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ULTRARELATIVISTIC ATOMIC COLLISIONS

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Ultrarelativistic Atomic Collisions

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ABSTRACT: Calculations of the coherent production of free pairs and of pair production with electron capture from ultrarelativistic ion-ion collisions are discussed. Theory and experiment are contrasted, with some conjectures on the possibility of new phenomena.

1. INTRODUCTION

The proposed new colliders designed to investigate the collisions of heavy ions at very high velocities have led to extensive theoretical speculation concerning possible new phenomena. In particular, the large and essentially classical electromagnetic fields associated with such collisions mean that the electromagnetic coupling to ordinary matter is greatly enhanced (Bottcher 1988). The production of multiple electron-positron pairs, muon pairs, vector bosons, and possibly the yet-unconfirmed Higgs boson all occur at nearly atomic distance scales (Bottcher 1989, 1990).

The process of electron-positron production with electron capture by one of the ions is the principal beam loss mechanism for highly charged ions in a storage ring and thus plays a central role in the design and the operation of these machines (Rhoades-Brown 1990). At present, there are no measurements of this process, and the various theoretical calculations seem to differ by as much as two orders of magnitude. These calculations encompass many different approaches – two-photon perturbation theory (Rhoades-Brown 1989), coupled-channel expansions (Rumrich 1991), distorted-wave techniques (Becker 1987), numerical lattice calculations (Strayer 1990), as well as use of the Weizsäcker-Williams approach (Bertulani 1988, Nikishov 1982, Rhoades-Brown 1991). Thus there is a pressing need for experiments to aid in resolving this ambiguity.

The production of electron pairs observed in the collisions of cosmic rays with nuclei was studied by Racah (Racah 1937) using the Weizsäcker-Williams, or equivalent photon method (Williams 1934, Weizsäcker 1934, Landau 1934). In more modern studies, this technique has been extensively used to study free pair production in different circumstances: electron pairs (Brodsky 1971, Bertulani 1988, Rau 1990), muon pairs, heavy vector bosons (Grabiak 1989), and possibly the Higgs boson (Papageorgiu 1989, Grabiak 1989, Drees 1989, Cahn 1990, Müller 1990). In contrast, direct Monte Carlo integration of perturbation theory has also been employed to study these systems. Although the total cross sections are in reasonable agreement between different approaches, details of differential cross sections, spectra, and impact parameter dependence are notably different (Bottcher 1989, 1990, 1990a, 1990b). A critical review of these different approaches will be given elsewhere (Bottcher 1992). A comparison of three different equivalent photon calculations with the Monte Carlo integration of the exact two-photon terms, as shown in Fig. 1, illustrates the point that the total cross sections obtained from the different approaches are nearly the same. The Lorentz factor, γ, is nearly equal to the energy per nucleon of the beam in GeV.

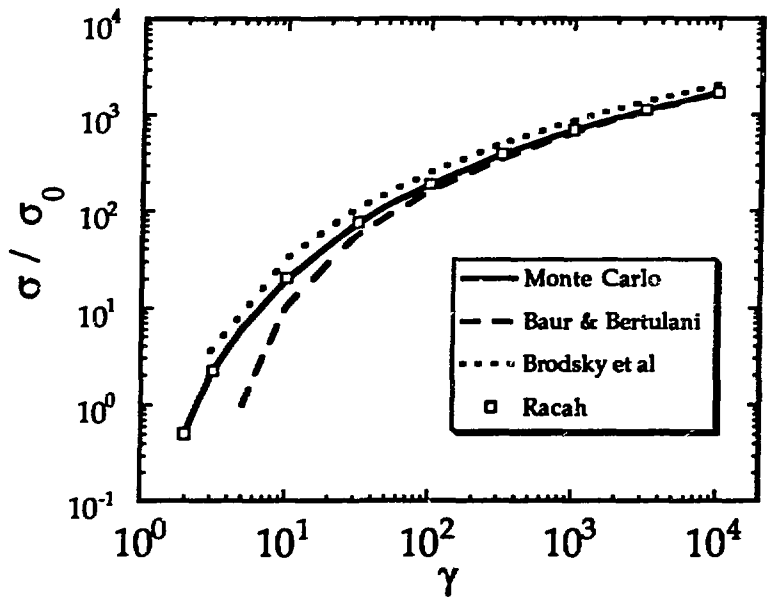

Fig. 1. The cross section for the production of electron pairs from the symmetric collision of Au nuclei as a function of the Lorentz γ of each beam. The constant σ_0 is given in the text.

The relation between this Lorentz factor in the center-of-mass system and the factor in an equivalent fixed-target frame is

$$\gamma_T = 2\gamma^2 - 1. \tag{1}$$

Equation (1) expresses the important result that collisions in colliders correspond to fixed-target experiments at very high energies. The curves in Fig. 1 are taken, respectively, from the references Brodsky (1971), Baur (1987), Racah (1937), and the Monte Carlo curve from Bottcher (1989) is given by the formula

$$\sigma = \sigma_0 C_\infty \frac{(\ln\gamma)^3}{1 + a_1 y + a_2 y^2 + \cdots} \quad ; \quad y = \ln[1 + (g_0 / \gamma)^2], \tag{2}$$

where the constants in Eq. (2) are given below

$$C_\infty = 3.40,\ a_1 = 0.4570,\ a_2 = 0.0222,\ g_0 = 10^3.$$

The scale for the cross sections, assuming point nuclear charge densities, is expressed in terms of the charge of the ion, and the Compton wavelength of the electron, and for Au ions is

$$\sigma_0 = (Z\alpha)^4 \lambda\!\!\!^{-2} \approx 165\, b. \tag{3}$$

A brief discussion of the classical field theory and the Monte Carlo method is discussed in the next section. Details of the method are given elsewhere (Bottcher 1989, Wu 1990).

2. TWO-PHOTON PAIR PRODUCTION

Most modern approaches to pair production begin with quantum field theory (Berestetskii 1979). However, for the collisions of highly charged ions it is possible to treat the electromagnetic field classically. We begin with the S-matrix

$$\mathcal{S} = \mathrm{T}\exp[-i \int d^4x\, \mathcal{H}_{\mathrm{int}}(x)] , \tag{4}$$

expressed in terms of the time-ordered interaction Hamiltonian which governs the coupling of electrons to the electromagnetic field

$$\mathcal{H}_{\mathrm{int}}(x) = e :\overline{\Psi}(x)\gamma^\mu \Psi(x) A_\mu(x): + e :J^\mu(x) A_\mu(x):. \tag{5}$$

In Eq. (5) Ψ denotes the electron field operator and the electromagnetic current and field operators are, respectively, J^μ and A^μ. The classical field model is then obtained by replacing J^μ and A^μ with their classical counterparts. Under these assumptions, the second term in Eq. (5) can be ignored since it only contributes a c-number phase to the S-matrix in Eq. (1).

The electromagnetic potentials come from the motion of the two ions

$$A^\mu(x) = A^\mu_a + A^\mu_b , \tag{6}$$

where a,b label the two ions. These potentials can be calculated from the nuclear charge density, ρ, of each ion expressed as a function of the four-momentum transfer $-q^2 > 0$

$$f(q^2) = \frac{F(-q^2)}{-q^2} = \frac{4\pi}{-q^2}\int_0^\infty \frac{r dr}{q}\sin(qr)\rho(r). \tag{7}$$

In the rest frame of each ion only the time component is nonzero, yielding solutions to Maxwell's equations as

$$A^0_{a,b}(q) = 2\pi e Z_{a,b}\,\delta(q^0) f_{a,b}(q^2)\exp(\pm i\vec{q}\cdot\vec{b}/2), \tag{8}$$

where b is the impact parameter separating the two ions. Lorentz boosting these potentials into the center-of-velocity frame of the a+b system gives the result

$$A^\mu_{a,b}(q) = 2\pi e Z_{a,b}\, u^\mu_{a,b}\,\delta(q^0 \mp \beta q^z) f_{a,b}(q^2)\exp(\pm i\vec{q}\cdot\vec{b}/2), \tag{9}$$

as a function of the "boost" four-velocities

$$u^\mu_{a,b} = (1,0,0,\pm\beta). \tag{10}$$

The total cross section is obtained from Eq. (1) by integrating over all impact parameters and summing over all final states

$$\sigma = \int d^2b \sum_{\mathrm{f}} \left|\langle \mathrm{f}|S|0\rangle\right|^2. \tag{11}$$

The lowest-order contributions to the S-matrix which create pairs out of the vacuum are the second-order terms given by

$$\left\langle \vec{k}_1 s_1 \vec{k}_2 s_2 \middle| S^{(2)} \middle| 0 \right\rangle = -ie^2 \int \frac{d^4k}{(2\pi)^4}\bar{u}(k_1 s_1)\{\not{A}_a(q_a)\frac{1}{\not{k}-m}\not{A}_b(q_b) + \not{A}_b(q_b)\frac{1}{\not{k}'-m}\not{A}_a(q_a)\} v(k_2 s_2). \tag{12}$$

The four momenta q_a, q_b are the momenta carried by the fields of each ion, and k, k' are the corresponding intermediate momenta for the direct and crossed terms

$$k = k_1 - q_a = -k_2 + q_b\,; \quad k' = k + q_a - q_b\,.$$

The slashes denote the usual Feynman contractions with the γ matrices (Berestetskii 1979). After a modest amount of analysis, the resulting total pair production cross section can be reduced to an eight-dimensional integration as follows

$$\sigma = \frac{Z_a^2 Z_b^2 (4\pi\alpha)^4}{4\beta^2} \int \frac{d^3k_1 d^3k_2 d^2q_{a\perp}}{(2\pi)^8 2E_1 2E_2} (f_a(q_a) f_b(q_b))^2$$
$$\times \sum_{s_1 s_2} \left| \bar{u}(k_1 s_1)(\not{u}_a \frac{1}{\not{q}-m} \not{u}_b + \not{u}_b \frac{1}{\not{q}'-m} \not{u}_a) v(k_2 s_2) \right|^2 . \qquad (13)$$

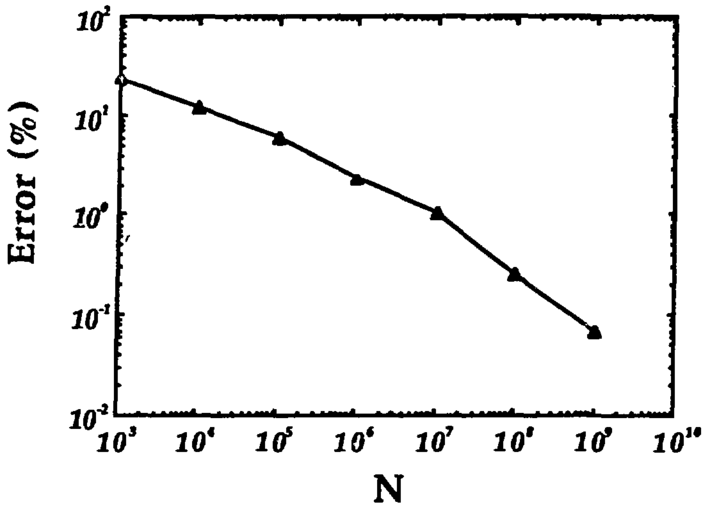

Fig. 2. Error in the Monte Carlo calculation for the case studied in Fig. 1 at an energy per nucleon of about 100 GeV as a function of the number of Monte Carlo points used in the integration.

In the above equation the longitudinal and time-like components of q_a and q_b are fixed by the delta function in Eq. (9) for the classical fields, while the transverse parts must be integrated over.

The eight-dimension integration needed to evaluate the above cross section is carried out using the Monte Carlo method. The integrand is nonsingular and positive definite so that simple mappings can be devised to bias the Monte Carlo sampling near the peaks in the integrand. This results in rapid convergence of the integration. The error in evaluating the cross section in Fig. 1 is shown in Fig. 2 as a function of the number of Monte Carlo points. We observe that the behavior of the errors goes as $N^{-1/2}$ for the larger values of N. In evaluating the curve in Fig. 1, we typically have used more than 10^7 points with a corresponding error of better than about one percent.

The algorithms that are needed to evaluate these expressions can easily be optimized on supercomputers with either vector instructions like the Cray 2, or massively parallel architectures like the Intel 860 hypercube. In Table 1 we compare timing measurements for a typical Monte Carlo calculation on these two machines. The calculation on the hypercube is repeated with a different number of nodes, demonstrating that the effects of message passing and communication among the nodes is not important. Note that the speed on the Cray 2 is about 150 MFLOPS, or about three times faster than most large scientific calculations. The overall speed on the Intel hypercube of about 1.6 GFLOPS is more than ten times faster than the Cray 2.

Nodes	16	32	64	128	Cray 2
Time (Sec)	65.7	29.8	14.2	6.9	74.7
MFLOPS	170	375	787	1620	150

Table 1. Comparison of the speed of Monte Carlo integration for a typical case having 2 x 10^6 points on the Cray 2 and on the Intel 860 hypercube with a different number of nodes.

Differential distributions can easily be calculated with this method by simply binning the integrand in terms of the requisite variables, thereby avoiding the expensive calculation of Jacobians. The doubly differential cross section in terms of the opening angle of the pair, Θ, and the angle specifying the total momentum of the pair with respect to the beam axis, θ, are shown in Fig. 3 for the case of symmetric Au collisions at a beam energy per nucleon of 100 GeV. In the figure the different curves correspond to histograms with very closely spaced angular bins, and thus appear as smooth curves.

Only recently has it become possible to measure this type of pair production at relativistic energies. Presently, the highest energy heavy-ion experiments are those of the WA90 collaboration at CERN (Vane 1991) using the 200 GeV per nucleon beams on fixed targets. Measurements of electron-positron pair production have been reported for collisions of S+Au, and future experiments with Pb beams are scheduled for 1993.

The experiment detects both electrons and positrons within an angular region of 20 degrees with respect to the beam and for a range of momenta between 0.9 and 7 MeV/c. In Fig. 4 the positrons have been summed over all angles, and the corresponding electrons have been summed over all angles and momenta in the range. The solid curve is the two-photon Monte Carlo calculations, and the dashed curve are the results of an equivalent photon calculation (Eby 1989). The experimental cross section is reported as 75±25 b as compared to the Monte Carlo result of 51 b. While these results are preliminary, there is an indication that theory and experiment may disagree for the higher momentum events.

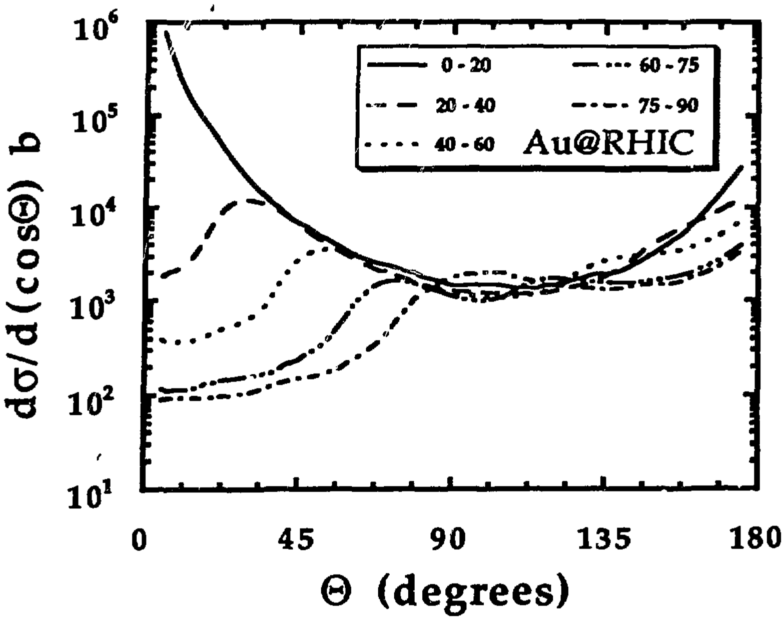

Fig. 3. Electron pair differential cross sections with respect to the opening angle of the pair, Θ, for various cuts on the pair orientation angle, θ, given in degrees. The case studied is the same as in Fig. 2.

3. PAIR PRODUCTION WITH CAPTURE

As previously stated, electron pair production with electron capture is an important process for designing heavy-ion colliders. The mechanism is a second-order one where the pair is first formed and the electron subsequently captured into an orbit of one of the ions. This formulation has been followed using the two-photon Monte Carlo method (Rhoades-Brown 1989). From Eq. (11) and Eq. (12), the cross section for capture into the bound state, Φ_T, can be written as

$$\sigma_c = \int d^2b \sum_s \int \frac{d^3k}{(2\pi)^3} \left| \langle \Phi_T \vec{k}s | S | 0 \rangle \right|^2 \quad . \tag{14}$$

The S-matrix in the equal velocity frame is given by Rhoades-Brown (1989) and is not repeated here. The results of evaluating this cross section using the Monte Carlo method for a series of target and projectile combinations is shown in Fig. 5 as a function of beam energy. For the case of an energy of 100 GeV per nucleon, the cross section for Au beams is about 80 b and would yield a beam lifetime of about 10 hours in the RHIC collider. These results are within ten percent of those of Lee and Weneser (Lee 1986).

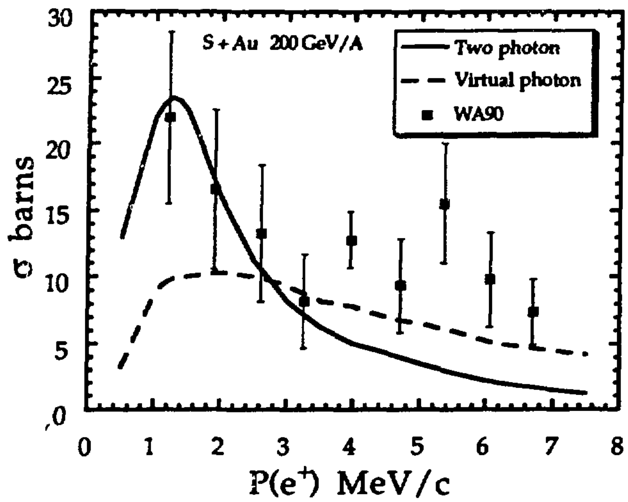

Fig. 4. The distribution of positrons as a function of momentum for collisions of S+Au at a laboratory energy per nucleon of 200 GeV.

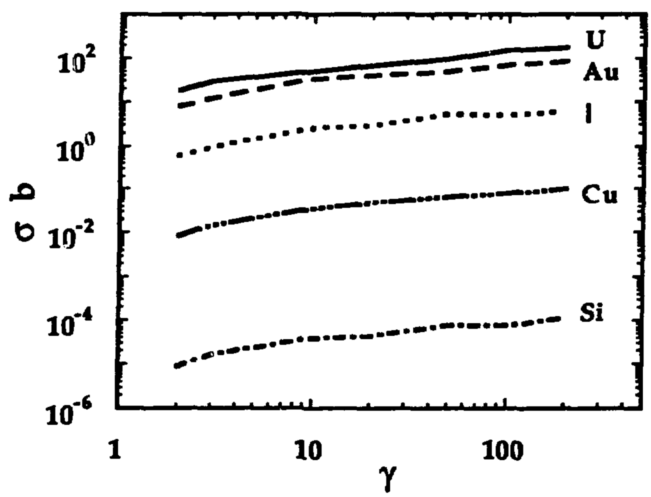

Fig. 5. The cross section for pair production with electron capture for symmetric collisions as a function of beam energy.

At lower energies, a variety of methods have been employed and are shown in Table 2 for comparison. Table 2 shows a variation in the capture cross section by more than a factor of ten and underscores the need for reliable experiments to help clarify the situation.

Method	Reference	σ (b)
Equivalent Photon	Baur 1987	2.8
Two Photon	Rhoades-Brown 1989	4.6
Distorted Wave	Becker 1987	0.3
Coupled Channel	Rumrich 1991	1.5

Table 2. Different capture calculations for collisions of Pb+Pb at an energy of 1.2 GeV per nucleon.

3. SUMMARY

This paper has addressed the phenomena of electron pair production and pair production with electron capture from the coherent fields associated with relativistic collisions of highly charged ions. We have discussed calculations of these phenomena using methods of quantum field theory and modern Monte Carlo algorithms for numerical integration. In discussing the production of electron pairs, we have compared our results with the new measurements of the WA90 group at CERN. Generally these measurements and the theory are in reasonable agreement. However, experiments with heavier beams are needed in order to test higher-order effects such as multiple pair production. Pair production with electron capture to one of the ions has not been measured at these energies, and theoretical calculations at lower energies differ by an order of magnitude. This disagreement could be more than a factor of 100 for the highest RHIC energies. Thus there is a clear need to obtain definite capture measurements as soon as possible.

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