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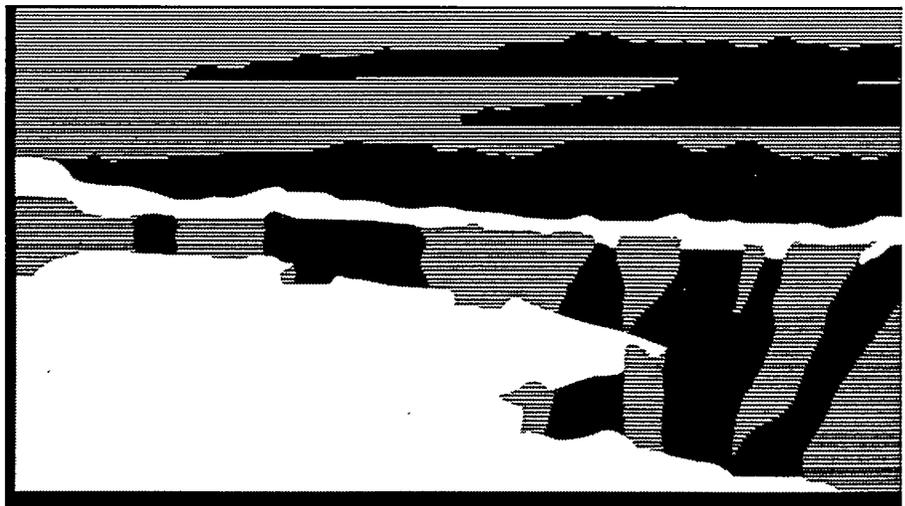
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New Approaches to Thermoelectric Cooling Effects in Magnetic Fields

A. Migliori¹, T.W. Darling¹, F. Freibert¹, S. A. Trugman¹, E. Moshopoulou¹, J.L. Sarrao²

ABSTRACT

We review thermoelectric effects in a magnetic field at a phenomenological level. Discussions of the limiting performance and problems with its computation for both Peltier and Ettingshausen coolers are presented. New principles are discussed to guide the materials scientist in the search for better Ettingshausen materials, and a brief review of the subtle measurement problems is presented.

Our intent in this paper is to provide a basic review of thermoelectric cooling in materials in which a strong magnetic field is present, and to indicate new directions in this old and extensively studied area of electronic transport in solids. The basic physical effects which describe heat transport by charge carriers in solids are the Peltier effect and the Ettingshausen effect. The Peltier effect, governing all modern thermoelectric coolers, is a thermal transport process requiring no magnetic field in which a thermal gradient is created parallel with an applied electric current. The Ettingshausen effect, a somewhat obscure but powerful refrigeration process, is a thermal transport process in which a thermal gradient is created perpendicular to an applied electric current and both these are perpendicular to an applied magnetic field. Because of the extensive literature on these processes and because of the detailed complexity of the new approaches developed at our laboratory, we will attempt only to present an accurate but simple overview. We begin with an historical perspective outlining briefly why there is some motivation to revisit this old problem.

For some period of time, there has been an

established interest in $\text{Bi}_{1-x}\text{Sb}_x$ alloys for use in electronic refrigerators. This appreciation was initiated by the experimental work of Jain [1] on the electrical properties of single crystalline $\text{Bi}_{(1-x)}\text{Sb}_x$ and inspired by the large magneto-thermoelectric figure of merit data [2] of Smith and the equally impressive thermomagnetic figure of merit data [3] of Cuff, et al. These data [2,3] have shown that the orientation of the applied magnetic field with respect to its rhombohedral crystal axes plays an important role in optimizing figure of merit values in $\text{Bi}_{(1-x)}\text{Sb}_x$ alloys. In fact the behavior of the thermopower in a transverse magnetic field for Bi [4] and $\text{Bi}_{(1-x)}\text{Sb}_x$ [5] is very different, dependent on the relative orientation of magnetic field and crystallographic axes and is closely related to the crystal symmetry and band structure of these materials. However, little has been published to rationalize physically this fact in terms of a microscopic picture. Goldsmid [6] has attempted to justify a preferred magnetic field orientation based on theoretical predictions for the infinite-magnetic-field thermomagnetic figure of merit as derived from experimentally determined physical parameters for single crystal Bi. Supported by this argument, the preferred orientation requires the electric current to flow along the trigonal direction, with the magnetic field along a bisectrix direction and the generated Ettingshausen heat flow along a binary direction. This experimental orientation has become the conventional

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orientation used in most experiments when determining the optimum operating efficiency of $\text{Bi}_{(1-x)}\text{Sb}_x$ alloys for use in an electronic refrigerator. However the apparent lack of understanding of the relationship between the band structure of Bi and its alloys with Sb, and the physics of thermoelectric and thermomagnetic effects leaves an interesting approach open, to be described below, that may provide substantially improved materials for Ettingshausen cooling.

ZERO MAGNETIC FIELD

The root of all thermoelectric effects is the small variation of the energy and momentum distribution of charge carriers caused by temperature gradients. Such variations produce, among other things, a non-zero electric field inside electric conductors - the Seebeck effect - and it is the coupling between this electric field and electric current that provides thermoelectric power generation or refrigeration (the Peltier effect). The size of the effect is dependent on the energy scales and temperature of the solid. In a degenerate metal, where only a few charge carriers near the Fermi energy ϵ_f (ϵ_f/k_b is of order 30,000K, k_b is Boltzmann's constant) are out of their ground state, the effects are small. In semiconductors and semi-metals where only a few charge carriers are present (Bi has 10^6 fewer charge carriers than Cu, and ϵ_f/k_b is $\sim 1000\text{K}$) and very few in the ground state, the effects are large.

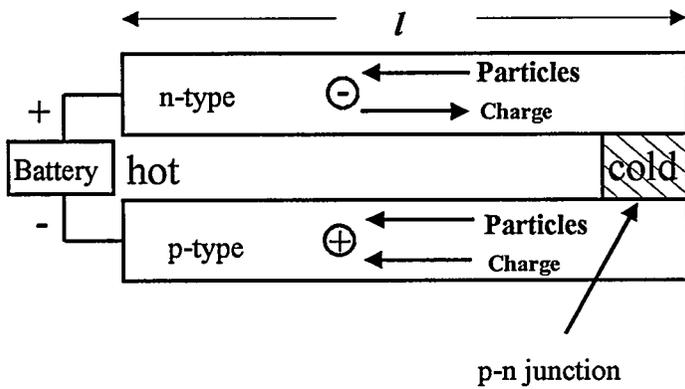


Figure 1. Shown is a simple schematic of a Peltier refrigerator consisting of a p-type and an n-type semiconductor bar joined at the cold end.

However all large thermoelectric effects, meaning those of sufficient strength to be of practical importance, arise only in solids that can be made to carry electric current via carriers (quasiparticles) of both negative (e or electron-like) and positive (h or hole-like) charge. Of

course, there are really only electrons, but holes in the electronic distribution act just like real particles with positive charge-that is they move in a direction opposite to electrons in an electric field. Because of this "reverse" motion, systems can be constructed where, although electric current circulates, "particle" current is unidirectional, and it is the particle current that carries heat. This is shown schematically in figure 1 for a Peltier cooler that consists of two legs, one of a p-type semiconductor (holes are the majority carrier) and one of an n-type semiconductor (electrons are the majority carrier). The details of the operation of this device have been reviewed extensively by Harman and Honig [7], but several key points, relevant to what follows, should be mentioned. One is that any holes in the n-type bar or electrons in the p-type bar will carry heat toward the cold end, degrading performance. Another is that ordinary thermal conduction is doing its best to carry heat from hot to cold. A pair of simple definitions [8] serve to quantify this as follows:

$$\bar{j} = \sigma(\bar{E} - S\bar{\nabla}T) \quad (1)$$

$$\bar{j}_q = \sigma ST\bar{E} + (\sigma S^2T - K)\bar{\nabla}T \quad (2)$$

where \bar{j}_q is the heat flux, \bar{j} is the electrical current, σ is the electrical conductivity tensor, K the thermal conductivity tensor, \bar{E} the electric field, S the thermopower tensor and T temperature. The thermal conductivity has two important components, K_p the phonon thermal conductivity and K_e the electronic thermal conductivity. The third equation of importance to a phenomenological understanding is

$$\bar{\nabla} \cdot \bar{j}_q - \bar{E} \cdot \bar{j} = 0 \quad (3)$$

which is a continuity equation for a time-independent temperature distribution inside the Peltier device, expressing the necessity for a balance between the difference between energy (heat) flowing into and out of a small volume (first term) and the electrical energy consumed in that volume (second term). Substituting (1) into (2) we obtain

$$\bar{j}_q = ST\bar{j} - K\bar{\nabla}T \quad (4)$$

which has the curious property that the total heat flow is the simple sum of the heat flow from ordinary thermal conduction and the heat pumped by the Peltier effect. For a one-dimensional system, (j is independent of position) and using (1), (3) and (4) we obtain

$$Tj \frac{dS}{dT} \frac{dT}{dx} - K \frac{d^2T}{dx^2} - \frac{j^2}{\sigma} = 0 \quad (5)$$

from which all the important quantities can be calculated. Note first that the second derivative of temperature with position is a critical piece, meaning

that no matter how short the Peltier device is, if it is operating in a useful region, we cannot neglect the variation of the temperature gradient. Second, in the usual treatments [6] the first term is neglected. This term is basically the Thomson heat, and for the cryogenic refrigerators our work is focused on where T varies substantially, it can be of comparable size to the other terms and should not be neglected. However, to get a feel for what goes on, let's neglect it anyway, as well as the temperature dependencies of all the other material properties and solve (5) using (1) and (4) with the hot end ($x=l$) anchored at T_h , and the cold end ($x=0$) at T_c . We also assume that the n-type and p-type semiconductors have the same material properties (true for Bi_2Te_3 , the most popular material, but difficult in general to achieve) except for the sign of S and that the total area of both materials is unity (it can be shown that nothing clever can be done by shaping the bars of semiconductor to improve things) [9].

Keeping in mind that Peltier refrigerators, though capable of coefficients of performance (COP) greater than unity, are never competitive on an energy usage basis with conventional refrigerators, we shall emphasize maximum heat pumped from the cold end, NOT maximum efficiency or COP . We find by twice integrating (5) that

$$T_h - T_{c0} = \frac{\sigma S^2}{K} T_c^2 \quad (6)$$

$$j_0 = \frac{\sigma S T_c}{l} \quad (7)$$

$$j_{q0} = S T_c j_0 - K \frac{T_h - T_c}{l} - \frac{1}{2} \frac{j_0^2 l}{\sigma} = \frac{K}{l} (T_c - T_{c0}) \quad (8)$$

$$W_0 = \frac{j_0^2 l}{\sigma} + S j_0 (T_h - T_c) = \frac{\sigma S^2 T_c T_h}{l} \quad (9)$$

and, finally

$$COP_0 = \frac{W_0}{j_{q0}} = \frac{1}{2} \frac{T_c - T_{c0}}{T_h - T_{c0}} \frac{T_c}{T_h} \quad (10)$$

where T_{c0} is the minimum possible temperature, j_0 is the current for maximum heat pumped j_{q0} (which, of course, is valid when $T_c = T_{c0}$ as well), W_0 is the electrical power per unit area required when maximum heat is pumped and COP_0 is the COP for maximum heat pumped. Note that if T_c is close to T_h , the best COP_0 is less than 1/2.

It is seen that everything depends more or less on T_{c0} which is a function of the dimensionless quantity, $\sigma S^2 T / K$, historically called ZT , a particularly unhelpful

appellation. Much more revealing is to note that $K / \sigma T_c$ has units of thermopower squared $(\mu\text{V}/\text{K})^2$ so that a critical thermopower S_0 can be defined such that

$$S_0 = \sqrt{\frac{K}{\sigma T_c}} \quad (11)$$

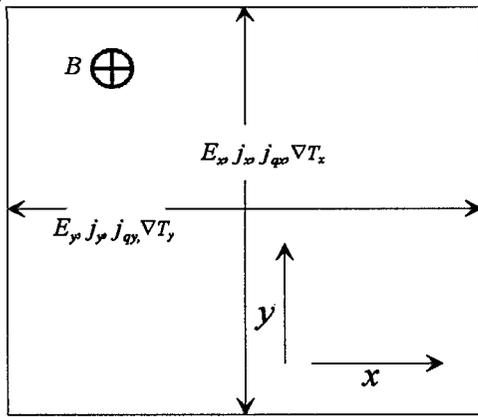
which expresses a value for the thermopower that must be achieved for any reasonable thermoelectric material. If the material were such that the phonon thermal conductivity could be neglected, the quantity inside the square root is the Lorenz number $L = (155 \mu\text{V}/\text{K})^2$, a constant that comes from the Wiedemann-Franz law, a very general relationship between the electronic thermal conductivity and the electrical conductivity [8]. Thus any material with a thermopower less than about $155 \mu\text{V}/\text{K}$ is not going to be a good candidate for a Peltier refrigerator, especially if K_p is not negligible. S_0 , computed from measured properties, is then a simple-to-measure break point for material searches because

$$\frac{T_h}{T_{c0}} = 1 + \frac{1}{2} \left(\frac{S}{S_0} \right)^2 \quad (12)$$

It can be seen that the search for good thermoelectric cooling materials is a battle to make S greater than S_0 ($S/S_0 = 1$ for very good materials) and minimize K_p (not much can be done about K_e because one is not going to beat the Wiedemann-Franz law substantially in a system with enough carriers to pump heat). Note that (12) neglects the variation of thermopower with temperature, as stated earlier. Some numerical studies have shown discrepancies greater than 10% [6] for (12), an error comparable to the target of many research programs in thermoelectrics.

EFFECTS IN A MAGNETIC FIELD

We are, however, not working on Peltier coolers, but on the more complex Ettingshausen cooler. The complexities come in because the material properties must be treated as tensors and the analysis is more difficult. Figure 2 exemplifies this. In figure 3 we show a schematic of what the charge carriers must do. Referring to figure 3 it can be seen that the idea is to use a single material in which both electrons and holes are present. An electric field in the y-direction makes the holes travel roughly upward and the electrons roughly downward. The magnetic field B , however, deflects both carriers to the positive x-direction, and it is this effect that, exactly analogous to Peltier cooler, pumps heat. Notice that just as in a Peltier cooler, the electric current carried by holes is opposite in direction



Ettingshausen Effect:
 input a voltage V_x
 set $\Delta T_x = 0$
 find ΔT_y

Nernst Effect:
 input a ΔT_x
 set $j_x j_y = 0$
 find E_y

Figure 2. Shown is a bar of material in a magnetic field, and the various transport quantities that contribute to thermoelectric cooling (the Ettingshausen effect) and thermoelectric power generation (the Nernst effect).

to the current carried by electrons but there is a net flow of particles toward the hot end.

Our goal here is to provide guidance in the selection of optimum Ettingshausen materials. It is clear that one quality of importance is to maximize the total number of particles moving toward the hot end. If the number and x-component velocity of electrons did not equal the same quantity for holes, then very quickly, charge would pile up at the hot end, creating a voltage (the Hall voltage) that would reduce (or stop if only one carrier were present) the flow of the majority carriers, degrading refrigeration. Therefore, the ideal Ettingshausen material must have equal numbers of electrons and holes with equal mobility. This is called 'e-h symmetry'. Such a system has ZERO thermopower and would produce no temperature drop if used in a Peltier cooler. The key guide for materials searches is, then, the electronic band structure. In what follows, we shall consider systems with parabolic bands in which the energy ϵ of the electron or hole is $p^2/(2m_i)$ where p is the momentum and m_i is the effective mass of the i^{th} carrier.

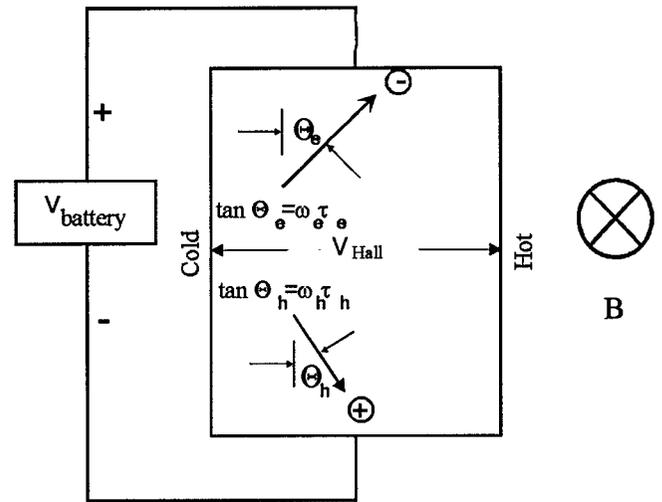


Figure 3. Shown is an Ettingshausen refrigerator. The angles will be discussed below.

This is the same dispersion curve as a free particle. However, the electron band, a parabola facing up (it would hold water) and the hole band, a parabola facing down, can overlap (the top of the hole parabola, or hole band edge, is above the bottom of the electron parabola or electron band edge) or not. If the bands overlap at the Fermi energy ϵ_f the material is a semi-metal. If they do not overlap, the material is a semiconductor. In both cases, ϵ_f lies between the band edges, its exact location dependent on the band masses m_i and temperature. The number of electrons available for conduction is the total number below ϵ_f and for holes it is the number above ϵ_f . In a semi-metal like Bi, ϵ_f crosses the two bands at a level such that there are exactly the same number of holes as electrons, but because the curvature of the parabolas (which determines the effective mass) is not the same, the masses are different, and so are the mobilities.

To get a feel for Ettingshausen cooling, let's proceed under the assumption that we have two bands, e and h, with the same masses and with the same mean free path—that is, perfect e-h symmetry. Equations (1)-(3) are still valid except that there is no ordinary thermopower and no Hall effect. This is expressed as follows

$$\sigma = \begin{pmatrix} \sigma_{11} & 0 \\ 0 & \sigma_{11} \end{pmatrix} \text{ and } S = \begin{pmatrix} 0 & S_{12} \\ -S_{12} & 0 \end{pmatrix} \quad (13)$$

where σ_{11} , S_{12} are the non-zero components of the

resistivity and thermopower tensors. We shall assume K to be a scalar. Noting that $j_x = E_x = dT/dy = 0$, we can obtain two more equations

$$j_y = \sigma_{11} \left(E_y + S_{12} \frac{dT}{dx} \right)$$

and

$$j_{qx} = \sigma_{11} S_{12} T E_y - (K - \sigma_{11} S_{12}^2 T) \frac{dT}{dx}$$

except that the current j is not independent of position, while E_y is. Thus (14) is not of the same form as (5), as is occasionally stated [6]. Another key point is that now $\sigma_{11} S_{12}^2 T / K$ cannot exceed unity because if it did, heat would flow from cold to hot with no energy input to the system. Thus the boundary of unity on a set of parameters not obviously constrained by physics is another key difference between the two refrigeration processes. The resulting differential equation for an Ettingshausen refrigerator, and its solution for temperature-independent parameters are

$$-E_y^2 \sigma_{11} = E_y T \frac{d(\sigma_{11} S_{12})}{dx} + \frac{d}{dx} \left[\left(K - \sigma_{11} S_{12}^2 T \right) \frac{dT}{dx} \right] \quad (15)$$

$\int \left(K - \sigma_{11} S_{12}^2 T \right) dT = \int \left(-xE_y^2 \sigma_{11} + const. \right) dx$
 where we dropped all terms arising from departures from e-h symmetry. The integral solution to the differential equation is of particular note because on the left is an integral with respect to T , not x , over materials properties, mitigating the huge problem of position integrals over material properties when T is not known.

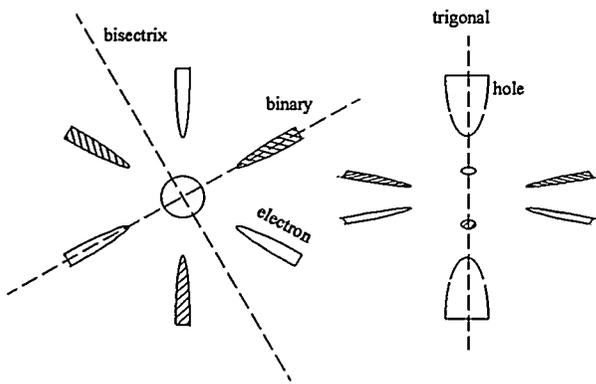


Figure 4. Shown is the Fermi surface of Bi. Under some doping conditions, it may be possible to eliminate the hole in the middle, so that at temperatures of order 100K, both electrons and holes would be where only the electron pockets are now.

Even though the conductivity appears on the right under the integral, this is only one material property to be dealt with. Unlike the Peltier case, the use of a temperature-independent thermal conductivity is justified because the magnetic field greatly reduces the electronic thermal conductivity which is usually proportional to temperature, leaving only the phonon term.

Because E and j are perpendicular to the heat flows, two or more Ettingshausen coolers (EC) in series can make electrical contact between the hot end of the smaller stage and the cold end of the larger stage, making it possible to produce a sequence of staged coolers simply by machining the correct shape from the bar of single material. Therefore, in a properly

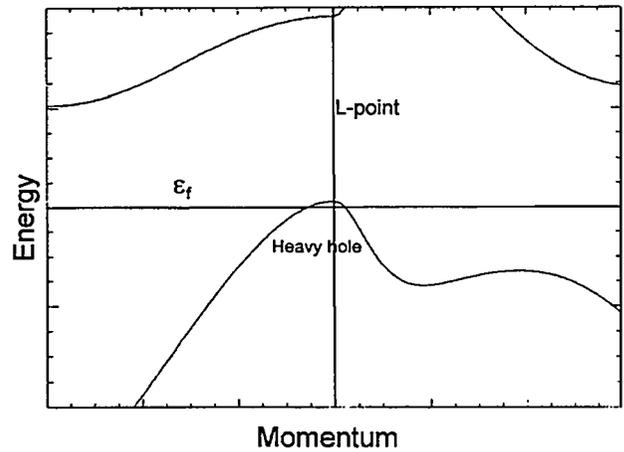
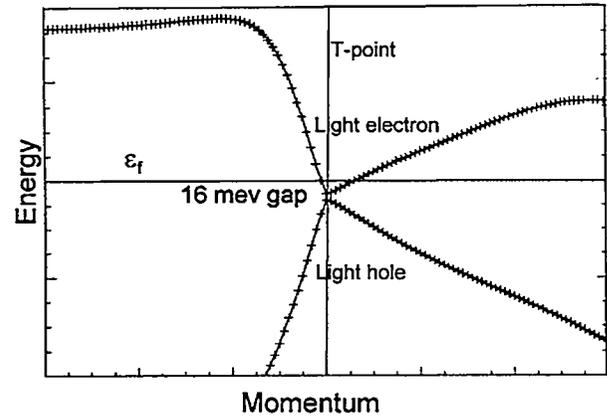


Figure 5. Shown are expanded views of the band structure of Bi near the heavy hole at the L-point and the light electron/light hole at the T-point. These calculations were provided by R.E.Allen and are the same values used in a recent publication [12].

engineered EC, there is no obvious minimum temperature, T varies strongly with length, and a set of formulae like those for the Peltier cooler is not very useful. Nevertheless, it is interesting to note that (15) can be solved for the minimum temperature if materials properties are independent of temperature- the equation is

$$\frac{T_{c0}}{T_H} = 1 - \frac{T_H}{2T_0} \quad \text{where} \quad (16)$$

$$T_0 = \frac{K}{\sigma_{11}S_{12}^2} > T_H \quad \text{and} \quad E_{y0} = \frac{S_{12}T_c}{l}.$$

Because of the form of (16) one must be very cautious about defining a figure of merit, but using T_0 as a parameter is at least simple. However, for the Ettingshausen cooler, the minimum value of $T_{c0}/T_H = 1/2$ when $T_0 = T_H$ [10].

As is obvious, no really simple direct comparison can be made because the Peltier figure of merit can be unbounded while the Ettingshausen equations apply only to an unrealistically simple geometry. Furthermore, the equations are fundamentally different, even though some authors attempt to map the EC problem onto the Peltier problem by using different definitions for K and σ [6]. Such attempts are no help to the materials scientist who is interested only in how well the refrigerator can be made to work. The best approach to developing an EC material is to use measured values of the conductivity tensor, the thermopower tensor and the thermal conductivity, and the complete version of (15) to optimize the shape and driving electric and magnetic fields. It is our belief that this has never been done as well as it is possible to do and that the real potential of EC's has not yet been realized [11].

OPTIMIZATION OF MATERIALS

Even without a complete solution to (15), there are very good ways to attack the material development problem based on a) the search for e-h symmetry, and b) the need to minimize the effective mass. We can quantify this second point, and at the same time provide a proper basis for the historical choices of B , E , and j in a Bi-based EC cooler. Then, we will use this and the band structure of Bi-Sb to suggest improvements. In figure 4 we show the Fermi surfaces of Bi. This set of surfaces is the intersection of ϵ_f with the dispersion curves of the electrons and holes. Even though there are several electron surfaces, the total number of electrons equals the number of holes. Where the surfaces are

narrow, the dispersion curves have a lot of curvature, yielding low effective masses. Remembering that one goal is to maximize the total flux of heat-carrying particles, can we find a rule that will tell us what directions B , E , and j must be in? If all the surfaces were spherical, then any direction is as good as another because the masses (inverse of the curvature) would all be the same. However, in a low-symmetry metal like the rhombohedral Bi, the electron masses vary with direction by a factor of 30, and the hole masses somewhat less. If we consider what happens when simple resistive transport occurs we find that for one of the carriers,

$$\begin{pmatrix} \frac{m_1}{ne^2\tau} & \frac{B}{nec} \\ \frac{B}{nec} & \frac{m_2}{ne^2\tau} \end{pmatrix} \begin{pmatrix} j_x \\ j_y \end{pmatrix} = \begin{pmatrix} E_x \\ E_y \end{pmatrix} \quad (17)$$

where n is the number of electrons (or holes) and c is the speed of light, e the charge on an electron, B is the magnetic field, τ is the mean time between scattering events, m_1 the carrier mass in the x -direction, m_2 in the y -direction. The low mass would occur for particles traveling across a narrow direction of the Fermi surface. A second version of (17) for the other carrier would look the same except that the signs of off-diagonal matrix elements are reversed. We can find the overall response by adding current densities linearly for the electrons and holes. If we have a system in which e-h symmetry is present, i.e. a good EC material, then when the full problem is solved, E_x would be zero (figure 2 gives the geometry) and so would the total j_x . However, what we are after is a very large positive value of j_x for one carrier and a large negative value for the other, summing the electric current to zero but maintaining a large particle flux toward the hot end. We can, therefore, solve each of the two version of (17) separately to maximize j_x/j_y , -that is we want the maximum possible transverse component of current for each carrier separately for a given current drawn from the battery. However, we know that E_x will be zero when both carrier contributions are summed, so we set it to zero in (17) to obtain

$$\frac{j_x}{j_y} = \frac{eB\tau}{m_1c} = (\omega\tau)_1 \quad (18)$$

where ω is the cyclotron frequency. This remarkable result shows that the particle current in the heat-carrying direction depends on the effective mass in that direction alone (figure 3), and that the angle that the current makes with the electric field is the Hall angle $\Theta = \tan(\omega\tau)$, the angle that the x - and y - components of

the electric field would make with each other if only one carrier were present and $j_x=0$ (figure 3).

Maximizing (18) is a very important (if $\omega\tau$ is of order unity the magnetic field is considered strong) priority but one cannot simply increase the magnetic field because if B exceeds 1T or so, permanent magnets can't be used and the system becomes a laboratory curiosity. Thus it is important for the effective mass to be very small, typically 0.01 or less than that of a bare electron. We can, however, keep electronic scattering down (and hence τ up) by minimizing alloy elements that change the electron count, and we can make sure that the temperature gradient points in the direction of lowest mass. This is the first rule. In Bi, the electron masses are much less than the hole masses so the hole pocket forces the temperature gradient to be perpendicular to the trigonal axis. The next rule is to note that in a magnetic field, electron orbits shift and re-quantize with an energy spacing $\hbar\omega$ (\hbar is Planck's constant) so it is important that the magnetic field not align with the low-mass direction or else large gaps will form in the energy spectrum (Landau levels), affecting the ability to get e-h symmetry because usually only one carrier will align. Thus the electrons, which are very light must not have the long axis of their pockets along B . This is the second rule. Thus B must be along the binary axis, leaving j along the bisectrix, as shown in figure 4.

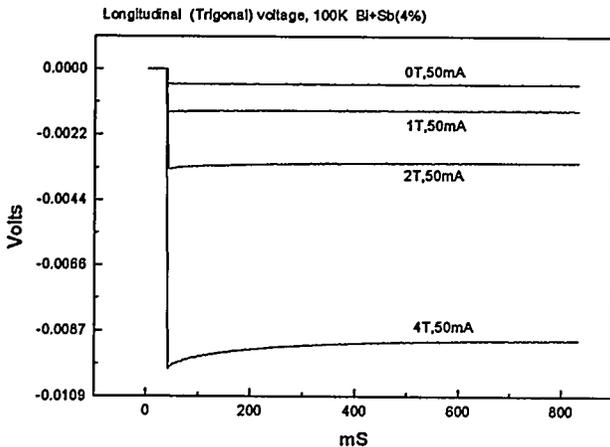


Figure 6. Shown is the response of a sample of $\text{Bi}_{0.96}\text{Sb}_{0.04}$ alloy to the application of a constant current. The current turns on at the beginning of the trace, about 40ms after the start of recording.

It is clear from figure 4 that it is very difficult to get e-h symmetry in Bi as it is. However, Bi is a low symmetry metal so that in certain symmetry directions the bands can cross or nearly cross at ϵ_f . This is illustrated in one of the best band structure calculations for Bi, done by Liu and Allen [12], where we show in figure 5 the very unusual, non-parabolic near-intersection of the light electron band with a normally unoccupied, perfectly symmetric light hole. If this light hole could be occupied equally with the light electron, perfect e-h symmetry would result. This has never been exploited, even though it is well known that the addition of Sb to Bi moves the heavy hole (the hole discussed above, which is normally the only hole present) down and out of the way, forming a semiconductor above 4%-7% Sb. If Sb is added, then the heavy hole moves away, the Sb scatters phonons well (its mass is very different from Bi), reducing the phonon thermal conductivity, and Sb has a minimum effect on τ (it does nothing to the electron density), keeping the Hall angle large. In fact, as the hole moves down and out of the way, the electron pockets shrink to zero. This might seem to be a problem, and it would be if the cooler were operated at a few K, however, at 100K or so, thermal excitations produce a good supply of e-h pairs, comparable in number to the number present in pure Bi at 0K, and, of course, the pairs are symmetric. A little doping will produce a system in which the very light electron and its symmetric hole are present in equal numbers. Under these circumstances, rule 1 tells us to put the temperature gradient along the trigonal axis, now picking up all three e-h pockets equally, instead of only part of one, as in pure Bi. Rule 2 says B must not form widely spaced Landau levels, so B goes along the binary axis, leaving j along the bisectrix. It is this that our work is aiming toward, with doping and Sb concentrations being investigated now. There is almost no systematic study of this region of the direction/concentration space [13] so much new data must be taken.

MEASUREMENTS

In order to explore carefully new alloys of doped Bi-Sb, a very tedious collection of somewhat difficult measurements must be made. These must include the resistivity tensor, the thermopower tensor and the thermal conductivity for many alloys over a broad temperature range in varying magnetic field. In addition, the measurement techniques are non-trivial.

The difficulty arises because a good thermoelectric material (Peltier or Ettingshausen) generates huge thermoelectric voltages and substantial temperature gradients when current is passed through it, mixing up thermopower and resistivity signals in a nearly impossible-to-unscramble way.

If we start with a uniform temperature and magnetic field in a sample to be measured, and drive a constant current through it, electric potentials are set up almost immediately, and then, with some time constant, thermal gradients develop which may change measured voltages. The measured voltage may be a strong function of time, with time constants less than 1 s. The constant voltage measured after a long time, when the gradients are stable, is the adiabatic value. Measurements made quickly give the isothermal values. Figure 6 shows a typical measurement of the time dependence of the longitudinal voltage on a Bi-Sb alloy. The curved region arises from a compounding of the Ettingshausen effect and the Nernst effect. The isothermal value of the voltage, combined with the known drive current, yields the resistance while the difference between isothermal and adiabatic voltages, combined with the thermal conductivity, yields the Nernst (or Ettingshausen) coefficient. This also provides a direct measure of the critical thermopower and T_0 in the material [14].

Two fast, ultra low noise amplifiers in a differential instrumentation amplifier configuration are used in the voltage measurements. The output of the amplifiers is digitized at rates near 1 kHz to measure over a time scale of approximately 800ms, by which time the voltage approaches a steady value. The timing of the current switch is controlled by a PC over a GPIB bus and the timing of the digitization cycle is adjusted to catch the voltage level before the current is switched on. Fitting a simple exponential growth to the curve gives a time constant of $\tau \sim 150$ ms, although the time dependence is not expected to be exactly exponential [13]. Such measurements, combined with static measurements of thermal conductivity, are required to predict the performance of a material used in an EC cooler.

SUMMARY

We have reviewed the Peltier and Ettingshausen effects, discussed accurate phenomenological equations to describe the Ettingshausen cooler and indicate how the Ettingshausen cooler can provide better performance

for the same effective material properties than a Peltier cooler via geometry optimization. Rules are described for determining the best directions to use in Ettingshausen materials, and the key difficulties with measurements are reviewed.

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