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SIMPLE ANALYTIC FORMULA FOR THE STRENGTH OF SPIN DEPOLARIZING RESONANCE*

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ABSTRACT

A simple analytic formula is derived to explain the periodicity of spin depolarizing resonance. The spin depolarizing resonance strengths of CPS and SPS at CERN and the lattices of meson factory at TRIUMF are used to compare with the analytic formula.

INTRODUCTION

Polarized proton is one of the important tools in the study of fundamental interactions. The polarized ion has to be accelerated through many spin depolarizing resonance to reach the energy of our interest. It is important to know the strength of these resonances.

Recently, strengths of the spin depolarizing resonances for most of the accelerators in the world has been calculated and compiled.[1] These calculations reveal many interesting features: (1) The resonances form families. The dominant resonance family are separated by a short and a long period, ΔK_S and ΔK_L, where ΔK_S is somewhat less than twice of the machine tune and ΔK_L equals to the twice of the number of bending cells. (2) At a separation from the dominant family equal to the total number of bending cells, there is another family with smaller resonance strength. (3) Between these dominant resonances, the depolarizing resonance strength are rather small.

In this paper, we shall derive a simple formula to describe these properties. The lattices of CPS, SPS and the lattice of complex γ_T designed for the meson factory at TRIUMF is used to demonstrate the validity of these simple formulas.

Equation of Motion For The Spin and Resonance Strength

The equation of motion for a spin particle in the circular accelerator can be expressed as[1-3]

MASTER

$$\frac{d\vec{S}}{d\theta} = \vec{S} \times \vec{\Omega}_a \qquad (1)$$

*Work performed under the auspices of the U.S. Department of Energy.

where

$$\vec{S} = S_1 \hat{x} + S_2 \hat{s} + S_3 \hat{y} \tag{2}$$

$$\vec{\Omega}_a = - t\hat{x} + r\hat{s} - \kappa\hat{y} \tag{3}$$

Here $\hat{s}$, $\hat{x}$ and $\hat{y}$ are respectively the unit tangent vector, the outward unit vector in the orbit plane perpendicular to s and the unit vector orthogonal to the plane of s and $\hat{x}$. $\kappa = \gamma G$ is the precession frequency of the spin, t and r are depolarizing coupling strength with

$$r = (1+\gamma G)y' - \rho(1+G)\left(\frac{y}{\rho}\right)' \tag{4}$$

$$t = (1+\gamma G)\rho y'' \tag{5}$$

where the prime denotes the derivative with respect to s. The resonance strength for a circular accelerator is then obtained by the Fourier series,

$$\zeta \equiv -t - ir \equiv \Sigma\, \epsilon_m e^{\pm iK_m\theta} .$$

where

$$\begin{aligned} \epsilon_{K_o} &= \frac{1}{2\pi} \oint \frac{\zeta(\theta)}{\rho(s)} e^{iK_o\theta(s)} ds \\ &\simeq - \frac{1}{2\pi} \gamma G \oint y'' e^{iK_o\theta(s)} ds \end{aligned} \tag{6}$$

where the fact that γG and ρ are large numbers in a large accelerator has been used in obtaining the approximated formula in Eq. (6).

Resonance Strength of Large Accelerators

A large accelerator has normally many FODO cells with dipoles then followed with straight sections for insertions (interactions or utilities). Let us assume that the accelerator is composed of M FODO cells in an arc and P super periods. Let the total tune of the machine be ν and the phase advance per cell be μ. Then $\nu_B \equiv MP\mu$ will be the total phase advance across the total FODO bending cells. We

assumed also that all the bending dipoles located at these regular FODO cells. Each straight insertion section has the phase advance of $(\nu-\nu_B)/P$ without any bending dipoles. The spin resonance strength in the leading order can be expressed as

$$\varepsilon_K = -\frac{1}{2\pi}\gamma G\int y''e^{iK\theta}ds$$

$$= -\frac{G\sqrt{\gamma\varepsilon_N/\pi}}{4\pi}\int\frac{\frac{\partial B_y}{\partial x}}{B\rho}\sqrt{\beta_y}(e^{i\psi(\theta)}+e^{-i\psi(\theta)})e^{iK\theta}ds \tag{7}$$

where ε_N is the normalized emittance, $\psi(\theta)$ is the betatron phase of the circular accelerator. Using the thin lense approximations, we obtain

$$\varepsilon_K = \frac{-G}{4\pi}\cdot\sqrt{\frac{\gamma\varepsilon_N}{\pi}}\left\{E_P^+(E_M^+\left(g_F\sqrt{\beta_y(F)}\right.\right.$$

$$\left. + g_D\sqrt{\beta_y(D)}\, e^{i\frac{\nu_B+K_o}{MP}\pi}\right) + X_{INS}^+)$$

$$+ E_P^-(E_M^-\left(g_F\sqrt{\beta_y(F)} + g_D\sqrt{\beta_y(D)}\, e^{+i\frac{K_o-\nu_B}{MP}\pi}\right) + X_{INS}^-) \tag{8}$$

where g_F and g_D are the integrated quadrupole field strengths for focusing and defocusing quadrupoles respectively, $X_{INS}^{\pm}$ are contributions from the insertion region and $E_P^{\pm}$ and $E_M^{\pm}$ are respectively the enhancement factor for the superperiodicity P, and M cells, i.e.

$$E_P^{\pm} = \frac{1-\exp\left(i(K_o\pm\nu)2\pi\right)}{1-\exp\left(i\frac{K_o\pm\nu}{P}\cdot 2\pi\right)} \tag{9}$$

$$E_M^{\pm} = \frac{1-\exp\left(i\frac{K_o\pm\nu_B}{P}\cdot 2\pi\right)}{1-\exp\left(i\frac{K_o\pm\nu_B}{MP}\cdot 2\pi\right)} \tag{10}$$

Some observation can be drawn from Eqs. (9-10):

(1) The enhancement factors $E_P^{\pm} = P$ at $K_o = kP \pm \nu$ and $E_M^{\pm} = M$ at $K_o = kPM \pm \nu_B$, where k is an integer. In a large accelerator, there are many more cells than the superperiodicity. The superperiodicity is also likely to be destroyed by the different physics requirements in different insertions. The enhancement factor $E_P^{\pm}$ is rather unimportant. The dominant resonance position appears at

$$K_o = k\ PM \pm \nu_B \quad (k \text{ integer}) \quad . \tag{11}$$

At this resonance, each cell contributes coherently to give rise to a large resonance strength

(2) Since the field gradients g_F and g_D are opposite in sign ($g_F \simeq -g_D$) and $\sqrt{\beta_y(D)} \simeq 2\sqrt{\beta_y(F)}$, we expect that the interference effect will be important also in the square bracket [] of Eq. (8). When the condition,

$$K_o = (2k+1)\ PM \pm \nu_B \quad k = \text{integer} \tag{11a}$$

$\Delta K_o = 2\ PM = 2 * \text{No. of bending cells}$. Assuming $\sqrt{\beta_y(D)} \simeq 2\sqrt{\beta_y(F)}$, we obtain also

$$\frac{\varepsilon[@((2k+1)PM \pm \nu_B)]}{\varepsilon[@(2kPM \pm \nu_B)]} \simeq 3 \ .$$

A Simple Demonstration

Let us consider a simple demonstrative example. We assumed that an accelerator is composed of 48 FODO cells, where 36 FODO cells has dipoles and the other 12 FODO cells are straight cells without dipoles for the consideration of injection, ejection, R.F. Stations etc. This simple accelerator can be arranged in many different ways. Let us disregard all the considerations on the orbital dynamic and dispersion functions. The tune of the machine, ν, is chosen to be 8.8 for comparison with $\nu_B = 6.6$. Table 1 lists four of many possible organizations of the accelerator.

For machine a, the enhancement factor of Eq. (8) is dominated by M than by P, we expect that the most important resonance appear at

$$\gamma G = MP \pm \nu_B = 36 \pm \nu_B \ .$$

Figure 1 shows that the peak position agrees exactly.

For machine b, the enhancement factor $E_P^{\pm}$ and $E_M^{\pm}$ are both impor-

tant. Because the E_M has larger window than E_P, the dominate resonance will be shifted from $36 \pm \nu_B$ (which is $34 + \nu$ and $38 - \nu$) to $32 + \nu$ and $40 - \nu$ (Eq. is enhanced). (See Fig. 1.)

For machines c and d, we clearly see that $E_P^{\pm}$ enhancement is very important. Reminant of $E_M^{\pm}$ enhancement is still there. This shows up in the fact that the dominant resonance apears at $36 \pm \nu$ and $108 \pm \nu$,

Table 1. Possible Arrangement of FODO Cells

Machines	P	Arrangement in a Superperiod	
		FODO cells with Dipole (M)	FODO cells without dipole
a	2	18	6
b	4	9	3
c	6	6	2
d	12	3	1

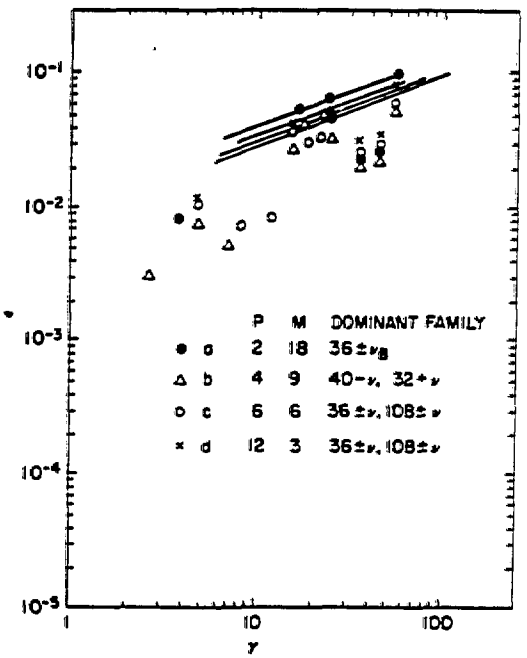

Fig. 1. Demonstration of the dependence of the spin depolarizing resonance strength on the superperiod, P, and FODO cells (M). Table 1 gives a definition of machines a, b, c and d respectively.

where both focusing and defocusing quadrupoles in the bending cells contributes additively.

We observe that the machine with large enhancement to few resonances has also fewer number of dominant resonances. Machines with less concentrated enhancement has more number of equally strong resonance strengths.

SPS and CPS at CERN

Figure 2 shows the spin resonance strength of PS. The PS is composed of 10 superperiod, where each superperiod has 5 cells. This corresponds to P = 10, M = 5 and $\nu = \nu_B$. The most important resonance appears at 50 ± ν, where every quadrupole in each half cell contributes coherently. The secondary resonance appears at 0 + ν, 100 ± ν etc., where every cell contributes coherently, while the focusing quadrupole and defocusing quadrupoles cancel somewhat. This structure is similar to that of AGS, where 60 ± ν is the primary resonance and 0 + ν is the secondary series. The resonance strength of these two machines are in fact comparable. At the PS energies, the snake requires a very large aperture. Therefore viable method of correction schemes will be similar to that ZGS and AGS.

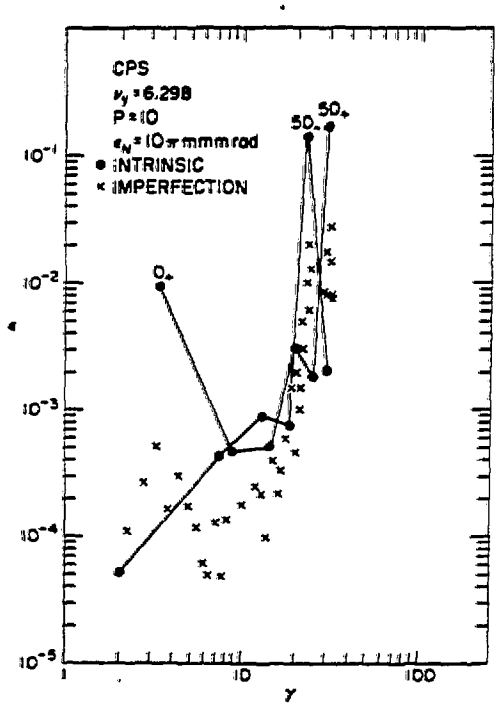

Fig. 2. The spin depolarizing resonance strength for PS.

Figure 3 shows the resonance strength of SPS. The lattice of SPS has 4 almost identical sextant, each composed of 15.5 bending cells. The other two sextant has an insertion of $\beta_H^* = 2$ m, $\beta_V = 1$ m. The number of the bending cells in this two sextant is also 15.5. Following paramenters of Eq. (8) can be used:

$$\nu = 27.7$$

$$\nu_B = 22.5$$

$$\nu - \nu_B \simeq 5$$

$$M = 15.5$$

$$P = 6$$

$$MP = 93$$

Thus the dominant families of resonance strength appears at $93 \pm \nu_B$ (or $88 + \nu$, $98 - \nu$); $3 * 93 \pm \nu_B$ ($274 + \nu$, $284 - \nu$) and $5 * 93 \pm \nu_B$ ($460 + \nu$, $470 - \nu$). The secondary families will be located at ν_B, 2

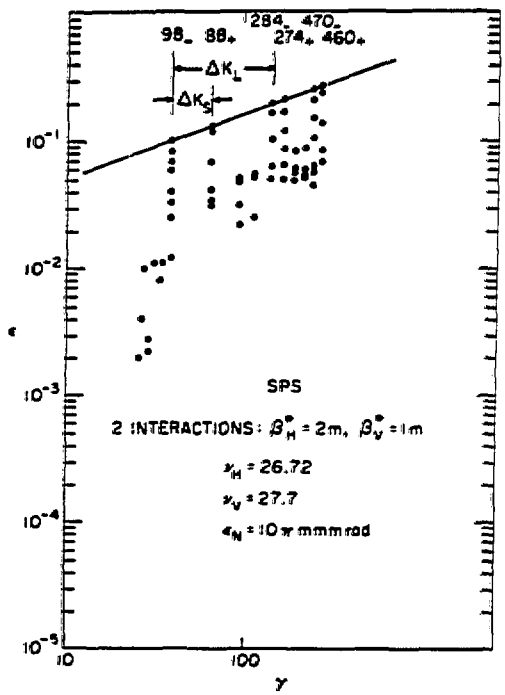

Fig. 3. Spin resonance strength for SPS.

* 93 ± ν_B and 4 * 93 ± ν_E, etc. The ratio of the strengths of the primary family to that of the secondary family will be about 3. These features are clearly seen in Fig. 3.

The resonance strength is about the same as that of RHIC. At this resonance strength, two snakes can cure these resonances very well provided that the snake resonances are avoided.

TRIUMF Meson Factory

The lattice design of meson factory proton synchroton at TRIUMF (TPS) has a feature that the transition energy is pushed into the complex plane. Whence the acceleration of the particle to the top energy does not cross the transition energy, where the microwave instability becomes important due to lack of Landau damping. The lattice of TPS is shown in Fig. 4 for one superperiod. The betatron functions are well-behaved and regular while the dispersion function is however somewhat large.

Figure 5 shows the resonance strengths of the 30 GeV (Fig. 3) proton synchrotron and its 3 GeV booster. Because of the missing dipoles in the first two half FODO cells, the coherent factor, M in

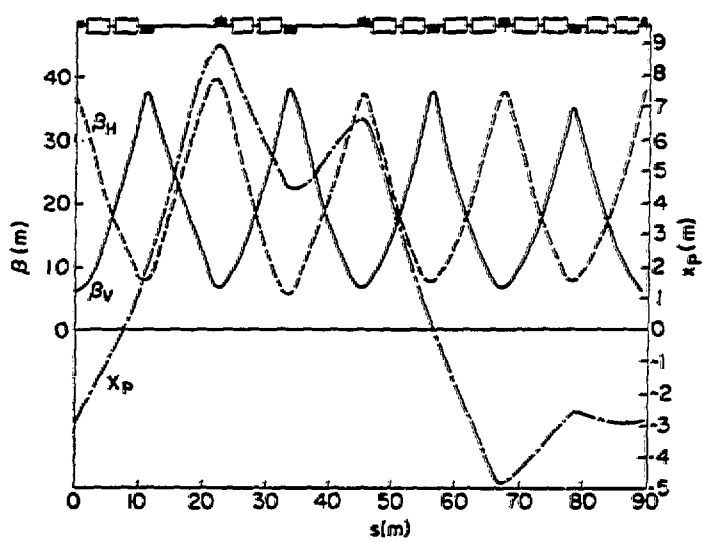

Fig. 4. The betatron functions of the lattice for the meson factory at TRIUMF. Note that the distribution of quadrupoles and dipoles give rise to complex γ_T.

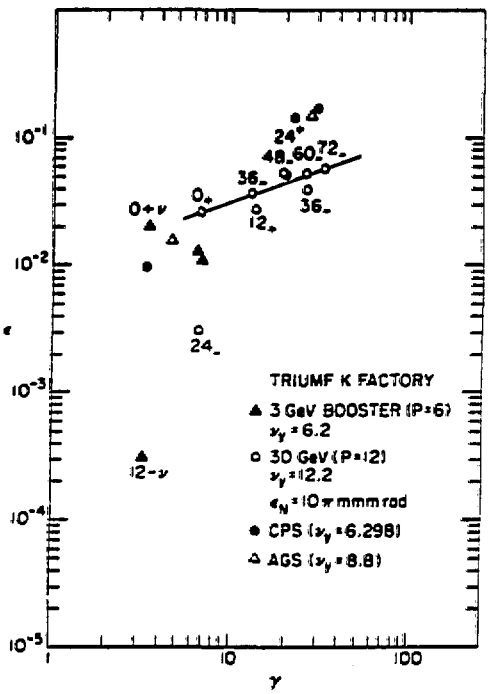

Fig. 5. Spin resonance strength for the meson factory lattice.

Eq. (8) disappears and the resonance strengths are equally strong at all kP ± ν (except $\sqrt{\gamma}$ factor).

The resonance strength of AGS and CPS are also shown for comparison. We observe that the AGS and CPS has fewer important resonance than the lattice of TRIUMF Kaon Factory. But the resonance strength of the kaon factory lattice is slightly smaller. These resonance strengths are already very large if we assumed the same particle acceleration rate as that of AGS (160 keV/turn). Therefore operation of the polarized proton beam in the Kaon Factory lattice may be slightly more difficult.

Discussion

The spin depolarizing resonance strength for the lattices of CPS, SPS and the TRIUMF Kaon Factory is discussed. The behavior of these resonance strength can be understood analytically in terms of some simple formula. This features has also been observed in the previous calculations.

We found that the strength of the maximum resonance strength is characterized by two enhancement factors: the regularly bending FODO cells and the superperiodicity of the machine. Machines, at high energies, such as SSC, Tevatron, SPS, HERA and RHIC, the number of

bending cells in the arc becomes very large, the resonance strength is therefore dominated by the coherent contributions of each bending cells. Numerical calculation agrees very well with the analytic formula.

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