Proceedings, XIth International Conference on
High Energy Accelerators, Geneva, Switzerland,
July, 1980

BNL-28006

CONF-800740--5

RESONANT IMPEDANCE OF BELLOWS ABOVE CUTOFF*

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ABSTRACT

The perturbation method of Chatard-Moulin and Papiernik is used to calculate the longitudinal and transverse impedances, $Z(\omega)$ and $Z_\perp(\omega)$, of a bellows. The bellows shape is defined by its radius $a(z) = a\,(1 + \varepsilon s(z))$, where a is the mean radius, ε a small parameter, and $s(z)$ describes the convolution of the bellows. We consider a finite wall conductivity and determine the resonant contribution to the impedance above the cutoff frequency of the unperturbed chamber, obtaining analytic approximations to the resonant frequencies, quality factors, and shunt impedances. The relation $Z_\perp(\omega) = (2c/a^2)Z(\omega)/\omega$, of course, does not hold as an identity, but it is found to be a useful relation for the shunt impedances, holding exactly for one family of transverse modes and providing an upper bound on the shunt impedances of the second set of transverse modes.

MASTER

INTRODUCTION

Following Chatard-Moulin and Papiernik[1], we consider a vacuum chamber with circular cross-section and radius varying with the axial coordinate z as

$$a(z) = a(1+\varepsilon s(z)), \text{ where } s(z) = \sum_{p=-\infty}^{\infty} C_p\, e^{2\pi i p z/L} . \qquad [1]$$

The perturbation parameter is $\varepsilon \ll 1$, the circumference of the ring is L, and the perturbation of the wall is taken to have zero mean, i.e. $C_0 = 0$. The longitudinal impedance $Z(\omega)$ is determined from the linear response to a current density

$$J_z = \frac{I(\omega)}{\pi\rho^2}\, \theta(\rho - r)\, e^{-i\omega\tau}, \quad \text{with } \tau = t - \frac{z}{v} . \qquad [2]$$

The azimuthal curvature of the ring is neglected, and we employ the usual cylindrical coordinates (r,ϕ,z). We denote the radius of the electron beam by ρ. The phase velocity is v, the angular frequency ω, and $\theta(x)$ vanishes for $x<0$ and has value unity for $x>0$. The current density will generate an axial electric field $E_z(r,z)e^{-i\omega\tau}$, where $E_z(r,z+L) = E_z(r,z)$, and the longitudinal impedance is defined by[2]:

$$\int_0^L dz\, \frac{1}{\pi\rho^2} \int_0^\rho 2\pi r dr\, E_z(r,z) = -Z(\omega)I(\omega) . \qquad [3]$$

Carrying out the perturbation calculation described in Ref. 1, one finds $Z(\omega) = Z^{(0)}(\omega) + \varepsilon^2 Z^{(2)}(\omega) + \ldots$, where $Z^{(0)}(\omega)$ is the impedance[2,3] of a smooth chamber of constant radius a, and the term linear in ε vanishes, because the integral from $z = 0$ to L of $E_z^{(1)}$ is zero.

A clear discussion of the definition of the transverse impedance $Z_\perp(\omega)$ and its calculation for a smooth cylindrical chamber of constant radius is given by Cotter[4].

* Research supported by the U.S. Department of Energy.

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He considers a coasting beam of uniform current density $J_z = I_o/\pi b^2$, for $r<b$, and assumes a coherent vertical oscillation of the center of mass $\bar{y} = \Delta\exp(ikz-i\omega t)$. Then,

$$Z_\perp(\omega) = \frac{-i}{e\beta I_o \Delta}\int_0^L \langle F_y\rangle\, dz, \qquad [4]$$

where e is the charge, β is the ratio of the particle velocity to the speed of light, and $\langle F_y\rangle$ is the vertical force $F_y = e(E_y + Z_o H_x)$ averaged over the beam cross section. We apply the method of Ref. 1 to find $Z_\perp(\omega) = Z_\perp^{(o)}(\omega) + \varepsilon^2 Z_\perp^{(2)}(\omega)+\ldots$, where $Z_\perp^{(o)}(\omega)$ is the transverse impedance[4,5] of a smooth chamber of constant radius a.

DISCUSSION OF RESULTS

To simplify the results we confine our attention to the case of

$$p/a \ll 1, \quad \beta = v/c \approx 1, \quad \omega a/c\gamma \ll 1, \qquad [5]$$

where $\gamma = (1-\beta^2)^{-\frac{1}{2}}$. For infinite wall conductivity, $\sigma = \infty$, we obtain

$$\frac{Z^{(2)}(\omega)}{\omega} = \left(\frac{-iZ_o L}{2\pi c}\right)\sum_{p=-\infty}^{\infty} |C_p|^2 \left(\frac{2\pi p a}{L}\right)^2 \frac{I_1(u)}{uI_o(u)}, \qquad [6]$$

and

$$Z_\perp^{(2)}(\omega) = \frac{2c}{a^2}\left(\frac{-iZ_o L}{2\pi c}\right)\sum_{p=-\infty}^{\infty} |C_p|^2 \left(\frac{2\pi p a}{L}\right)^2 \left[\frac{I_1'(u)}{uI_1(u)} - \frac{I_1(u)}{u^3 I_1'(u)}\right], \qquad [7]$$

where

$$u = \Lambda a, \qquad \Lambda^2 = \left(\frac{2\pi p}{L}\right)^2 + \left(\frac{2\pi p}{L}\right)\left(\frac{2\omega}{c}\right), \qquad Z_o = 377\Omega\ . \qquad [8]$$

Note that Λ and hence $u = \Lambda a$ depend on the summation index p. The modified Bessel functions are denoted $I_m(u) = i^{-m} J_m(iu)$, and their derivatives $I_m'(u)$. The resonant frequencies of the longitudinal modes are determined by[1] ($n = 1,2,3,\ldots$):

$$u^2 = -x_{on}^2, \quad J_o(x_{on}) = 0, \qquad [9]$$

and the two familes of transverse modes are specified by

$$\text{(a) } u^2 = -x_{1n}^2, \quad J_1(x_{1n}) = 0; \quad \text{(b) } u^2 = -x_{1n}'^2, \quad J_1'(x_{1n}') = 0. \qquad [10]$$

When the chamber wall has a finite conductivity σ, the behavior of the longitudinal impedance near the resonances of Eq. (9) is

$$\frac{Z(\omega)}{\omega} \approx \frac{-iZ_o\varepsilon^2|C_p|^2 p}{\omega-\omega_{onp}+i\Gamma(\omega_{onp})}\ . \qquad [11]$$

In Eq. (11), $Z_0 = (\mu_0/\varepsilon_0)^{1/2}$ is the impedance of free space,

$$\omega_{onp} = -\left(\frac{\pi pc}{L} + \frac{Lcx_{on}^2}{4\pi pa^2}\right) , \qquad [12a]$$

and

$$\Gamma(\omega_{onp}) = \frac{L|\omega_{onp}|}{2\pi|p|a}\left(\frac{|\omega_{onp}|\varepsilon_0}{2\sigma}\right)^{1/2}\left(1 - i\,\mathrm{sgn}\,(\omega_{onp})\right) . \qquad [12b]$$

As is usually done, we assume that the dimensionless "surface resistivity", $(|\omega|\varepsilon_0/2\sigma)^{1/2}$, is small.

Near the first family of transverse resonances (10a),

$$Z_\perp(\omega) \approx \frac{-iZ_0\varepsilon^2(2c/a^2)|C_p|^2 p}{\omega - \omega_{1np} + i\Gamma(\omega_{1np})} \qquad [13a]$$

and in the vicinity of the second family (10b),

$$Z_\perp(\omega) \approx \frac{-iZ_0\varepsilon^2(2c/a^2)|C_p|^2 p}{\left(x_{1n}'^2-1\right)\left(\omega-\omega_{1np}'\right) + i\left(1+\alpha_{np}^2\right)\Gamma\left(\omega_{1np}'\right)} . \qquad [13b]$$

In Eqs. (13a,b) ω_{1np} and ω_{1np}' are obtained from the expression (12a) for ω_{onp}, by replacing x_{on} by x_{1n} and x_{1n}', respectively. Also, we have used

$$\alpha_{np}^2 = \left(x_{1n}'^2 - 1\right)\left(cx_{1n}'/a\omega_{1np}'\right)^2 . \qquad [14]$$

Comparing Eqs. (11) and (13a), it is seen that the equation[5]

$$Z_\perp(\omega) = \frac{2c}{a^2}\,\frac{Z(\omega)}{\omega} , \qquad [15]$$

provides a relation between the shunt impedances of the longitudinal modes and the first family (10a) of the transverse modes. Since α_{np}^2 is always positive, Eq. (15) yields an upper bound on the shunt impedances of the second class of transverse modes (10b).

Let us return for a moment to discuss the expression (11) for the resonant behavior of the longitudinal impedance. The shunt impedance is defined by $R_{np} = Z(\omega_{onp})$ and the quality factor by $Q_{np} = |\omega_{onp}|/2\mathrm{Re}\Gamma(\omega_{onp})$. Then introducing the resistive wall impedance Z_{Res} and an impedance Z_c characteristic of the distorted wall geometry,

$$Z_{Res}(\omega) = Z_0\left(\frac{L}{2\pi a}\right)\left(\frac{|\omega|\varepsilon_0}{2\sigma}\right)^{1/2} \quad \text{and} \quad Z_c(p) = Z_0|pC_p|c , \qquad [16]$$

we can write

$$R_{np} = \frac{Z_c^2(p)}{Z_{Res}(\omega_{onp})}, \quad Q_{np} = \frac{|p| Z_o}{2\, Z_{Res}(\omega_{onp})}, \qquad [17]$$

where we have ignored the small frequency shift due to the imaginary part of Γ. The form of Eqs. (16) and (17) is reminiscent of the results of Ruggiero[7], who applied a transmission line analysis to a somewhat different problem than that considered here. One conclusion of Ruggiero, which is exhibited by Eq. (17), is that both the shunt impedance and the quality factor are reduced when the conductivity of the chamber wall is decreased, for example, by using stainless steel rather than copper.

The resonant frequency[1] ω_{onp} of Eq. (12a) can be written

$$\omega_{onp} = -\left(\frac{\pi p a}{L x_{on}} + \frac{L x_{on}}{4\pi p a}\right) \omega_n^{co}, \qquad [18]$$

where $\omega_n^{co} = x_{on} c/a$ is the cutoff frequency of the nth propogating mode in the unperturbed chamber. From (18) we see that $|\omega_{onp}|$ is always greater than the cutoff frequency ω_n^{co}, consistent with the resonant behavior corresponding to radiated energy loss.

SQUARE WAVE DISTORTION OF WALL

We should remark that the results of the perturbation analysis considered in this paper cannot be directly applied to the case of a square wave distortion of the chamber wall, considered for example by Keil and Zotter[2] and Month and Peierls[3]. The reason is simply that the perturbation expansion rests on the smallness of $\varepsilon a \cdot ds/dz$, but when $s(z)$ is a square wave, its derivative contains delta functions so $\varepsilon a \cdot ds/dz$ is not uniformly small as required.

The lack of validity of Eq. (6) for a square wave can also be seen by considering the low frequency behavior. In the limit $\omega \to 0$, the behavior of the order ε^2 term is:

$$\frac{Z(\omega)}{-i\omega} = \frac{2 Z_o \varepsilon^2 a}{c} \sum_{p=1}^{\infty} p |C_p|^2 \frac{I_1(2\pi p a/L)}{I_o(2\pi p a/L)}. \qquad [19]$$

For a square wave $\Sigma p|C_p|^2$ diverges, and hence, so does the sum in (19). We have checked that Eq. (19) is self-consistent when the wall perturbation is smooth enough so that $\Sigma p|C_p|^2$ converges, that is the low frequency behavior is properly related to the stored energy via Poynting's theorem.

MODEL OF A BELLOWS

As a model of a <u>single</u> bellows located in a storage ring, we consider the shape function specified by

$$s(z) = \begin{cases} \sin(2\pi z/\ell), & |z| \le m\ell/2 \\ 0, & m\ell/2 \le |z| \le L/2 \end{cases}$$

and $s(z+L) = s(z)$. In the above expression ℓ is the wavelength of the bellows convolutions and m is the number of wavelengths comprising the bellows. Taking the Fourier transform of $s(z)$, we find that the coefficients C_p of Eq. (1) are maximum for $|p| \approx L/\ell$, with $|C_p| \approx m\ell/L$. Using these values in Eqs. (12a), (16) and (17) we obtain

$$|\omega_n| \approx \frac{\pi c}{\ell} + \frac{\ell c x_{on}^2}{4\pi a^2}, \quad \frac{1}{Q_n} \approx \frac{\ell}{\pi a}\left(\frac{|\omega_n|\varepsilon_o}{2\sigma}\right)^{\frac{1}{2}}, \quad \frac{R_n}{Q_n} \approx 2Z_o m^2(\ell/L)\varepsilon^2 .$$

As a numerical example consider a ring of circumference $L = 200$m, so the revolution frequency $\nu_{rev} = c/L = 1.5$ MHz. Take the unperturbed radius $a = 4$ cm, the wavelength of the convolution $\ell = 1$ cm, the perturbation parameter $\varepsilon = 0.05$, and the number of full wavelengths in the bellows $m = 10$. Assuming the bellows to be made of copper, $\sigma = 5.9\times10^7\,\Omega^{-1}m^{-1}$, we find for $n = 1$ ($x_{o1} = 2.4$):

$$\nu_1 = \omega_1/2\pi = 15\text{GHz}, \quad n_{rev} = \nu_1/\nu_{rev} = 10^4, \quad Q_1 = 1.5\times10^5, \quad R_1/Q_1 = 9.4\times10^{-3}\Omega, \quad R_1/n_{rev} = 0.14\Omega .$$

The width of the resonance line $\Gamma_1/2\pi = \nu_1/2Q = 50$ KHz, while the separation of the resonances is $\Delta\nu_1 = \frac{c}{2L}\left[1-\left(\frac{x_{o1}\ell}{2\pi a}\right)^2\right] = 670$ KHz. The frequency interval between the $n = 2$ and $n = 1$ modes is much larger, $\nu_2 - \nu_1 = 587$ MHz. Let us close by noting that the resonant frequencies and quality factors are independent of the ring circumference, but the shunt impedance is inversely proportional to it. Hence, the shunt impedance divided by n_{rev} is inversely proportional to the square of the circumference.

ACKNOWLEDGEMENTS

I wish to thank A. G. Ruggiero for stimulating discussions, C. Pellegrini for bringing the paper of Chatard-Moulin and Papiernik to my attention, and J. Galayda for helpful comments and a reminder of the importance of Poynting's theorem.

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