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W. H. Gray and J. E. Akin

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FINITE ELEMENT STRESS ANALYSIS OF ORTHOTROPIC SOLENOIDS*

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Abstract

The mechanical behavior of superconducting magnets deviates from isotropy due to their construction techniques, which involve the layering of superconductor, insulation, and sometimes structural reinforcement within the windings. This paper describes a finite element stress analysis which has been extended to consider the effects of orthotropic material properties, as well as differential thermal contraction and spatially varying magnetic body forces. The procedure is applicable to all arbitrarily shaped axisymmetric magnets. A comparison between the finite element stress analysis and an analytical solution for a rotationally transversely isotropic solenoid is presented. Good agreement is obtained within the limits of the analytical solution.

Introduction

The design of high field solenoid magnets is a necessary ingredient in the technology of fusion reactors and high energy physics. The stress analysis of such magnets is often based on simplified analytical models.[1-3] While such models are very useful in preliminary designs, they have some important disadvantages. The most significant shortcoming of these formulations is the fact that the axial loads on the conductors resulting from the radial component of the magnetic flux density, $\vec{B}$, are neglected. For some geometries, the stresses associated with these loads can be significant. A second disadvantage is the difficulty of considering the effects of structural supports.

This paper presents a general analysis based on a finite element formulation which utilizes isoparametric quadrilateral elements. Since a standard finite element procedure automatically allows for arbitrarily shaped nonhomogeneous materials, it is possible to consider the effects of the conductor, insulation, and reinforcement distribution throughout the magnet. In addition, the above materials can have orthotropic constitutive relationships.

The following sections of this report present our finite element formulation to this problem, as well as a comparison with an analytical solution presented in Ref. 3.

Finite Element Formulation

The equilibrium equation, resulting from the principle of virtual work as applied to structural analysis, relates the nodal displacements, {u}, the nodal forces, {F}, and the structural stiffness, [K], according to the relationship

$$[K]\{u\} = \{F\}. \qquad (1)$$

Both [K] and {F} are obtained by assembling[4] the corresponding element contributions, $[K^e]$ and $\{F^e\}$.

Within an element the displacement field, $\{\delta\}$, is approximated by using the values of the nodal displacement, $\{u^e\}$, and an interpolation polynomial called a shape function, $[N^e]$:

$$\{\delta\} = [N^e]\{u^e\}. \qquad (2)$$

A strain-displacement matrix, $[B^e]$, is formulated by differentiating Eq. (2) with respect to the coordinate axes,

$$\{\varepsilon\} = [B^e]\{u^e\}. \qquad (3)$$

The stress is the matrix product of the material constitutive relationship and Eq. (3),

$$\{\sigma\} = [D^e]\{\varepsilon\} = [D^e][B^e]\{u^e\}. \qquad (4)$$

The stiffness matrix is written in matrix form as

$$[K^e] = \int_{V^e} [B^e]^T[D^e][B^e]\, dV. \qquad (5)$$

In axisymmetric structural analyses, compatibility conditions limit the number of nonzero stresses and strains to four. They are

$$\{\sigma\} = \begin{Bmatrix} \sigma_{RR} \\ \sigma_{ZZ} \\ \sigma_{RZ} \\ \sigma_{\theta\theta} \end{Bmatrix} = [D] \begin{Bmatrix} \varepsilon_{RR} \\ \varepsilon_{ZZ} \\ \varepsilon_{RZ} \\ \varepsilon_{\theta\theta} \end{Bmatrix}. \qquad (6)$$

The coordinate system in Fig. 1 shows the directions in which these stresses act. (The Z-axis is perpendicular to the plane of the diagram.) The constitutive matrix, [D], is formed from inverting the compliance matrix, [C]. In general, for an axisymmetric orthotropic solenoid, seven material constants exist and therefore the compliance matrix is[3]

$$[D] = [C]^{-1} = \begin{bmatrix} \frac{1}{E_{RR}} & \frac{-\nu_{RZ}}{E_{RR}} & 0 & \frac{-\nu_{R\theta}}{E_{RR}} \\ \frac{-\nu_{RZ}}{E_{RR}} & \frac{1}{E_{ZZ}} & 0 & \frac{-\nu_{R\theta}}{E_{ZZ}} \\ 0 & 0 & \frac{1}{G_{RZ}} & 0 \\ \frac{-\nu_{R\theta}}{E_{RR}} & \frac{-\nu_{Z\theta}}{E_{ZZ}} & 0 & \frac{1}{E_{\theta\theta}} \end{bmatrix}, \qquad (7)$$

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where E_{RR}, E_{ZZ}, and $E_{\theta\theta}$ are the Young's moduli of the three independent directions; G_{RZ} is the shear modulus relating the R and Z material planes; and ν_{RZ}, $\nu_{Z\theta}$, and $\nu_{R\theta}$ are the three independent Poisson's ratios. For the special case of rotational transverse isotropy, only five constants exist independently, because

$$\nu_{R\theta} = \nu_{Z\theta} \tag{8a}$$

and

$$E_{RR} = E_{ZZ}. \tag{8b}$$

Rotational transverse isotropy is representative of a wide variety of layer- or pancake-wound magnets.

Generally, Eq. (5) is evaluated by numerical integration such that

$$[K^e] = 2\pi \sum_{i=1}^{g} W_i [B_i^e]^T [D_i^e][B_i^e] R_i |CJ_i| , \tag{9}$$

where i denotes a Gaussian integration point and W_i is the corresponding Gaussian weighing function. There are g Gaussian integration points. The relationship between the element and the structural coordinates is defined by the coordinate transformation Jacobian, [CJ], and its determinate |CJ| is used in the above equation.

Finite Element Magnetic Load Matrix

The contribution to the total matrix due to body forces acting on an element is

$$\{F^e\} = \int_{V^e} [N^e]^T \{f\}\, dV, \tag{10}$$

where {f} is the array containing the components of the body force per unit volume and $[N^e]$ is the interpolation function matrix defining the displacements within the element in terms of its nodal displacements. The integral is evaluated numerically by Gaussian quadrature,

$$\{F^e\} = 2\pi \sum_{i=1}^{g} W_i [N_i^e]^T \{f_i\} R_i |CJ_i| . \tag{11}$$

The computation of f_i involves the cross product of the current density vector, $\vec{J}$, and the magnetic field vector, $\vec{B}$, or

$$\vec{f} = \vec{J} \times \vec{B}.$$

For this axisymmetric problem $\vec{J}$ has only one nonzero component,

$$\vec{J} = j_\theta \hat{i}_\theta ,$$

and the magnetic field has components in the R and Z directions,

$$\vec{B} = B_R \hat{i}_R + B_Z \hat{i}_Z .$$

Therefore,

$$\{f_i\} = \begin{Bmatrix} j_{\theta_i} B_{Z_i} \\ -j_{\theta_i} B_{R_i} \end{Bmatrix}.$$

The current is assumed to be constant within a typical element.

Application

As a typical example, consider the solenoid whose characteristics are listed in Table 1.

Table 1

Inner radius	0.1 m
Outer radius	1.0 m
Half height	1.0 m
Current density	10,000 A/cm²
E_θ	121 GPa
E_R	60.5 GPa
Poisson's ratio	0.33

Stresses and displacements were determined by two procedures for the purpose of comparison. The first method is based upon the theory presented in Ref. 3. This method assumes B_Z to be a linear function of radius. The second method involves using a computer program based upon the method described in the previous section. For this example, a uniform mesh of 252 nodes and 220 elements was utilized. This finite element computer code is coupled directly to a general-purpose solenoid field calculation code, and therefore the spatial variation of the magnetic field is used directly in the calculation.

Figure 2 presents a plot of the nondimensional hoop stress, σ_θ/jB_iR_i, vs the nondimensional radial position, $\gamma(R/R_i)$, at the center plane of the solenoid. The subscript i refers to values at the inner radial position. The solid line represents the solution found in Ref. 3 and the circles are values of stress at the Gauss points obtained from this analysis. Excellent agreement is shown.

Figure 3 shows a plot of nondimensional radial stress, σ_R/jB_iR_i, vs the nondimensional position, γ, at the center plane of the solenoid. As in Fig. 2, the solid line represents the analytical solution[3] and the circles are values of stress at the Gauss points of the finite element model. Here the agreement between the two solutions is not as good; however, it should be noted that if the Gauss point stresses are taken in pairs and averaged, the resulting values agree quite well with the analytical solution.

Conclusions

A finite stress analysis has been extended to consider the effects of orthotropic material properties and spatially varying magnetic body forces. The procedure is applicable to all arbitrarily shaped axisymmetric magnets. Comparison of this method with analytical theory yields good agreement within the limits of application of the analytical theory.

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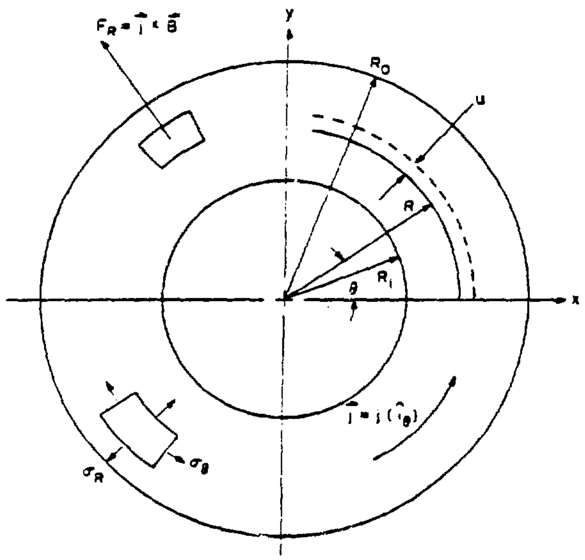

SOLENOID COORDINATE SYSTEM

Fig. 1. A solenoid coordinate system. The current is represented by $\vec{J}$, the force by F_R, the deflection by u, and the stress by σ_R and σ_θ, the radial and hoop components, respectively.

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Nondimensional Hoop Stress

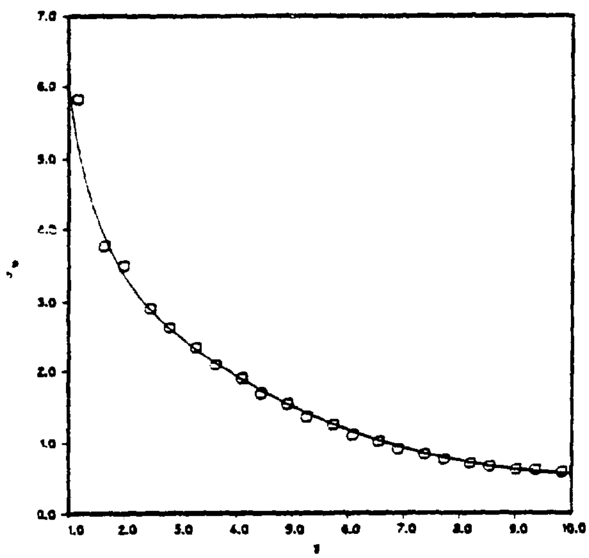

Fig. 2. A graph of the predicted nondimensional hoop stress for a solenoid of large radial build. The solid line represents a solution obtained from Ref. 3, and the circles represent data points obtained from the Gaussian integration points using the analysis presented in this report.

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Nondimensional Radial Stress

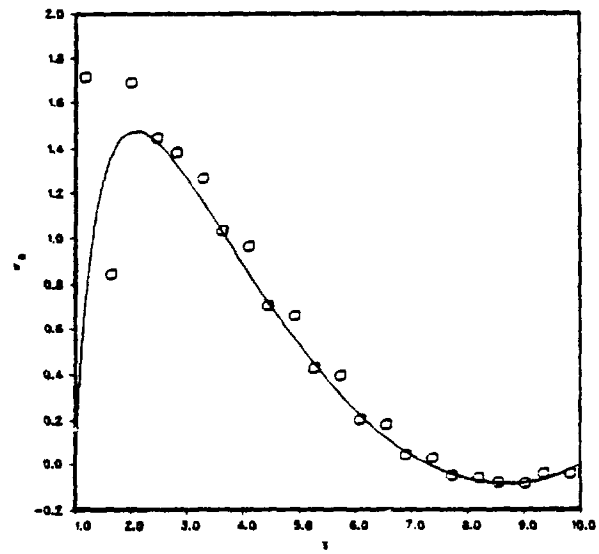

Fig. 3. A graph of the predicted nondimensional radial stress for a solenoid of large radial build. The solid line represents a solution obtained from Ref. 3, and the circles represent data points obtained from the Gaussian integration points using the analysis presented in this report. No attempt was made to smooth the stress predictions obtained from the finite element method even though there are several methods available. For example, a simple smoothing technique involves averaging two adjacent Gaussian integration points to obtain a centroidal element average.