a function of the invariant mass of the pair and the total rapidity. The beam rapidity, 2.39, is indicated by the arrow. Log_{10} of the contour values are included as labels.

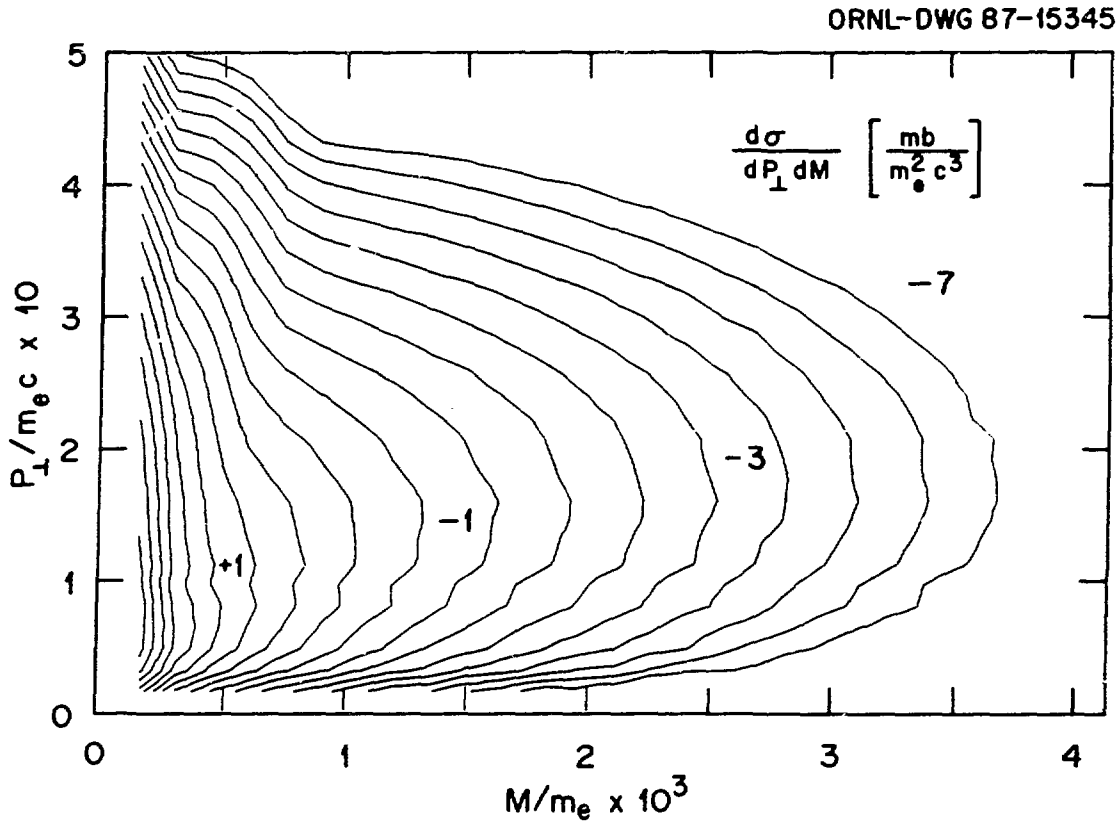

Fig. 6. Contours of the inclusive electron pair production cross section for the collision in Fig. 3 as a function of the total transverse momentum of the pair and the invariant mass. The beam rapidity, 2.39, is indicated by the arrow. Log_{10} of the contour values are included as labels.

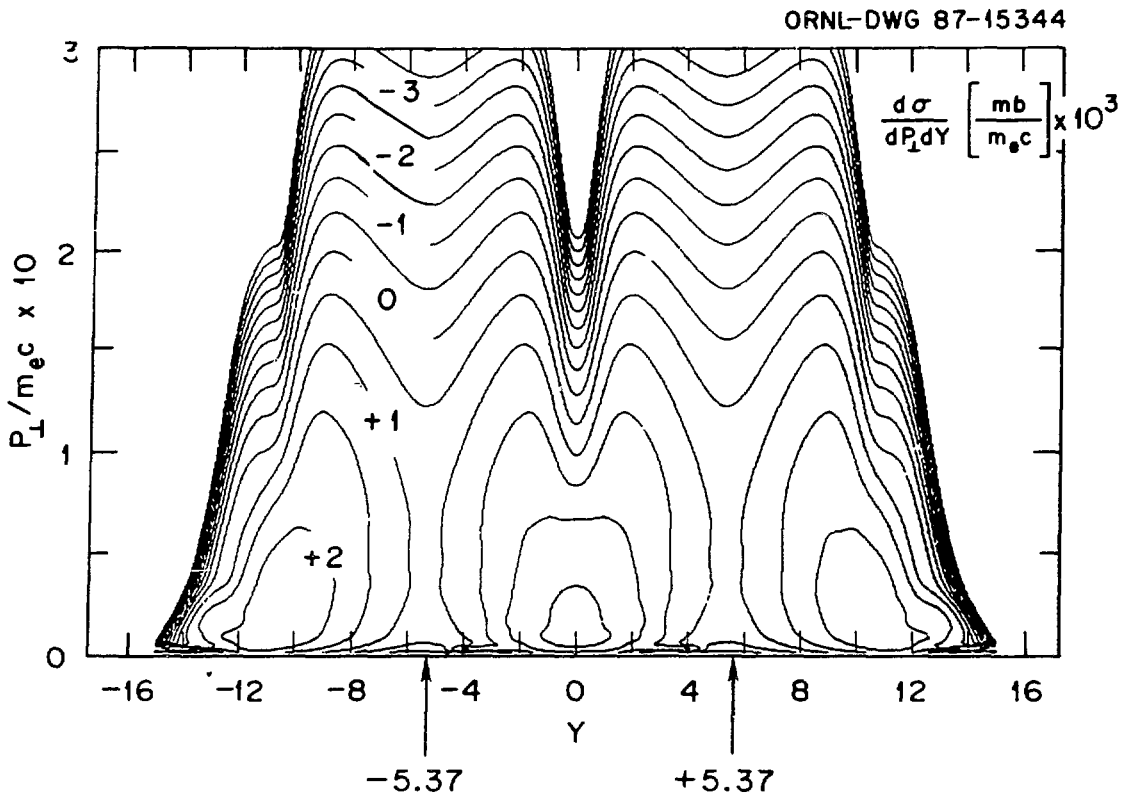

Fig. 7. Contours of the inclusive electron singles production cross section for the collision of Au+Au at a collider energy per nucleon of 100 GeV as a function of the electron transverse momentum and rapidity. The beam rapidities, ±5.37, are indicated by the arrows. Log_{10} of the contour values are included as labels.

the nuclei, and is sensitive to details of the nuclear charge distribution. Because of the relatively small Compton sizes of the mu and tau, it is probably important to give the nuclear charge form factor in terms of the quark distributions in the nucleus.

ii) The cross section yields for muons and tauons are large compared to those predicted by the Weizsäcker-Williams process, due to the coherence developed during the time evolution in the interior of the nucleus. The differential cross sections are strongly peaked at the projectile and target rapidity and have a spread in the transverse momentum variable on the order of the Compton momentum of the produced lepton.

In conclusion, we should like to emphasize that other particles should readily be produced by this mechanism, including the J/ψ and W-pairs, and possibly even magnetic monopoles.[11] There is some evidence that central collisions of heavy ions in this energy range will undergo tremendous deceleration forces. If this is the case, then the production of leptons as we have discussed will be substantially enhanced.

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