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MAGNETIC FIELD AND PHASE TRANSITION IN THIN FILM SUPERCONDUCTORS*

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The behavior of superconducting specimens in a magnetic field at temperatures close to the critical temperature has been successfully treated by means of the Landau-Ginzberg-Gorkov(L-G-G) theory.[1-3] At lower temperatures down to zero, on the other hand, the theory seems to be in a confused state at present, a number of papers claiming conflicting results.[4-7] We would like to consider here a thin film of superconductor (thickness L $\lesssim 10^{-5}$cm) allowing essentially complete penetration of the magnetic field which runs parallel to the film surfaces. Adapting and extending our previous calculations[5] done for bulk material to this particular condition, we conclude:

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a) The phase transition to the normal state due to the magnetic field should be of the second order at all temperatures.

b) A simple scaling rule exists concerning the field and temperature dependence of the energy gap.

c) The critical field H depends on thickness L and reduced temperature t like $H_c \sim L^{-3/2}(\ln 1/t)^{1/2}$ for not too thin films ($L \gtrsim 0.5 \times 10^{-5}$cm) and near t = 1 in accordance with the L-G-G theory, but deviates from this behavior for very small thicknesses or at moderately low temperatures. We note in particular that the conclusion a) is contrary to the calculations of Bardeen,[6] which predicts a first order transition for thin films and $t \lesssim 0.3$.

The details of the calculations will be published elsewhere. We give here an outline of the arguments leading to the above results. Instead of treating the magnetic field as a perturbation on the standard B.C.S. ground state, we first take the magnetic eigenstates $|n\rangle$ of single electrons and perform the pairing of these eigenstates. We may pair $|n\rangle$ and its spin- and space-reversed state $|\bar{n}\rangle$ as partners since our Hamiltonian in the London gauge and the boundary conditions are invariant under this operation. The energy gap equation at a temperature $T = 1/k\beta$ takes the form

$$\phi_n = \sum_m \frac{V_{nm}\phi_m}{2E_m} \tanh\left(\frac{1}{2}\beta E_m\right), \quad E_n = (\varepsilon_n^2 + |\phi_n|^2)^{1/2} \qquad (1)$$

where ϕ_n is the gap parameter, ε_n the single electron energy,

and V_{nm} the matrix element of the interaction. The result a) follows immediately from this since the free energy of the superconductive state becomes always lower than that of the normal state, except when all $\phi_n \to 0$.

A closer examination of Eq. (1) shows that the effect of the magnetic field on ϕ_n appears mainly through the field dependence of the matrix element V_{nm}. This means that the suitably defined average $\phi = \overline{\phi}_n$ depends on H only through a change in the effective coupling parameter $\rho(H) = N\overline{V}(H)$ characteristic of the B.C.S. theory. We may thus write the solution $\phi(t,H)$ as

$$\phi(t,H) = F(t)\,\phi(0,H)$$

$$\phi(0,H) = f[\rho(H)] \equiv R(H)\phi(0,0) \equiv R(H)\phi_0 \qquad (2)$$

where F(t) is the known function relating $\phi(t)$ to $\phi(0)$, which is independent of $\phi(0)$ and H. A superconductor in a magnetic field then behaves like a fictitious superconductor having a reduced gap parameter $\phi_0 = R\phi_0$ and a correspondingly reduced critical temperature $T_c' = RT_c$. For fixed t and increasing H, we shall have a phase transition at a critical field H_c for which $T_c/T_c = t$, namely

$$R(H_c) = t \qquad (3)$$

The reduction factor R(H) has been obtained by comparing our earlier perturbation formulation with the above general result. The perturbation calculation, when applied to our geometry, gives at t = 0

$$\delta\phi/\phi = -(8/\pi^6)(e^2H^2L^4/\hbar c)(p_F L/\pi\hbar)\,\mathrm{sh}^{-1}(\pi^2\hbar^2/m\phi c^2L^2) \qquad (4)$$

where m and p_F are electron mass and Fermi momentum. We find that the discreteness of the electron momenta and the use of the London gauge are essential for obtaining the correct result. A failure to follow these considerations would lead to a $\delta\phi$ consistent with Bardeen's calculations, which corresponds to replacing the sh^{-1} function in Eq.(4) by its argument and would predict a first order phase transition. It is actually not justifiable when ϕ and/or L become small. Eq.(4), on the other hand, is free from these difficulties, but still may not be adequate in making extrapolations to $\delta\phi$ which is comparable with ϕ itself. We should be on a safer ground in this regard if we would express Eq.(4) as a change $\delta V(H)$, substitute $V(H) = V(o) + \delta V(H)$ thereby obtained into Eq. (1), and solve it for ϕ without making an expansion with respect to $\delta\phi$. This is so because the magnetic field affects most of the individual electron wave functions only slightly for a thin film where the electrons cannot complete circular classical orbits, so that we may use perturbation theory as far as the magnetic eigenfunctions are concerned. The program can be carried out easily when one makes the further simplifying assumption that V is separable: $V_{nm} = S_n S_m^* V_o$, since Eq. (1) then reduced to the manageable form

$$1 = V_o \sum_n \frac{|S_n|^2}{2E_n}, \quad \phi_n = S_n\phi/(\overline{S^2})^{1/2} \qquad (5)$$

Tracing back the perturbation calculation which led to Eq.(4), we can identify $|S_n|^2$. It has the additional advantage in enabling

one to use physical insight and put $|S_n|^2 = 0$ in case we obtain a negative value as the perturbation becomes too strong. Such a situation arises for electrons running almost parallel to, and confined in the vicinity of, the film surfaces by the magnetic field. Although perturbation theory fails for these states, the pairing partners are now separated to the opposite sides of the film and will not contribute to pairing energy.

In this way we have determined the reduction factor R(H) of Eq.(2), which turns out to decrease roughly exponentially with H^2 without ever vanishing for finite H. More explicitly, R may be cast in the form

$$R(H) = \exp[-(8/\pi^4)\ (eH/\hbar c)^2\ \xi_0 L^3 \gamma] \qquad (6)$$

where ξ_0 is the coherence length and $\gamma(H)$ is a rather slowly varying function. Since our entire set of approximations used breaks down for very high fields, it is impossible to infer H_c at t = 0 from our formulas. But for laboratory temperature ranges Eqs.(2) - (6) seem to be adequate.

Fig. 1 shows the measurements of Douglass and Meservey[8], using tunnelling techniques, on the field dependence of energy gap for lead films of thickness 1000Å and at a reduced temperature t = 0.12, in comparison with our calculations for this particular case. On the theoretical side, our formulas are rather crude, and furthermore there may be some questions about their applicability to an anomalous superconductor like lead. On the experimental side, there are uncertainties in determining

the exact critical field, and in inferring the gap parameter from tunnelling current curves. The theoretical curve has been normalized to $H_c = 2350$ gauss suggested by the data, in contrast to a theoretical value of 3700 gauss.

We conclude that the agreement between theory and experiment is at least qualitatively satisfactory. In particular, the second order nature of the phase transition is strongly indicated by the data. Thermal conductivity measurements by Morris[9] for lead at similar temperatures also seem to show a good agreement with our theory.

We wish to thank Professors D. H. Douglass, Jr. and R. Meservey for permitting us to use their data before publication.

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Figure Caption.

The magnetic field dependence of energy gap. The theoretical curve is normalized to $H_c^2 = 5.5 \times 10^6$ gauss^2.

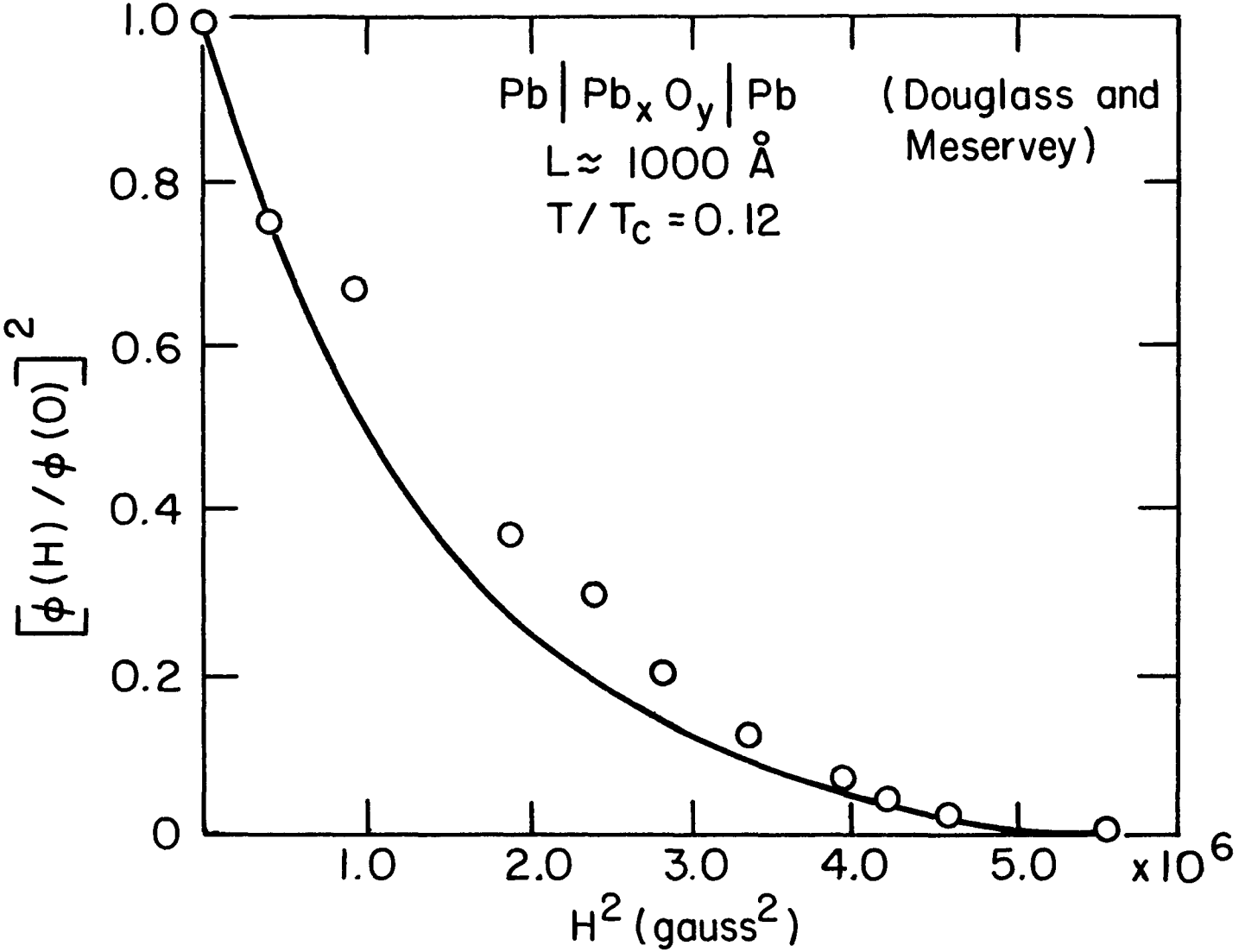
1.0
0.8
0.6
0.4
0.2
0
$[\phi(H)/\phi(0)]^2$
Pb | $Pb_x O_y$ | Pb
L ≈ 1000 Å
T/ T_c = 0.12
(Douglass and Meservey)
1.0
2.0
3.0
4.0
5.0
x 10^6
H^2 (gauss^2)

Fig.1