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CONF-871179--9-Vugraphs

Benchmarks of Neutral Beam Injection Codes for Stellarators*

R. H. FOWLER, J. A. ROME, R. N. MORRIS,

ORNL

K. HANATANI

Plasma Physics Lab., Kyoto University

CONF-871179--9-Vugraphs

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The 3-dimensional geometry of stellarators/heliotrons make it more difficult to calculate beam deposition and thermalization. Extensive comparisons between the Oak Ridge and Kyoto codes were undertaken, using a simplified model for the Heliotron-E plasma and magnetic field. The perpendicular injection geometry causes most of the energetic fast ions to be helically trapped. The high transform and shear of the heliotron configuration helps to confine these orbits, but energy losses in the model cases ranged from 20% to 50%. Although the codes use different coordinate systems and collision operators, when they solve the same problem, the answers are equivalent. For perpendicular injection, very little, if any, acceleration of the slowing process can be tolerated.

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BENCHMARKS OF NBI CODES FOR STELLARATORS

I. Introduction - motivation

II. Description of thermalization codes – orbit equations, collision operators, etc.

III. Sensitivity and/or Numerical Issues

IV. Benchmarks for Heliotron-E perpendicular injection with model field

V. Heliotron-E results using a filament model

VI. ATF results for perpendicular and tangential injection

VII. Conclusions

Importance of Benchmarking NBI Codes

- Beam inputs are a significant part of the power balance but are seldom directly measured

- Calculations serve as a guide to determine optimal injection angles

- Calculations are expensive, so the appropriateness of analytic moments should be determined

Codes and Cases Considered

HELIOS – Kyoto real space code

MAGCOM – ORNL magnetic coordinate code

DESORBS – ORNL real space code

CASES:

Perpendicular and parallel injection into Heliotron-E and ATF

ORNL MAGCOM CODE

- Boozer coordinates, natural for profiles and electric field treatment
- Only mod $B = \sum_{n,m} A_{n,m}(\psi)\cos(n\phi - m\theta)$ needed
- $A_{n,m}(\psi)$ are least-squares fitted
- Relatively fast but loss boundary cannot be extended much beyond outer flux surface

ORNL DESORBS CODE

- Real space cylindrical coordinates
- ψ is interpolated using 3-D splines
- For complex fields vector B is interpolated using 3-D splines
- More difficult to treat electric fields
- Slower than MAGCOM but allows treatment of complete geometry

Guiding Center Equations in Magnetic Coordinates

$$\dot{\psi} = (\dot{P}_\theta g - \dot{P}_\phi I)/\gamma, \qquad \dot{\rho}_c = [-(\rho_c g' - \iota)\dot{P}_\theta + (\rho_c I' + 1)\dot{P}_\phi]/\gamma,$$

$$\dot{\theta} = \left(\delta \frac{\partial B}{\partial \psi} + e\frac{\partial \Phi}{\partial \psi}\right)\frac{\partial \psi}{\partial P_\theta} + \frac{e^2 B^2}{m}\rho_c \frac{\partial \rho_c}{\partial P_\theta},$$

$$\dot{\phi} = \left(\delta \frac{\partial B}{\partial \psi} + e\frac{\partial \Phi}{\partial \psi}\right)\frac{\partial \psi}{\partial P_\phi} + \frac{e^2 B^2}{m}\rho_c \frac{\partial \rho_c}{\partial P_\phi},$$

where $\rho_c = mv_{\|}/eB$, B is obtained by FFT,

$$B = \sum_{n,m} A_{n,m}(\psi)\cos(n\phi - m\theta)$$

The toroidal current within a flux surface is $I(\psi)/(2\times 10^{-7})A$, and the poloidal current outside a flux surface is $g(\psi)/(2\times 10^{-7})A$.

$$\gamma = e[g(\rho_c I' + 1) - I(\rho_c g' - \iota)], \qquad \delta = e^2\rho_c^2 B/m + \mu.$$

$$\dot{P}_\theta = -\delta\frac{\partial B}{\partial \theta}, \qquad \dot{P}_\phi = -\delta\frac{\partial B}{\partial \phi}.$$

$$\frac{\partial \psi}{\partial P_\theta} = \frac{g}{\gamma}, \qquad \frac{\partial \rho_c}{\partial P_\theta} = -\frac{(\rho_c g' - \iota)}{\gamma},$$

$$\frac{\partial \psi}{\partial P_\phi} = -\frac{I}{\gamma}, \qquad \frac{\partial \rho_c}{\partial P_\phi} = \frac{(I'\rho_c + 1)}{\gamma}$$

Guiding Center Equations in Real Space

$$\frac{d\mathbf{R}}{dt} = \frac{\hat{\mathbf{B}}}{qB} \times \left(\mu \nabla B + m v_{\parallel}^2 \frac{\hat{\mathbf{B}}}{B} \cdot \nabla \mathbf{B}\right) + v_{\parallel} \hat{\mathbf{B}},$$

$$\frac{dv_{\parallel}}{dt} = -\frac{\mu}{m} \hat{\mathbf{B}} \cdot \nabla B,$$

$$\hat{\mathbf{B}} = \frac{\mathbf{B}}{B},$$

$$\mu = \frac{1}{2} m \frac{v_{\perp}^2}{B} = \frac{1}{2} \frac{m}{B} \left(v^2 - v_{\parallel}^2\right) ,$$

$$v_{\parallel} = \frac{d\mathbf{R}}{dt} \cdot \hat{\mathbf{B}}(\mathbf{R}).$$

Slowing Down of Fast Ions

$$\frac{\partial f}{\partial t} = \frac{1}{\tau_s v^2} \frac{\partial}{\partial v} \left(v^3 + v_c^3\right) f$$

$$f = \frac{\delta\left(v - v_o\right)}{v_o^2}$$

$$\frac{\partial \langle v \rangle}{\partial t} = -\frac{\left(v_o^3 + v_c^3\right)}{\tau_s v_o^2}$$

On plasma ions:

$$\Delta v_i = -\frac{v_c^3}{v_o^2} \frac{\Delta t}{\tau_s}$$

On plasma electrons:

$$\Delta v_e = -v_o \frac{\Delta t}{\tau_s}$$

Pitch Angle Scattering

$$\frac{\partial f}{\partial t} = \frac{1}{2\tau_s}\frac{v_c^3}{v^3}\frac{\partial}{\partial \zeta}\left[\left(1-\zeta^2\right)\frac{\partial t}{\partial \zeta}\right]$$

Sample $\zeta = \frac{v_\parallel}{v}$ from the Gaussian:

$$P(\zeta) = \frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{1}{2}(\zeta-\langle\zeta\rangle)^2/\sigma^2}$$

where $\sigma^2 = \langle\zeta^2\rangle - \langle\zeta\rangle^2$

let $f = \delta(\zeta - \zeta_o)$

$$\sigma^2 = \frac{v_c^3}{v^3}\frac{\Delta t}{\tau_s}\left(1-\zeta_o^2\right)$$

$$\langle\zeta\rangle = \zeta_o\left(1 - \frac{v_c^3}{v^3}\frac{\Delta t}{\tau_s}\right)$$

Charge Exchange with Re-absorption

$$\xi = \int_o^L e^{-n_o \sigma_{cx} l} n_o \sigma_{cx} dl$$

$$L n_o \sigma_{cx} = v \Delta t n_o \sigma_{cx} = \frac{\Delta t}{\tau_{cx}} > -ln\xi$$

The neutral is tracked by transforming from guiding center coordinates to particle coordinates with a random phase for the gyro-motion

Profile Parameters For Heliotron-E Cases

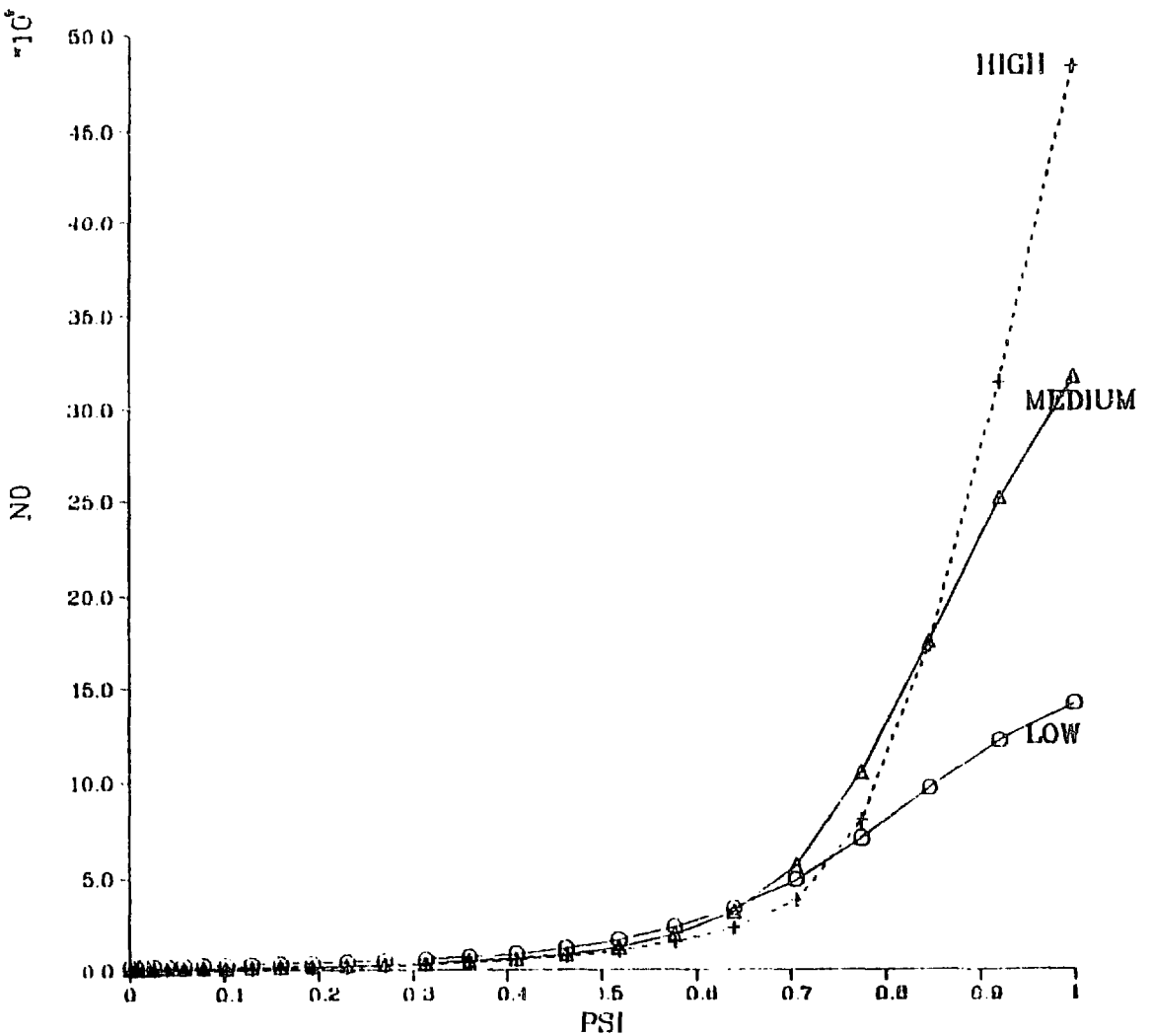
$$T_i(\psi) = T_e(\psi) = T_o(1 - \psi^{S_1})^{R_1} + T_t$$

$$N_i(\psi) = N_e(\psi) = N_o(1 - \psi^{S_2})^{R_2} + N_t$$

$$E_o = 26 \text{ keV}$$

	High Density	Medium Density	Low Density
R_1	0.5	2	1
R_2	0.86	1	1
S_1	1	2.575	1
S_2	1	1	1
T_0	376 eV	406 eV	540 eV
T_t	5 eV	10 eV	60 eV
N_o	1.37×10^{14} cm^{-3}	8.7×10^{13} cm^{-3}	4.45×10^{13} cm^{-3}
N_t	1.00×10^{11} cm^{-3}	0.3×10^{13} cm^{-3}	4.50×10^{11} cm^{-3}

HELIOTRON NEUTRAL DENSITY PROFILES



Sensitivity Questions

- Numerical/Statistical - How many particles are sufficient? Random numbers, etc.
- Magnetic field – Are simple models accurate?
- Boundary model used to determine orbit losses – Can the outer flux surface be used?
- Acceleration of slowing down process – Can computer time be saved without loss of accuracy?
- Frequency of collisions – What is a reasonable number of collisions? Should collisions be "independent" of orbit integration?

Reducing Collisions Saves A Small Amount of Computer Time — Is it Worth the Risk?

Model magnetic field (NF 21(1981) 1067).
Heliotron-E low density case N0 = 4.45e13, T0 = 540 eV, τ_s = 12 ms
100 particles
Boundary at last closed surface

Collisions (1000's)	Orbit Loss(%)	GE(%)	Gi(%)
2	39.9	42.6	15.7
10	52.7	35.3	11.0
20	51.8	35.5	12.2
30	51.5	35.7	12.4
40	45.4	39.2	14.8
143	45.6	39.1	14.5
	47.8	37.9	13.4

- Collisions should be every time step in order to accurately sample at turning points where trapping and detrapping occurs

Acceleration Techniques Drastically Reduce Computational Time But When are They Valid?

Scheme 1

Compute τ_s for problem. Then choose $n_c = \#$ of collisions. The Δt for the collision operator is given by

$$\Delta t_c = \frac{\tau_s}{N_c}$$

Δt_c, N_c are the same for all acceleration factors; i.e., there is always N_c collisions with Δt_c for each collision. The "acceleration" is given by changing the orbit following time $t_{\text{orbit}} = \frac{\tau_s}{N_g}$, N_g = acceleration factor.

N_c collisions during t_{orbit} time.

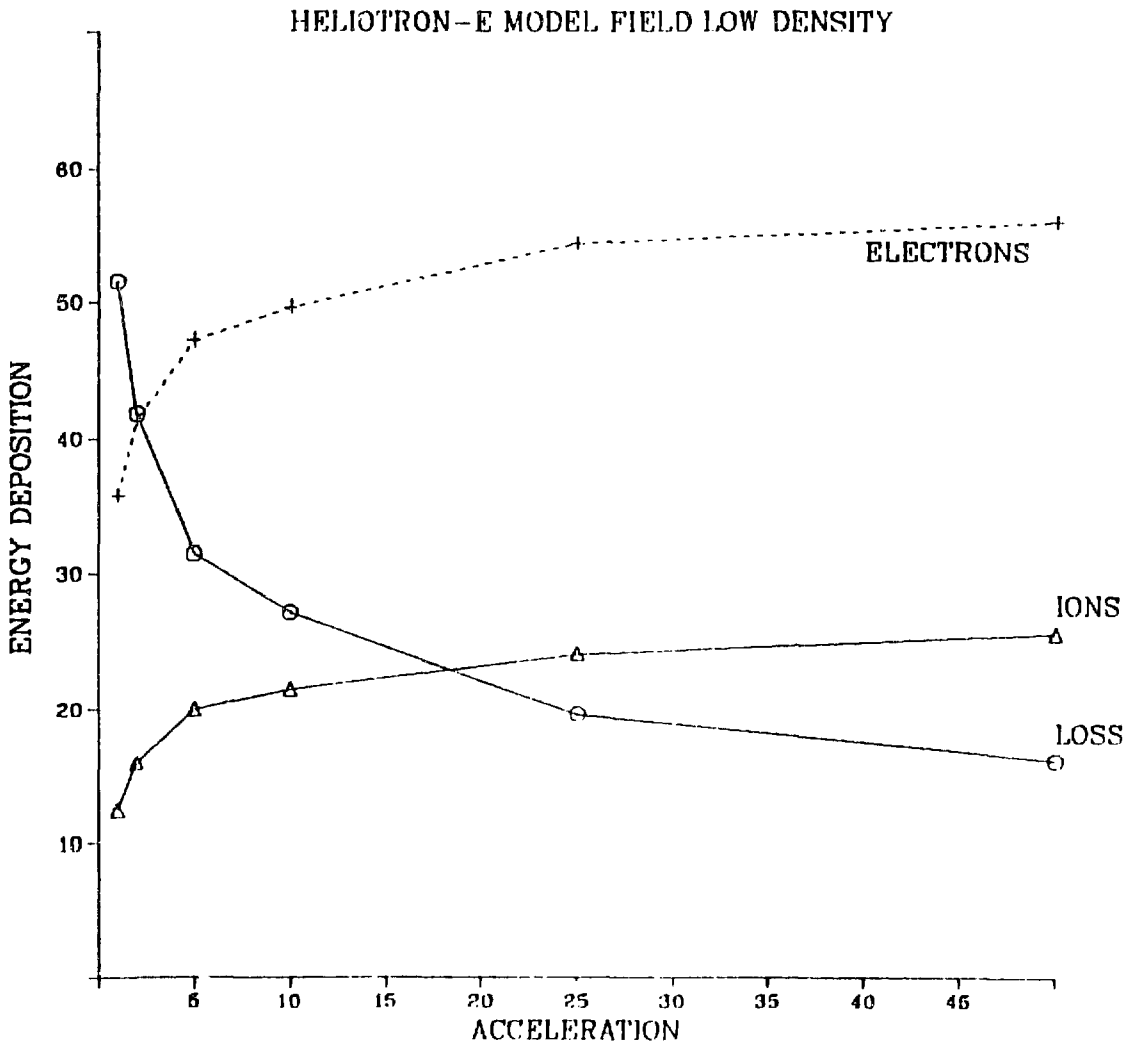
Scheme 2

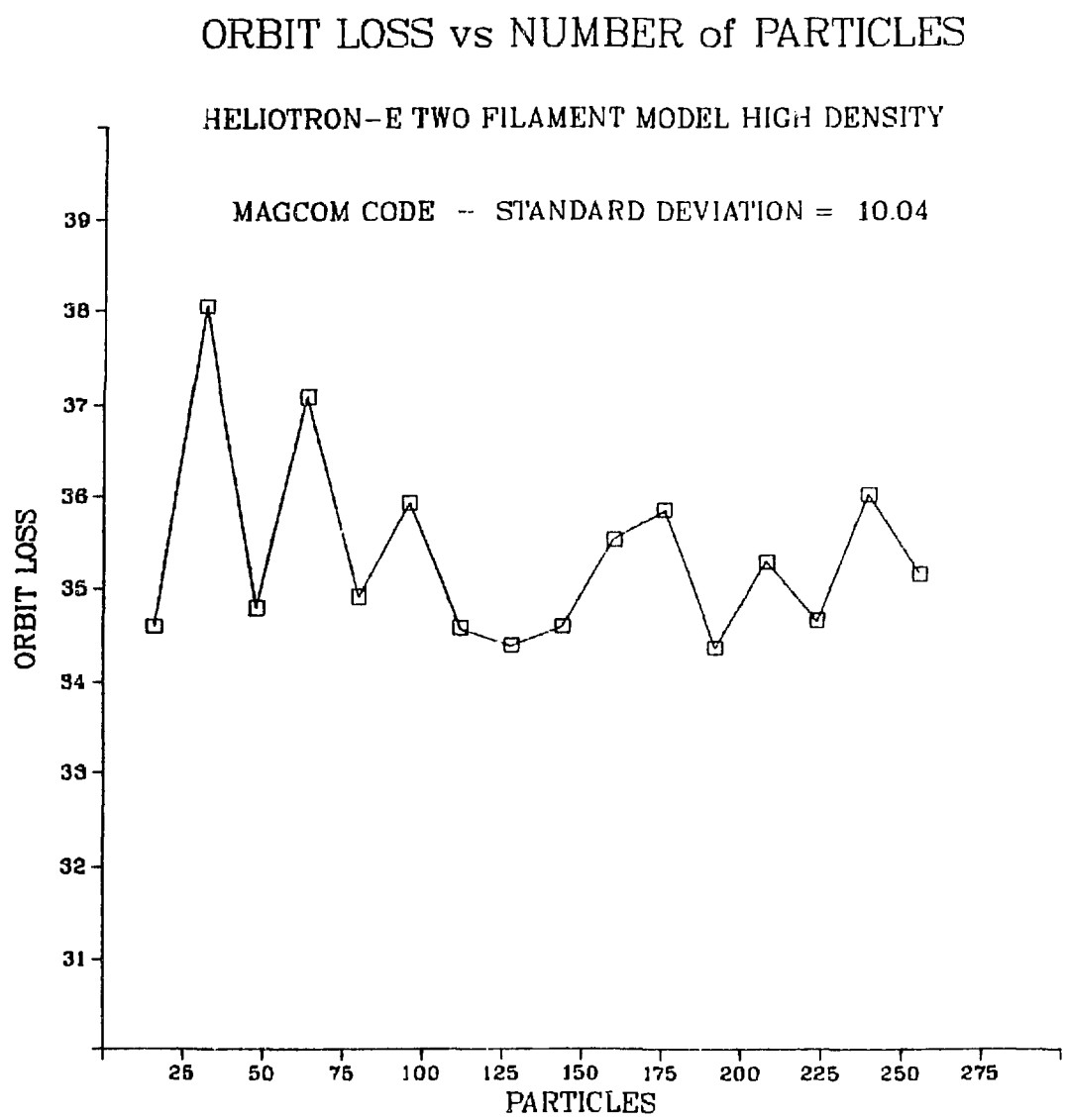
Let Δt_{DE} = integration time step of GC eqs. Collision after each step with $N_g \Delta t_{DE} = \Delta t_c$ for the collision operator (pitch angle and slowing down).

N_g = goose factor

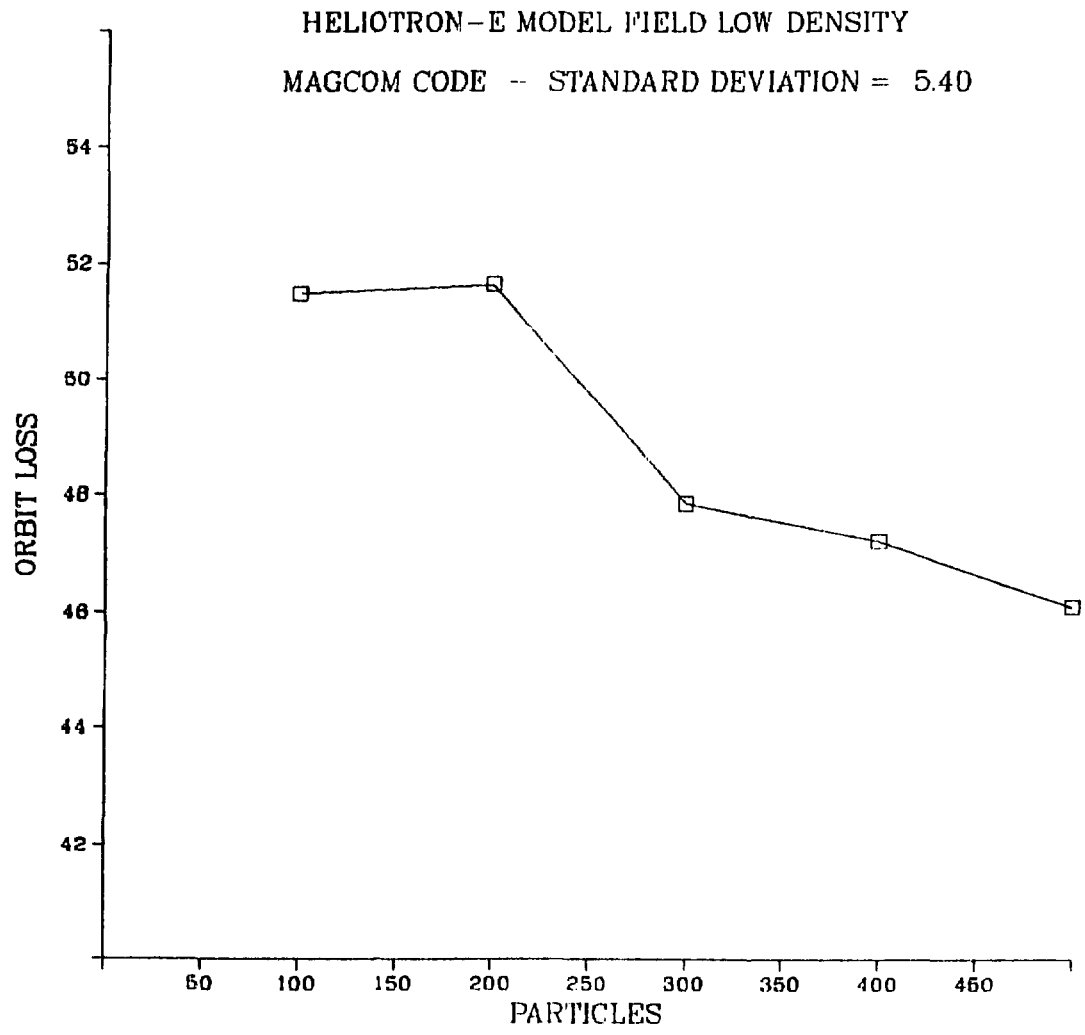
t_{orbit} = time for all particles to thermalize or be lost.

ACCELERATION of SLOWING DOWN PROCESS




ORBIT LOSS vs NUMBER of PARTICLES
HELIOTRON-E TWO FILAMENT MODEL HIGH DENSITY
MAGCOM CODE -- STANDARD DEVIATION = 10.04
ORBIT LOSS
39
38
37
36
35
34
33
32
31
25
50
75
100
125
150
175
200
225
250
275
PARTICLES

ORBIT LOSS vs NUMBER of PARTICLES



Model Helical Field Used for Benchmarks

Hanatani, Wakatani, Uo, Nucl. Fusion 9(1981)1067

$$\vec{B} = B(0)\vec{h} + \nabla\psi \times \vec{h}$$

$$\vec{h} = (\hat{z} + \rho\hat{\theta})/(1+\rho^2)$$

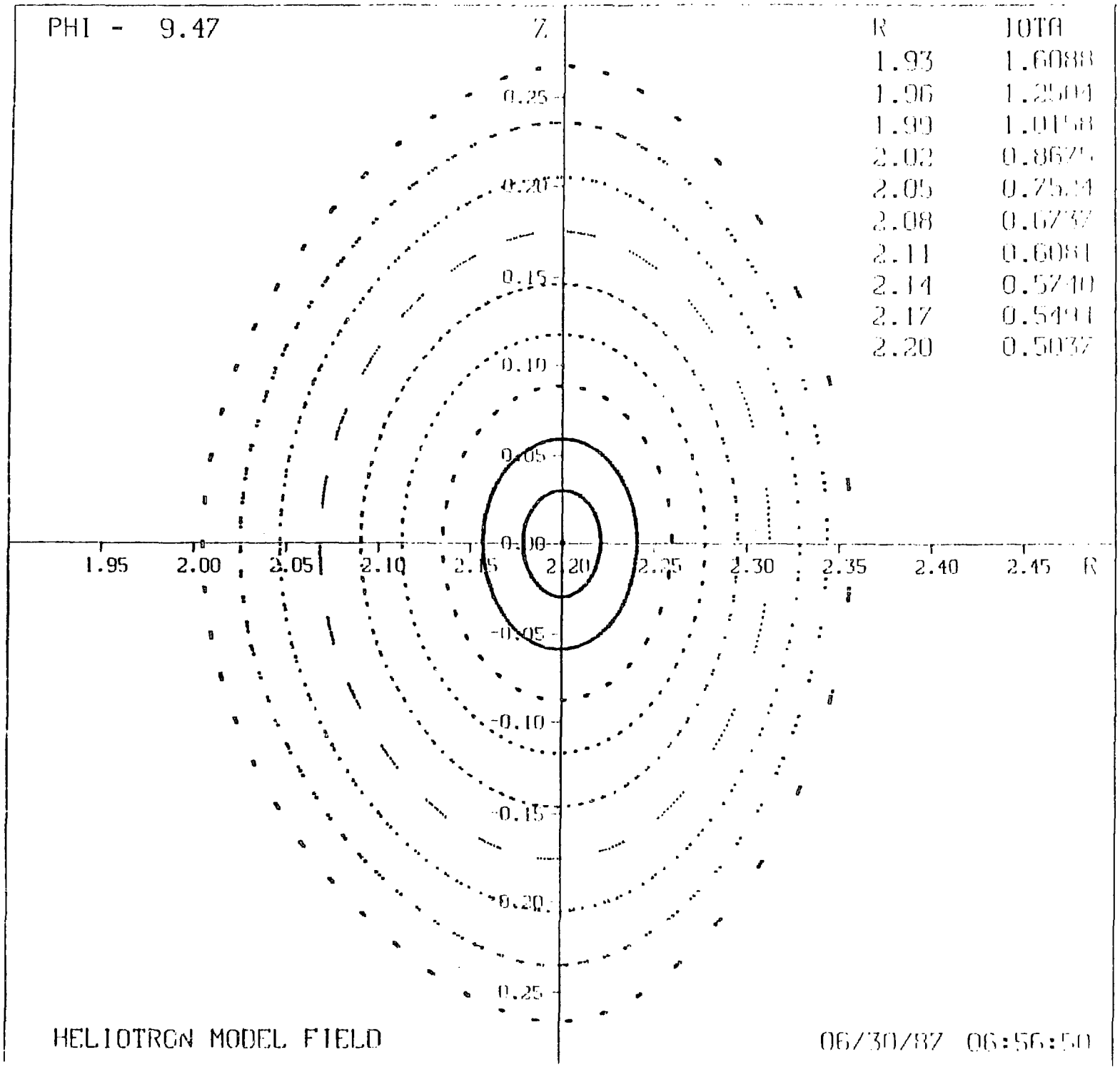
$$\psi = -\frac{\pi}{L} B_{cz}(0)\{\alpha^\star + e_0 - e_1\rho_c K_2'(2\rho_c)\cos 2(\theta - \frac{2\pi z}{L})\}\rho^2$$

$$\rho_c = 2\pi a/L, \ \rho = 2\pi\tilde{r}/L$$

a = minor radius

L = pitch length

toroidal correction factor $(1 - r/R\cos\theta)$


PHI - 9.47
Z
R
1.93 1.6088
1.96 1.2504
1.99 1.0158
2.02 0.8675
2.05 0.7534
2.08 0.6737
2.11 0.6081
2.14 0.5740
2.17 0.5411
2.20 0.5032
0.25
0.20
0.15
0.10
0.05
0.00
-0.05
-0.10
-0.15
-0.20
-0.25
1.95
2.00
2.05
2.10
2.15
2.20
2.25
2.30
2.35
2.40
2.45
R
HELIOTRON MODEL FIELD
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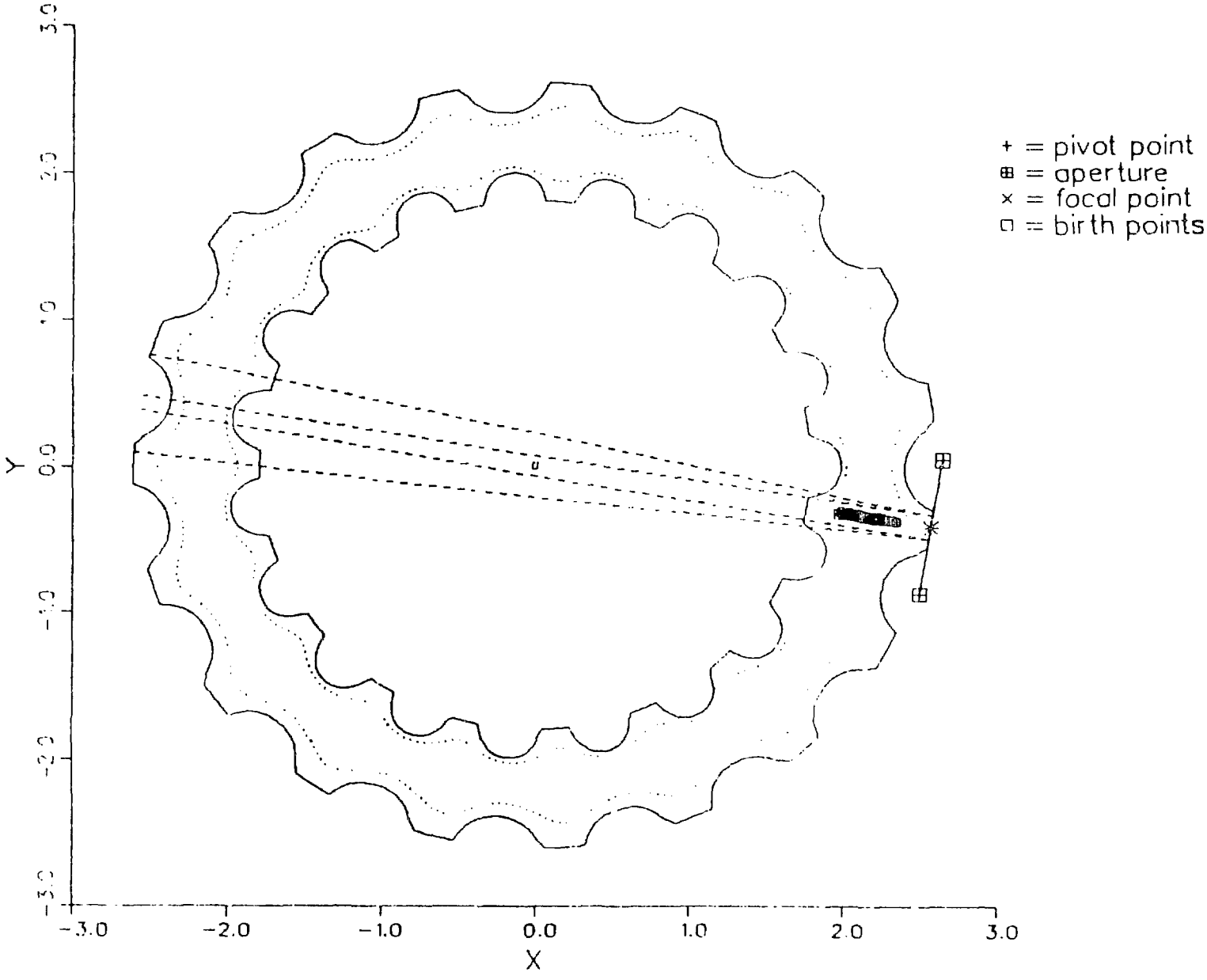
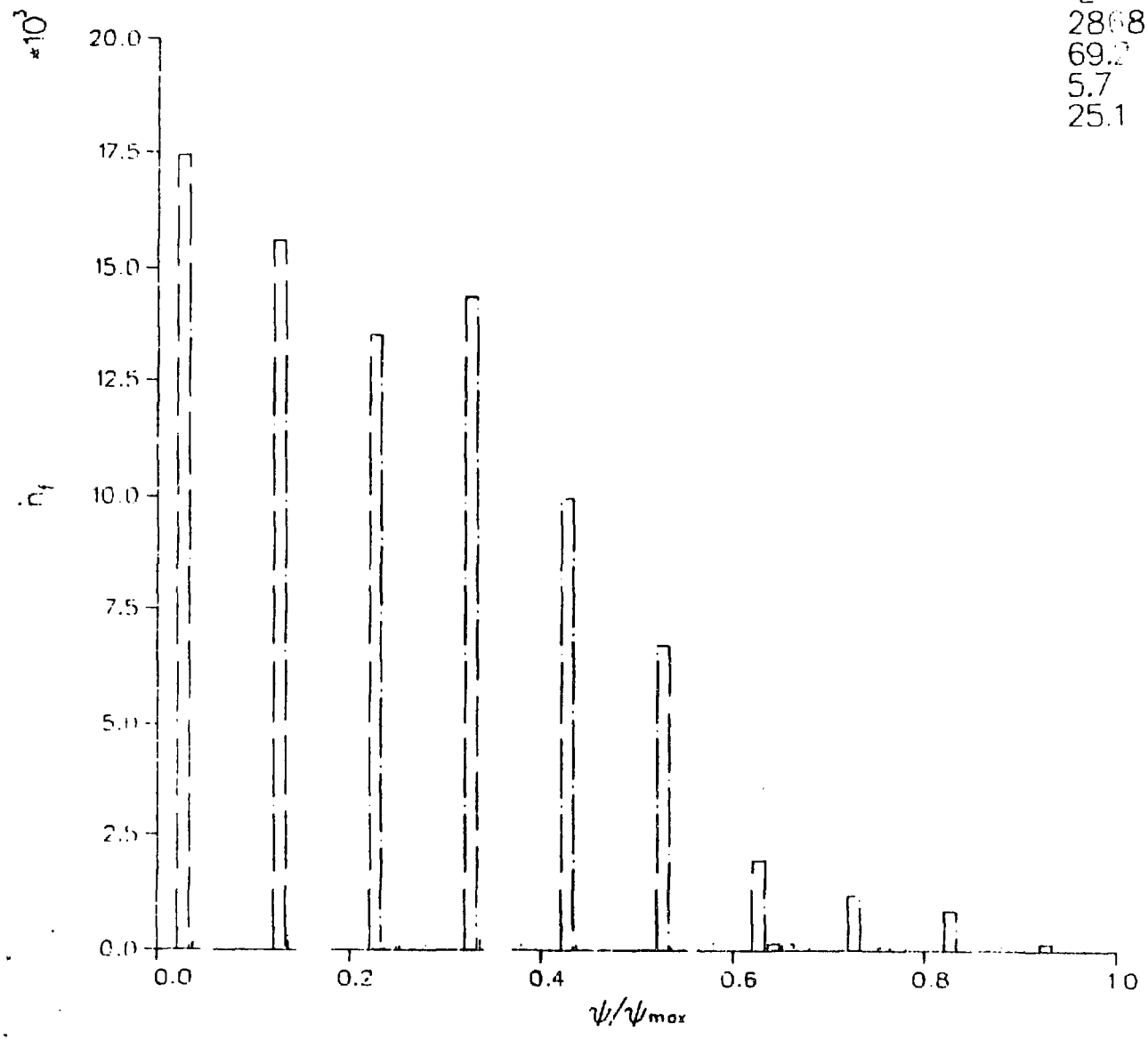
Beam Line

E Component

Top View



Fast Ion Density



	E	
	2868	Total Part.
	69.2	Prct. Deposit
	5.7	Prct. Shine
	25.1	Prct. Blocked
bhzfoc=		396.0
bvtfoc=		396.0
bdvghz=		1.2
bdvgvt=		1.2
angle3=		0.0
phirbd=		−9.5

E Component

Benchmark STEPS

Kyoto and ORNL CODES

- Cases benchmarked – Perpendicular injection into high (I) and low density (II) Heliotron-E plasmas with a model magnetic field (Hanatani, et al., Nucl. Fusion 9 (1981) 1067)

- Beam deposition agrees – 92% (I) and 65% (II) beam absorption, shinethrough agrees

- Collisionless orbit loss (I) agrees – 28.5% with boundary at last closed flux surface

- Numerically calculated collisionless orbit shift of deeply trapped particles agrees with analytical prediction

- Collisionless orbits match for Kyoto and ORNL flux coordinate codes

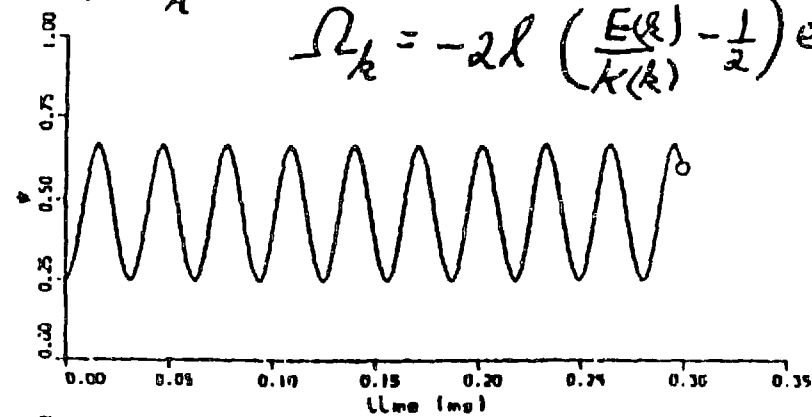
Shift of deeply trapped particles

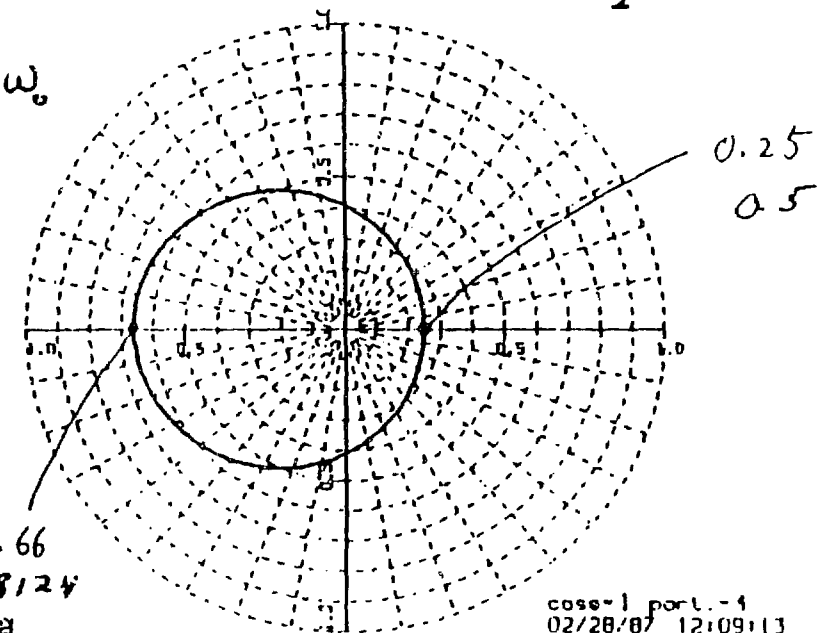
$$\Delta = \frac{-V_\perp}{\Omega_k} \qquad V_\perp = -\epsilon_t \omega_0 r \qquad \epsilon_t = a/R_0$$

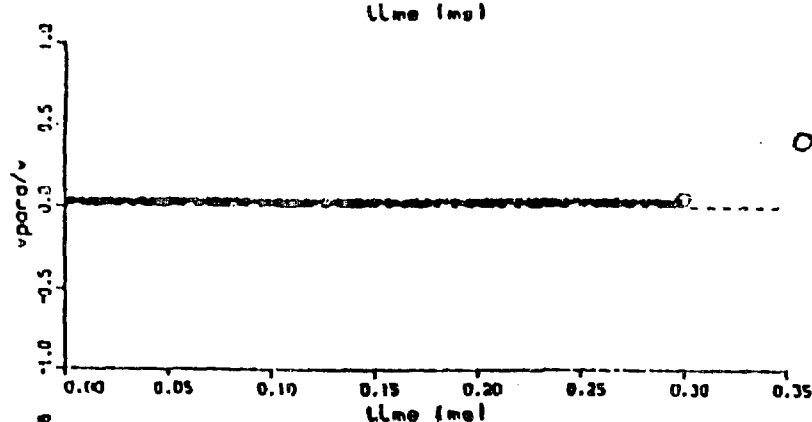
$$\Omega_k = -2\ell\left(\frac{E(k)}{K(k)} - \frac{1}{2}\right)\epsilon_h \omega_0$$

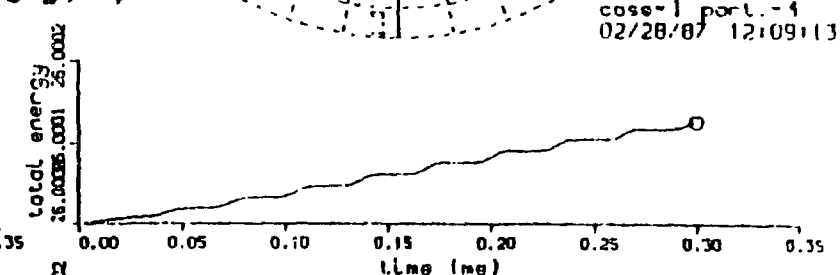
$$\frac{\Delta}{a} = -0.15 \text{ analytic}$$

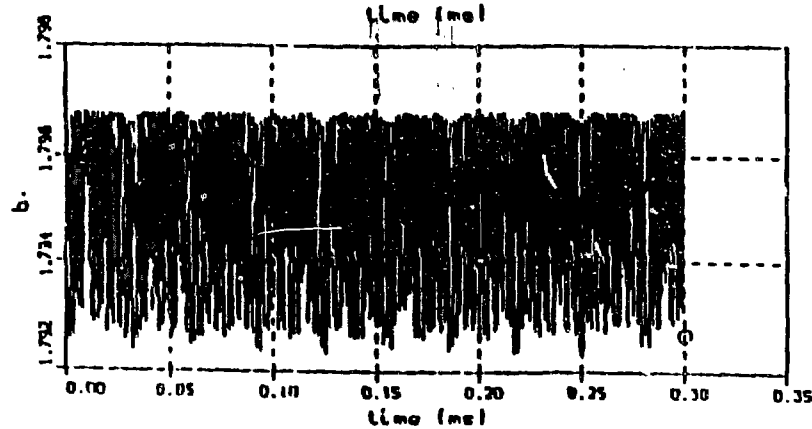
$$\Delta' = \frac{0.8124 - 0.5}{2} = 0.156$$

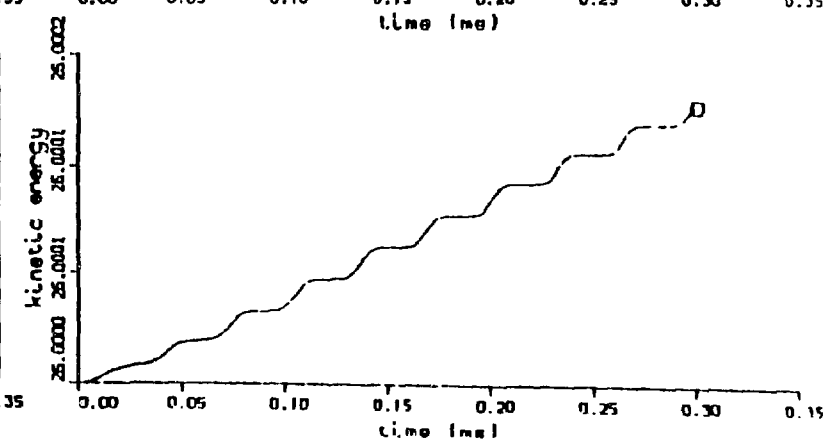




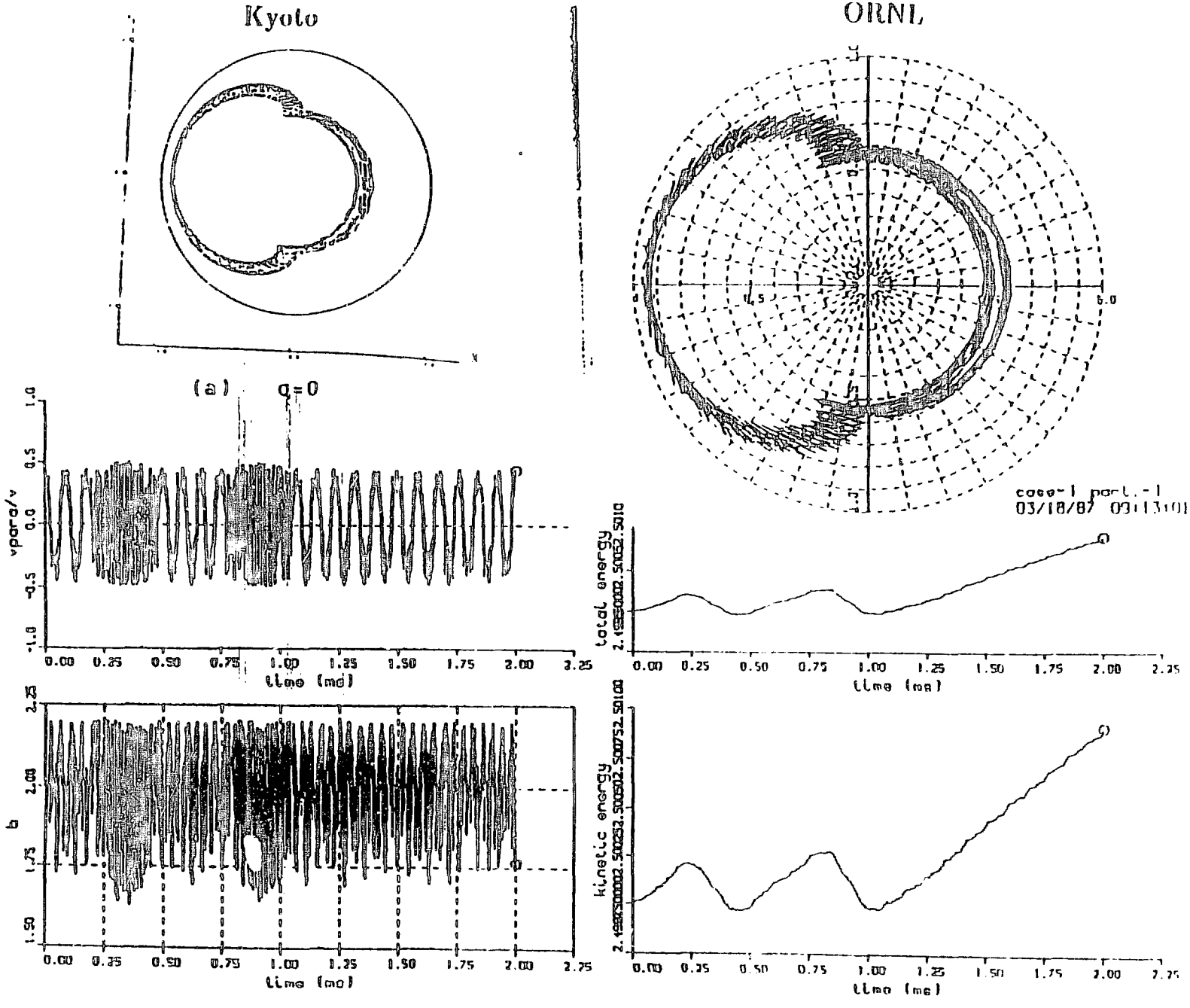






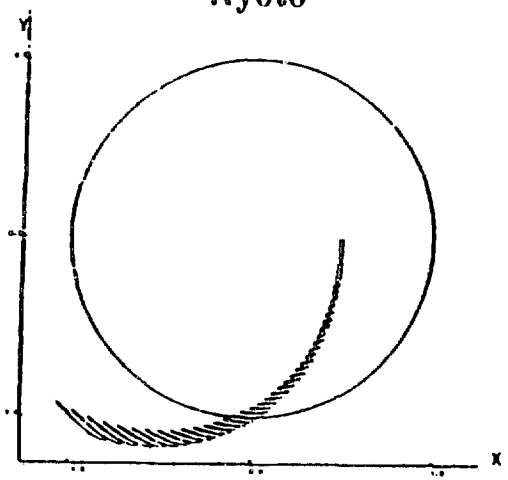


Collisionless orbit



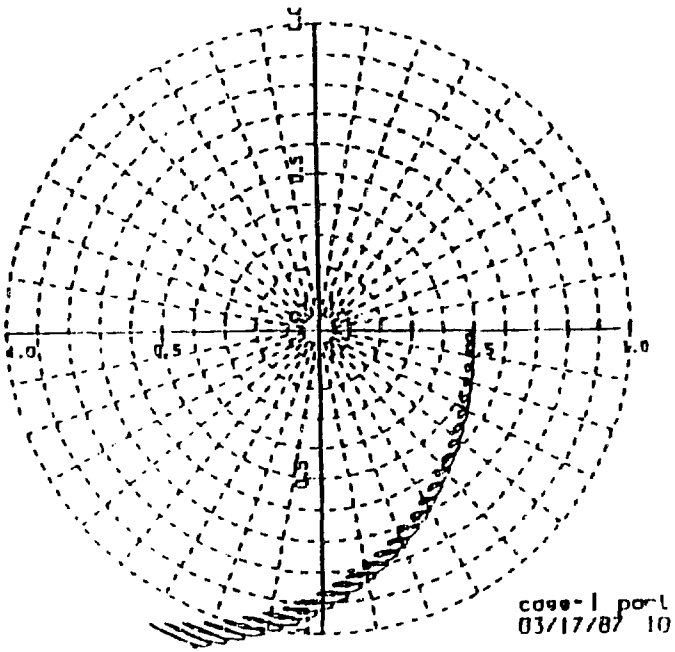
Collisionless orbit

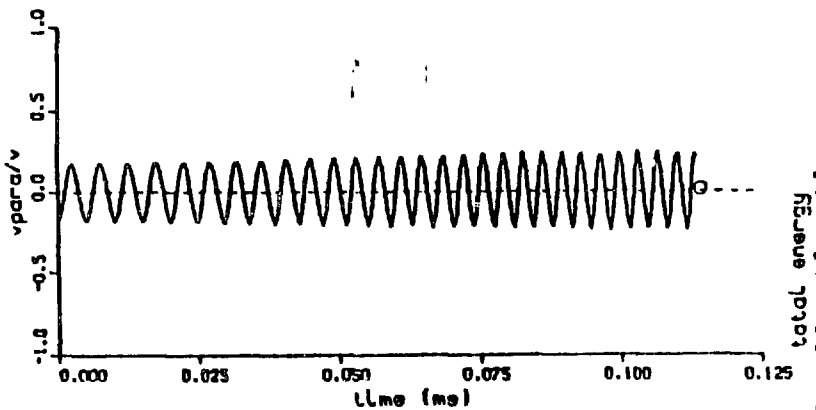
Kyoto

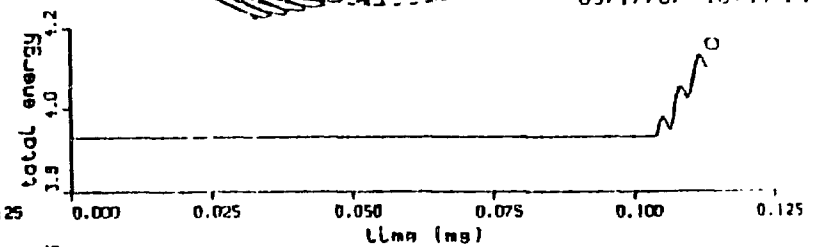


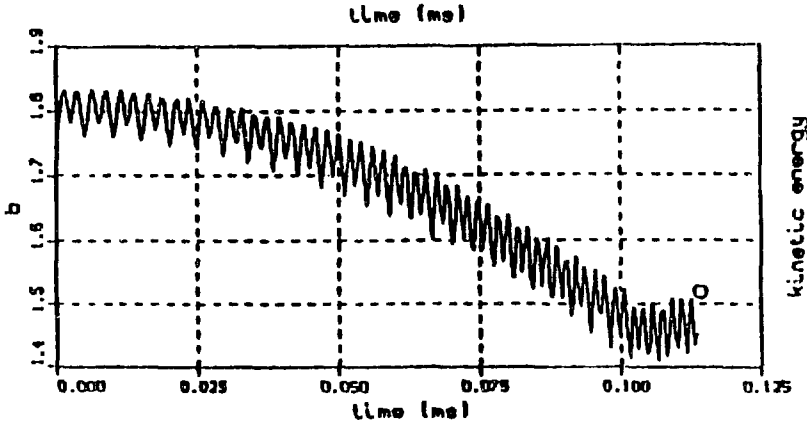
(b) $\Phi = \rho^2$ (kV)

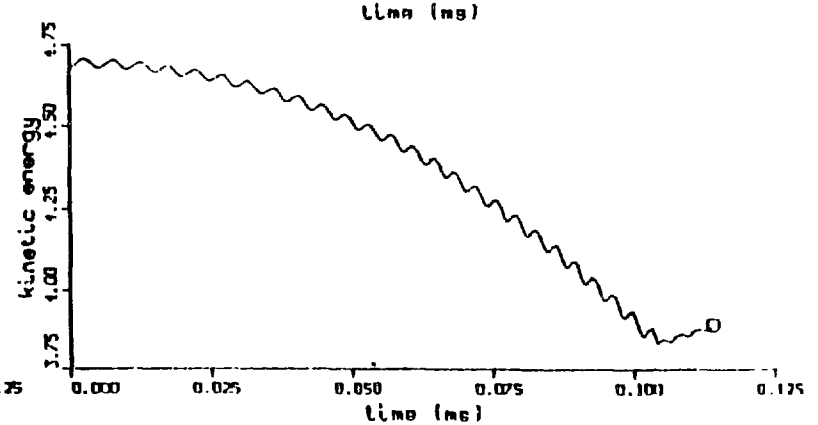
ORNL











- Thermalization at birth positions for Case I

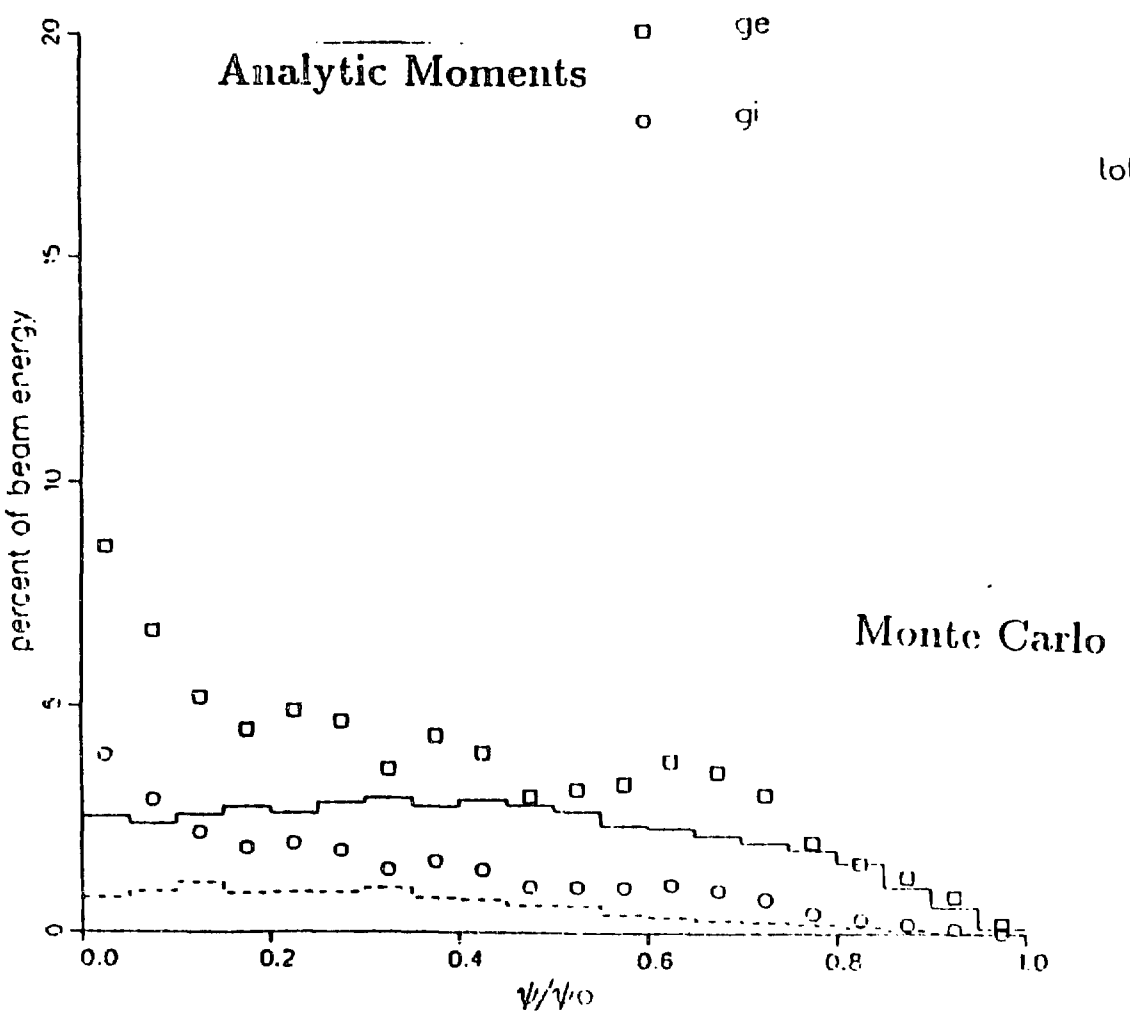
	Number of Fast Ions	Energy to Electrons	Energy
HELIOS	1000	73.3%	26.7%
MAGCOM	512	71.8%	23.3%

- Complete thermalization process, no charge exchange, boundary at last closed flux surface

Code	Density Case	Number of Fast Ions	Energy Orbit Loss	Energy to Electrons	Energy to Ions
HELIOS	High	100	40.3%	46.8%	13.0%
MAGCOM	High	100	43.1%	44.8%	11.0%
DESORBS	High	100	42.8%	44.1%	11.4%
HELIOS	Low	512	47.4%	39.2%	13.3%
MAGCOM	Low	500	46.1%	39.9%	13.6%

Neutral Beam Energy Deposition

Heliotron-E Perpendicular Injection



High Density

MAGCOM

totals

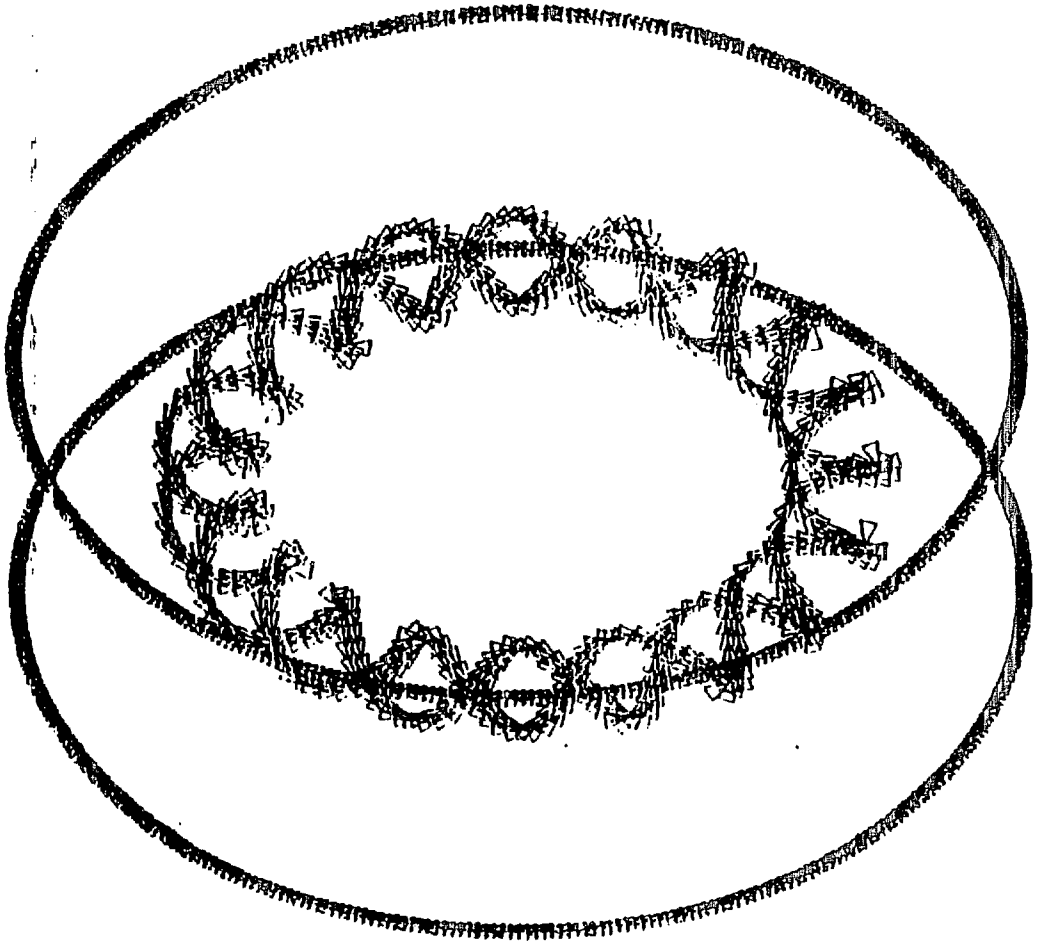
electrons	44.8
ions	11.0
therm	0.5
lost	43.1
potential	0.0
charge ex	0.0

Heliotron-E

HF coils

Two filaments, 380 segments

$I_H = 1.16$ MA (2 T)

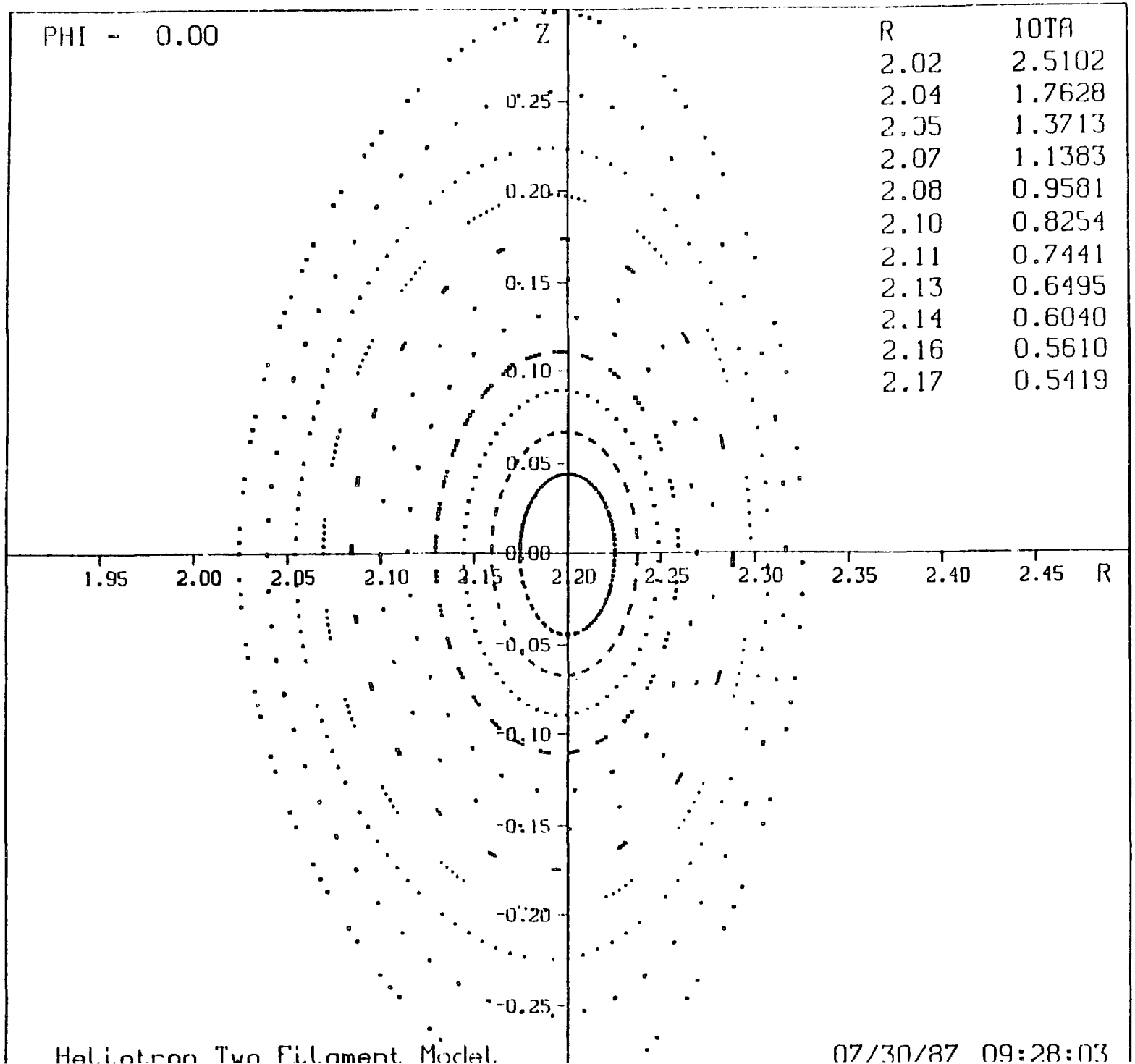


VF coils

$Z_v = \pm 1.2$ m

$r = 3.5$ m

$I_v = -0.9894 I_H$


PHI - 0.00
Z
R
IOTA
2.02 2.5102
2.04 1.7628
2.05 1.3713
2.07 1.1383
2.08 0.9581
2.10 0.8254
2.11 0.7441
2.13 0.6495
2.14 0.6040
2.16 0.5610
2.17 0.5419
0.25
0.20
0.15
0.10
0.05
0.00
-0.05
-0.10
-0.15
-0.20
-0.25
1.95
2.00
2.05
2.10
2.15
2.20
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2.30
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2.45
R
Heliotron Two Filament Model
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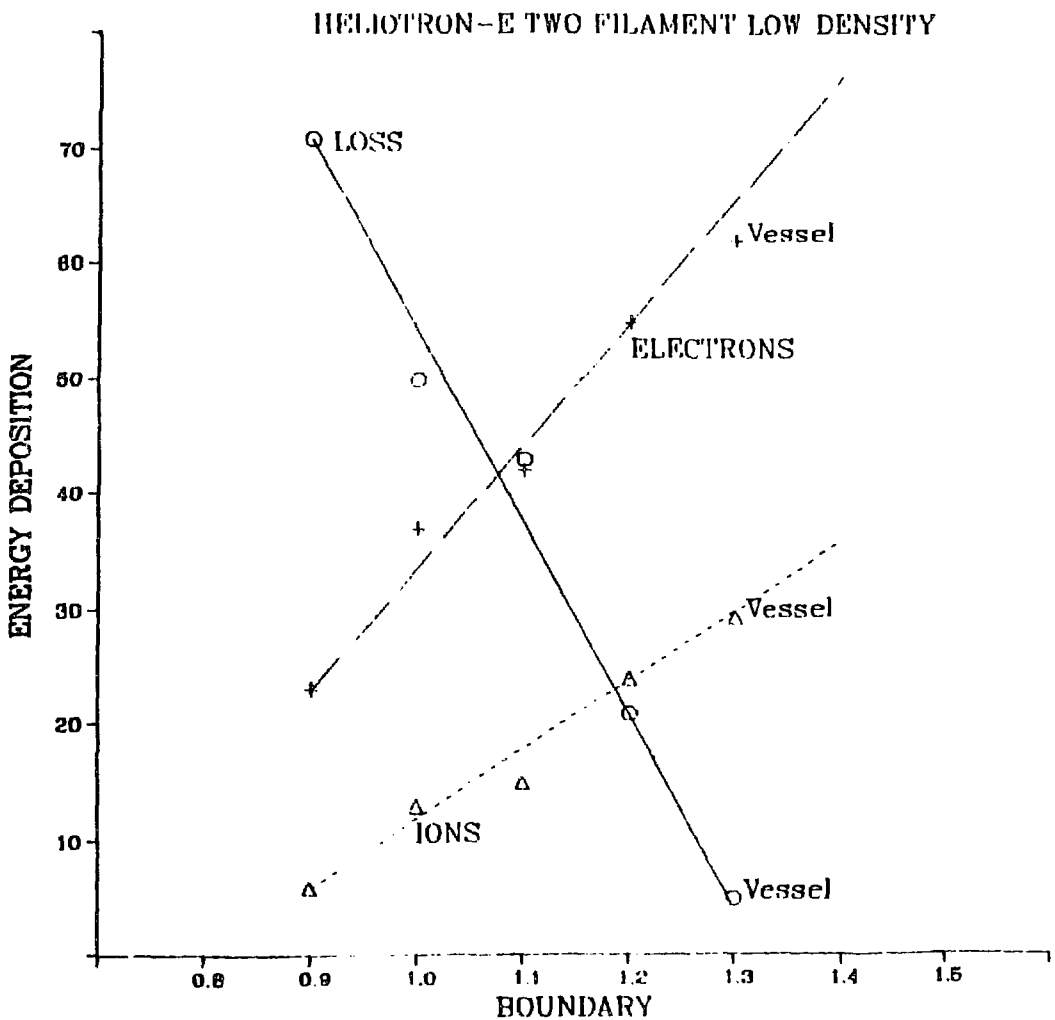
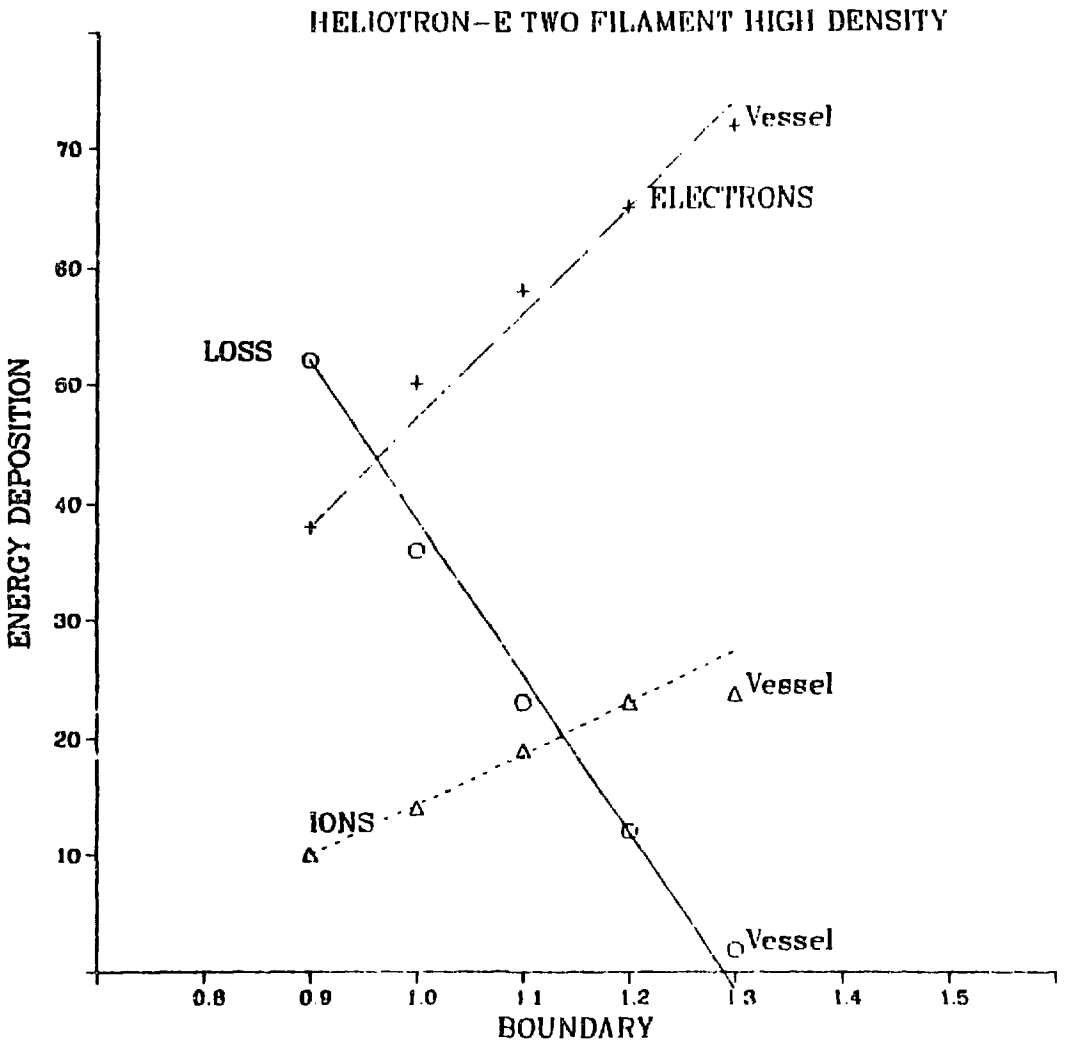
ENERGY DEPOSITION vs BOUNDARY

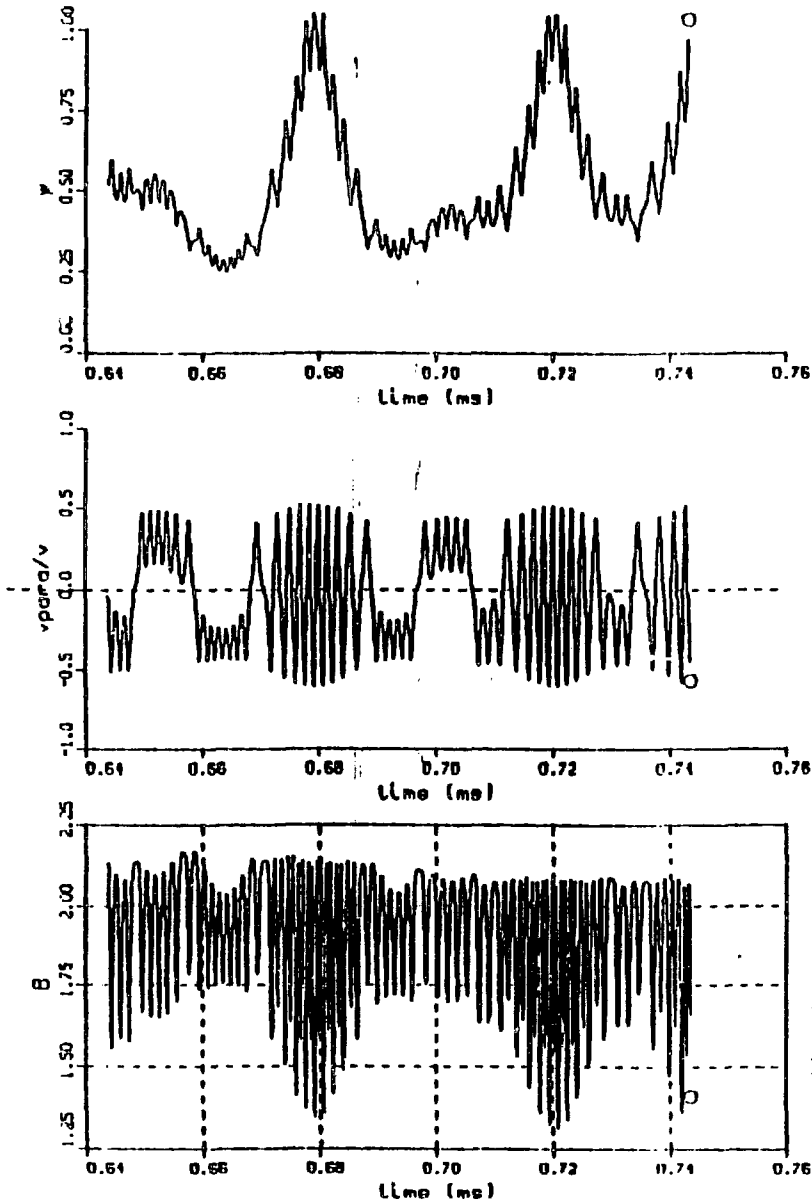


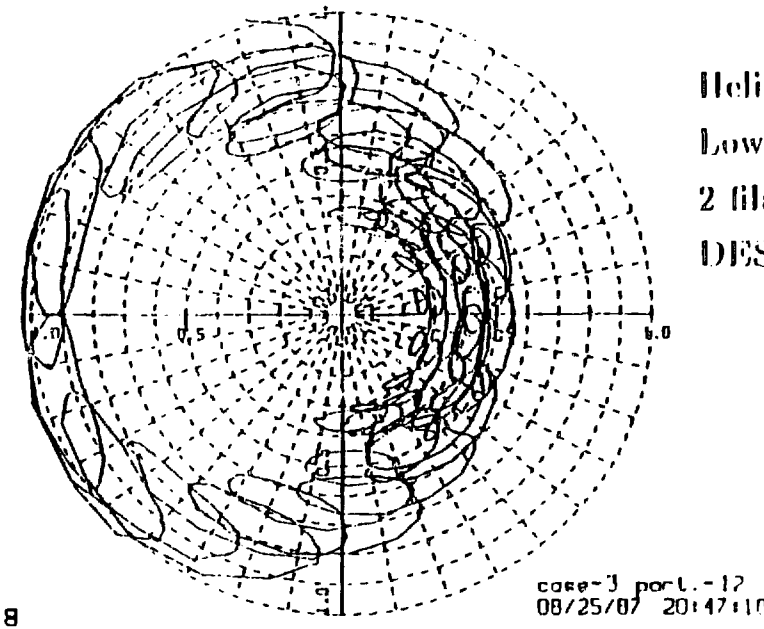
ENERGY DEPOSITION vs BOUNDARY



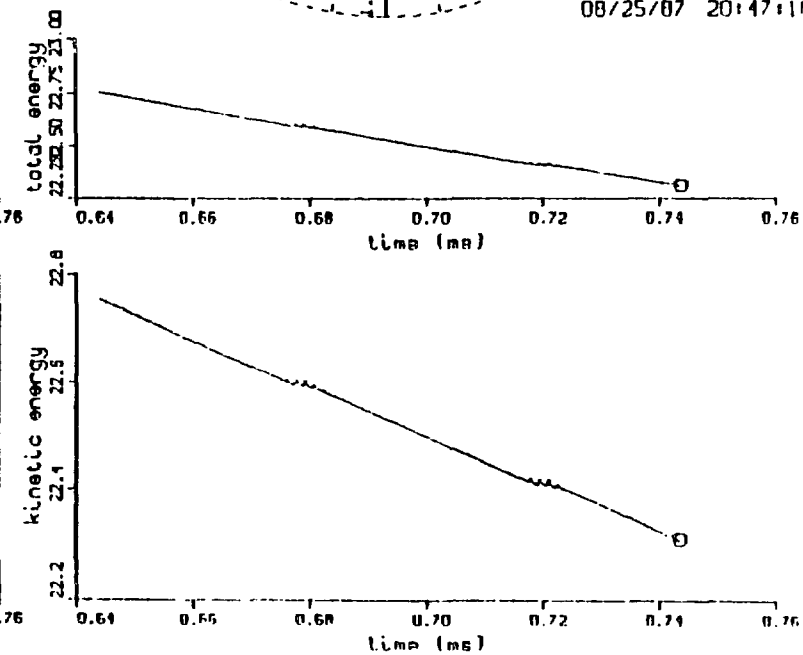
Many helical orbits leave and re-enter the outer flux surface.

Charge exchange loss





Heliotron-E
Low Density
2 filament model
DESORBS



Heliotron-E

Two Filament Model

Perpendicular Injection

Vacuum Vessel Geometry

Density Case	Energy Orbit Loss	Charge Exchange Loss	Energy to Electrons	Energy to Ions
High	0.08%	10.3%	66.1%	21.5%
Medium	0.06%	12.2%	64.5%	21.2%
Low	7.32%	21.8%	49.9%	19.2%

Profile Parameters For ATF Cases

$$T_i(\psi) = T_e(\psi) = T_o(1-\psi) + T_t$$

$$N_i(\psi) = N_e(\psi) = N_o(1-\psi) + N_t$$

$$\Phi = \Phi_o(1-\psi)$$

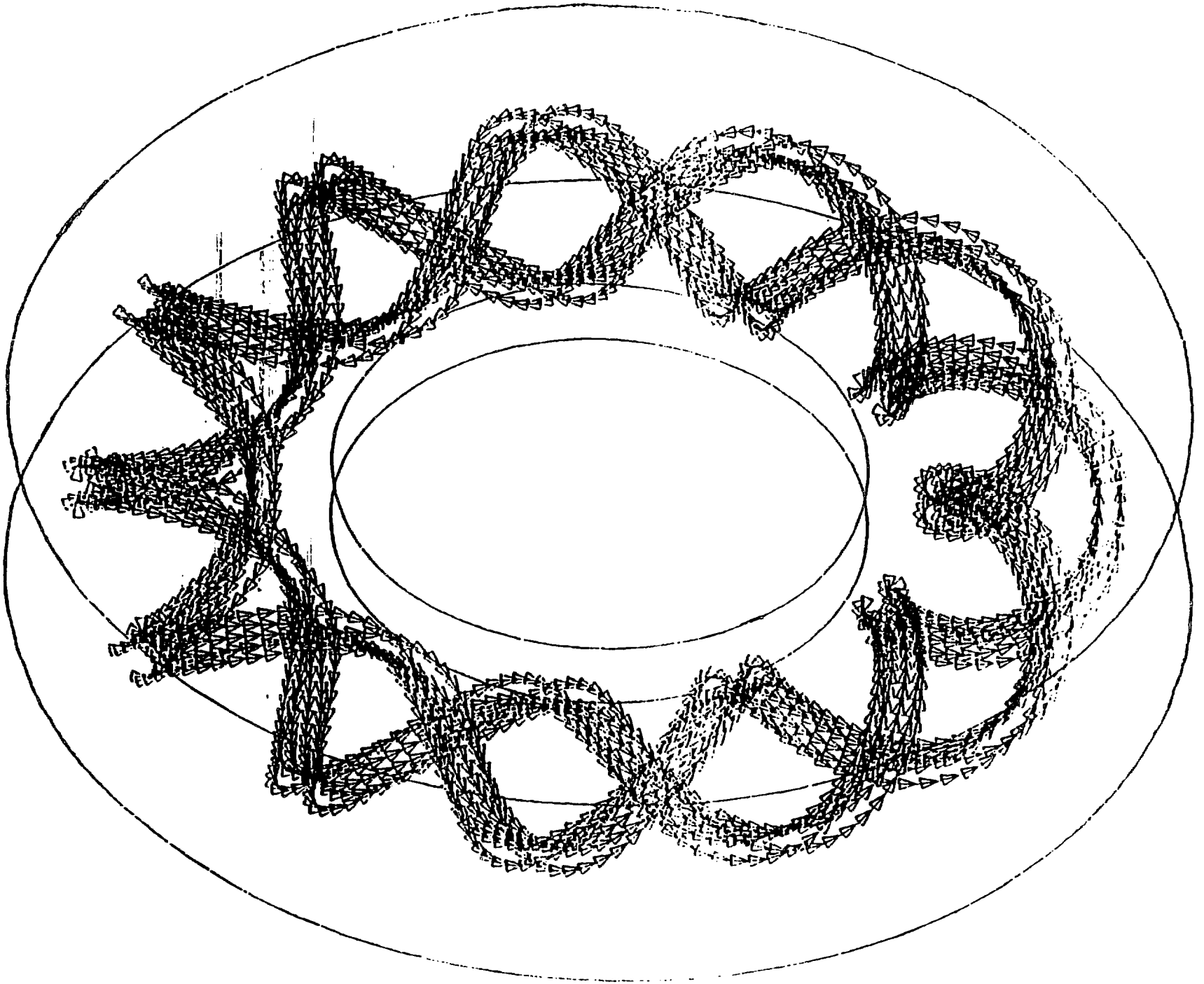
$$T_o = 1000\ eV,\ T_t = 50\ eV$$

$$N_o = 4 \times 10^{13}\mathrm{cm}^{-3},\ N_t = 1 \times 10^{12}\mathrm{cm}^{-3}$$

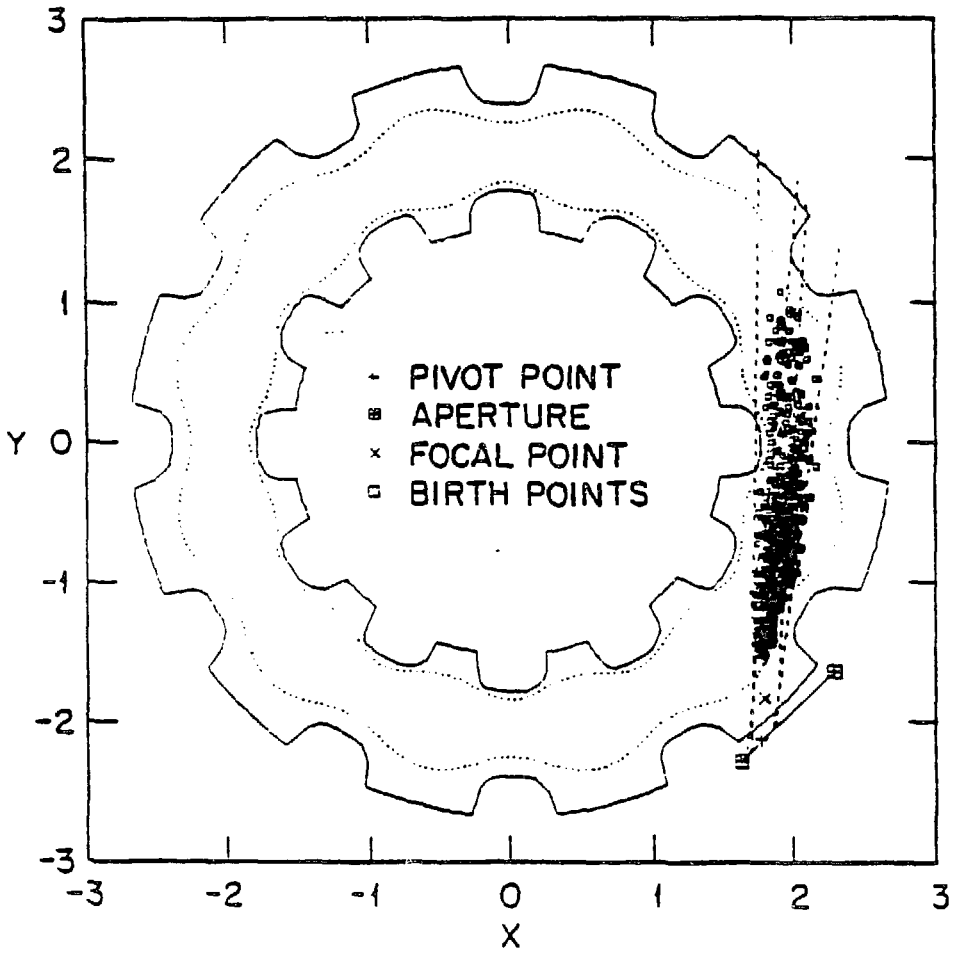
$$\Phi_o = 0 \text{ or } 2\ kV$$

$$E_o = 40\ \mathrm{keV}$$

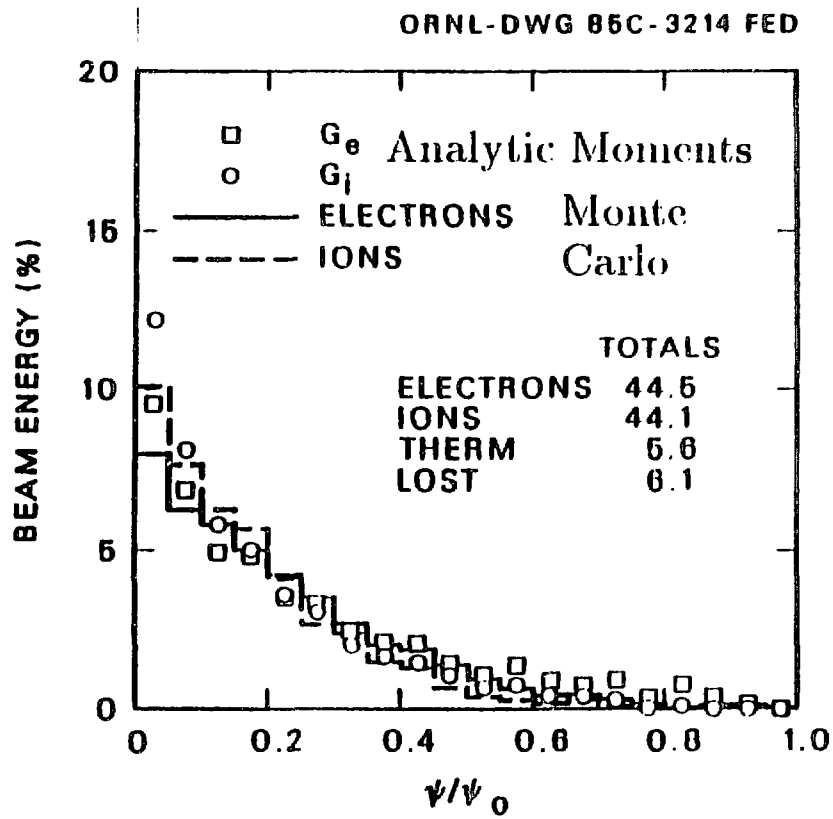
ATF Six Filament Model



ORNL-DWG85-2505 FED



ATF Tangential Injection



ATF

Six Filament Model

Vacuum Vessel Geometry

Injection	Energy Orbit Loss	Energy to Electrons	Energy to Ions
Perpendicular	64.2%	26.9%	7.53%
Tangential	7.99%	49.1%	36.5%

Conclusions

- Results are very sensitive to:
  - Magnetic field model. Also the fields in real space and flux space must correspond exactly to get agreement.
  - Outer boundary model for perpendicular injection where many helical orbits leave and re-enter the outer flux surface and sample large regions of the plasma. However, outer flux surface is adequate for ATF tangential injection.
- The number of collisions should be large enough to accurately sample at turning points where trapping and detrapping occurs.
- Acceleration of slowing down process should not be used for perpendicular injection which produces large numbers of helical orbits. Acceleration can be used for ATF tangential injection which produces mostly passing particles.
- Perpendicular injection into Heliotron-E for our "best" model produces a small orbit loss while for ATF the orbit loss is ~65%.
- The analytic moments method can be used for ATF tangential injection but not for any perpendicular injection cases.