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By

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H. Sakata† and L.C. Schmid

INTRODUCTION

Three vertical safety rods have been provided in the D_2O moderated Plutonium Recycle Critical Facility[1] (PRCF). The reactivity worth of each rod and the combined worth of three rods have been measured[1] by correlating the flux transient which results when the rods are dropped in the reactor with that calculated for a given step change of reactivity. The experiments in the PRCF were performed with the three safety rods distributed symmetrically in the core at a radius of 16 inches. The average worth of a safety rod is 26.1 mk and the worth of all three rods inserted simultaneously is 34% larger (antishadowing effect) than the sum of each individual rod.

The interaction effects have been calculated by the method of Nordheim and Scalettar[2] as a function of radius and compared with the experimental result. A 31% antishadowing effect is calculated for a radius of 16 inches and the effect is dependent on the location of the rods in the core.

CALCULATIONAL TECHNIQUE

The worth of the safety rods is determined by comparing the buckling B^2 of the reactor with the rods inserted and that B_1^2 with the rods removed. For simplicity, the core is assumed to be bare, homogeneous, and cylindrical

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† Work conducted while on leave from Japan Atomic Energy Research Institute, Tokai-Mura, Naka-gen, Ibaraki-ken, Japan.

in shape. The three safety rods are place symmetrically on the same radius as shown in Figure 1. According to one-group, perturbation and diffusion theory, the critical equation is

$$k_\infty = 1 + L^2B^2 \tag{1}$$

and a change of buckling $\Delta B^2 = B_1^2 - B^2$ is compensated by a change in Δk for the reactor to remain critical.

The reactivity change caused by the insertion of a safety rod is

$$\Delta k = (k_\infty - 1)\,\frac{\Delta B^2}{B^2} = \frac{2(k_\infty - 1)}{\left(1 + \frac{B_z^2}{B_r^2}\right)}\,\frac{\Delta B_r}{B_r} \tag{2}$$

Where B_r^2 and B_z^2 are the radial buckling and axial buckling respectively and k_∞ is the multiplication factor for an infinite reactor. The change ΔB is calculated from solutions of the reactor equations as obtained by Nordheim and Scalettar.

According to the Nordheim and Scalettar treatment[3, 4] of absorbing rods the general solution of the reactor equations is expressed by a sum of regular and irregular parts which are independent of the azimuthal angle and which contain singularities at the positions of the rods. The irregular solution predominates in the neighborhood of each rod and the regular solution predominates everywhere else. The complete solution is forced to satisfy the boundary conditions that it vanish at effective surfaces of the rods and at the reactor boundary. With this procedure the equation for a critical reactor with one off center safety rod is

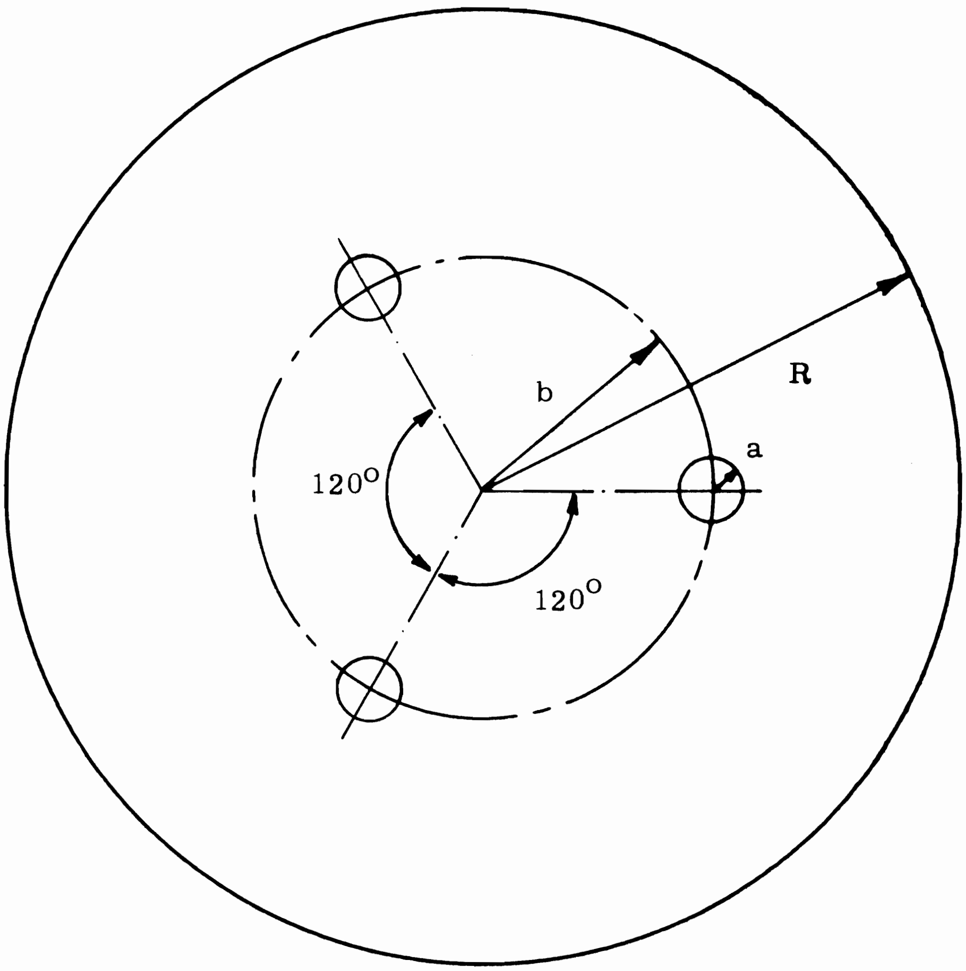

FIGURE 1

Arrangement of Safety Rods in the Reactor. In the Experimental Situation R = 91.51 cm and b = 40.64 cm

$$Y_o\,(B_{1r}a) = \frac{Y_o(B_{1r}R)J_o^2(B_{1r}b)}{J_o(B_{1r}R)} \tag{3}$$

where the J_o and Y_o are Bessel Functions of the first and second kind of zero order. The Bessel Functions are expanded in a Taylor Series about B_r in order to obtain ΔB_r. Since $B_{1r}-B_r$ is small only the first two terms of the expansion need to be retained. Thus,

$$\left(\frac{\Delta B_r}{B_r}\right)_1 = \left(\frac{\Delta B_r}{B_r}\right)_0 \left\{1 - \left(\frac{\Delta B_r}{B_r}\right)_0 \quad G_1(b)\right\}^{-1} \tag{4}$$

where

$$\left(\frac{\Delta B_r}{B_r}\right)_0 = \frac{Y_o(B_rR)J_o^2(B_rb)}{B_rR\;Y_o(B_ra)J_1(B_rR)} \quad . \tag{5}$$

In Equation (4)

$$G_1 = \frac{1}{2} - B_rR\,\frac{Y_1(B_rR)}{Y_o(B_rR)} \;-\; 2\,B_rb\,\frac{J_1(B_rb)}{J_o(B_rb)}$$

$$-\;2\,B_rR\,\frac{J_1(B_rR)}{Y_o(B_rR)} \sum_{n=1}^{\infty} \frac{Y_n(B_rR)\;J_n^2(B_rb)}{J_n(B_rR)\;J_o^2(B_rb)} \tag{6}$$

and

R = Effective core radius including the extrapolation distance.

$B_rR = 2.405.$

a = Effective radius of the safety rod.

b = Distance from core center to safety rod center.

The procedure to calculate ΔB_r for the insertion of three safety rods is identical to the calculation for the one eccentric rod. The main differences in the calculation is that now there are three irregular solutions to the reactor equations, one for each rod. The general solution is

$$\psi = Y_o(B_{1r}r_1) + Y_o(B_{1r}r_2) + Y_o(B_{1r}r_3) + \sum_{n=-\infty}^{\infty} A_n J_n(B_{1r}r)e^{in\theta} \tag{7}$$

where the first three terms are the irregular solutions (one for each safety rod) and the fourth term is the regular solution which predominates in regions removed from the safety rods.

Now

$$Y_o(B_{1r}a) + 2\,Y_o(B_{1r}\sqrt{3}\,b) - \frac{3Y_o(B_{1r}R)\,J_o^2(B_{1r}b)}{J_o(B_{1r}R)} = 0 \tag{8}$$

after applying the boundary conditions and the fact that when $r_1 = a$ then $r_2 = r_3 \simeq \sqrt{3}\,b$. After expanding Equation (8) in a Taylor Series about B_r and solving for ΔB_r

$$\left(\frac{\Delta B_r}{B_r}\right)_3 = 3\left(\frac{\Delta B_r}{B_r}\right)_o \left[1 - 3G_3\left(\frac{\Delta B_r}{B_r}\right)_o\right]^{-1} \tag{9}$$

is obtained, where

$$G_3 = \frac{1}{2} - B_rR\,\frac{Y_1(B_rR)}{Y_o(B_rR)} - 2B_rb\,\frac{J_1(B_rb)}{J_o(B_rb)} + \frac{2}{3}\,B_rR\,\frac{J_1(B_rR)\;Y_o\;(\sqrt{3}\;B_rb)}{Y_o(B_rR)\;J_o^2\;(B_rb)}$$
$$- 2\,B_rR\,\frac{J_1(B_rR)}{Y_o(B_rR)}\sum_{m=1}^{\infty}\frac{Y_{3m}(B_rR)\;J_{3m}^2(B_rb)}{J_{3m}(B_rR)\;J_o^2\;(B_rb)} \tag{10}$$

The worth of the rods when simultaneously inserted relative to the sum of each rod alone is the interaction effect and is given by Equations (4) and (9) as

$$\frac{(\Delta k)_3}{3(\Delta k)_1} = 1 + F \qquad (11)$$

where

$$F = \frac{(3G_3 - G_1)\left(\frac{\Delta B_r}{B_r}\right)_o}{1 - 3G_3\left(\frac{\Delta B_r}{B_r}\right)_o} \qquad (12)$$

The interaction effect and the reactivity worth of each safety rod are calculated as a function of radius b using Equation (2), Equation (4), Equation (9) and Equation (11). The description and the nuclear characteristics of the PRCF are listed in Tables I and II, respectively. The nuclear constants given in Table II were computed for the condition in which the reactor is critical with all control and safety rods removed from the core

TABLE I

DESCRIPTION OF PRCF

Core	Fuel	1.8 w/o Pu-Al (~ 6% Pu^{240}), UO_2 (natural uranium)
	Moderator	D_2O (99.75 mol/o)
	Dimension	Radius: 90.40 cm (35.6")[a] Height: 222.5 cm (87.6")[b]
Safety Rod	Material	Cd covered Al cylinder in 8.88 cm (3.50") O.D. hole.
	Dimension	Diameter: 6.96 cm (2.74") O.D. Cd Thickness: 0.0635 cm (0.025") Length: 215.9 cm (85")
	Position	40.64 cm (16") from core center

(a) (Radius) = (Effective radius) - (Extrapolation distance)
(b) (Height) = (Effective height) - (Reflector savings)

TABLE II

CHARACTERISTICS OF PRCF

Effective Radius	R	91.51 cm
Extrapolation Distance	λ_e	1.11 cm
Radial Buckling	B_r^2	6.9082×10^{-4} cm^{-2}
Axial Buckling[a]	B_z^2	1.3876×10^{-4} cm^{-2}
Diffusion length[b]	L^2	165.39 cm^2
Multiplication Factor[c]	k_∞	1.1372

(a) $B_z^2 = \left(\frac{\pi}{H + 2\lambda_R}\right)^2 = \left(\frac{\pi}{222.5 + 44.2}\right)^2$

(b) Calculated by three group diffusion theory for the condition where all control and safety rods are removed from the core.

(c) $k_\infty = 1 + (B_r^2 + B_z^2)\, L^2$

RESULTS

The reactivity worth of one rod and the combined worth of the three rods were calculated as a function of $\frac{b}{R}$ using Equation (2),(4), and (9) and are shown in Figure 2. The interaction effect was calculated for values of b/R using an effective radius a = 1.67 cm for a safety rod and Equation (11). The results are plotted in Figure 3. The calculations were made using Program GIESR[(5)] which was written for the purpose. For the calculations terms were summed through the Bessel functions of 27th order (i.e., $m = 9$ and $n = 27$). This was more than adequate since values of G_1 and G_3 were converged to five places with terms through the 21st order.

The value used for the effective radius is a semiempirical value determined by equating $(\Delta k)_1$ obtained from Equations (2) and (4) to the experimental value 26.1 mk and solving for a. The values of $J_0(B_r b)$

in Equation (5) represent values of the neutron flux at positions of radius b from the core center. The experimental values φ_{exp} obtained from measurements of the flux distribution[6] do not have the $J_o(B_r b)$ shape. In addition to values $J_o(B_r r)$, the corrected values

$$J_o(B_r r) \times \frac{\varphi_{exp}(B_r r)}{\varphi_{exp}(o)\ J_o(B_r r)}$$

have been used in the calculations using Equation 5. Results of using the correction are also shown in Figures 2 and Figure 3 as Curves A1 and A2 and Curve A.

Using this technique the value 1.67 cm is obtained for the effective radius of a safety rod using the measured flux distribution. Using the same technique and the $J_o(B_r r)$ distribution, the value 3.53 cm is obtained for the effective radius of a safety rod.

A value which is obtained analytically, for the effective radius, is 2.09 cm. In the case of a rod[3] which completely absorbs thermal neutrons

$$a = a_o \exp \left| - \frac{d}{a_1} \right|$$

where a_1, a_o, and a are respectively the physical radius of the rod, the physical radius of the channel the rod is in, and the effective radius of the combination. The d is the extrapolation length into the rod and is given[7], in terms of the mean free path λ_{tr} of the surrounding medium, by

$$\frac{d}{\lambda_{tr}} = \frac{4}{3} - 4\ T\ (a_1) + f\ (a_1) = 0.899 \qquad (14)$$

since $4T$ (3.48 cm) = 0.500 and f (3.48 cm) = 0.066. Hence, for a_o = 4.44 cm, a_1 = 3.48 cm and λ_{tr} = 2.91 cm, the value of a from Equation (13) is 2.09 cm.

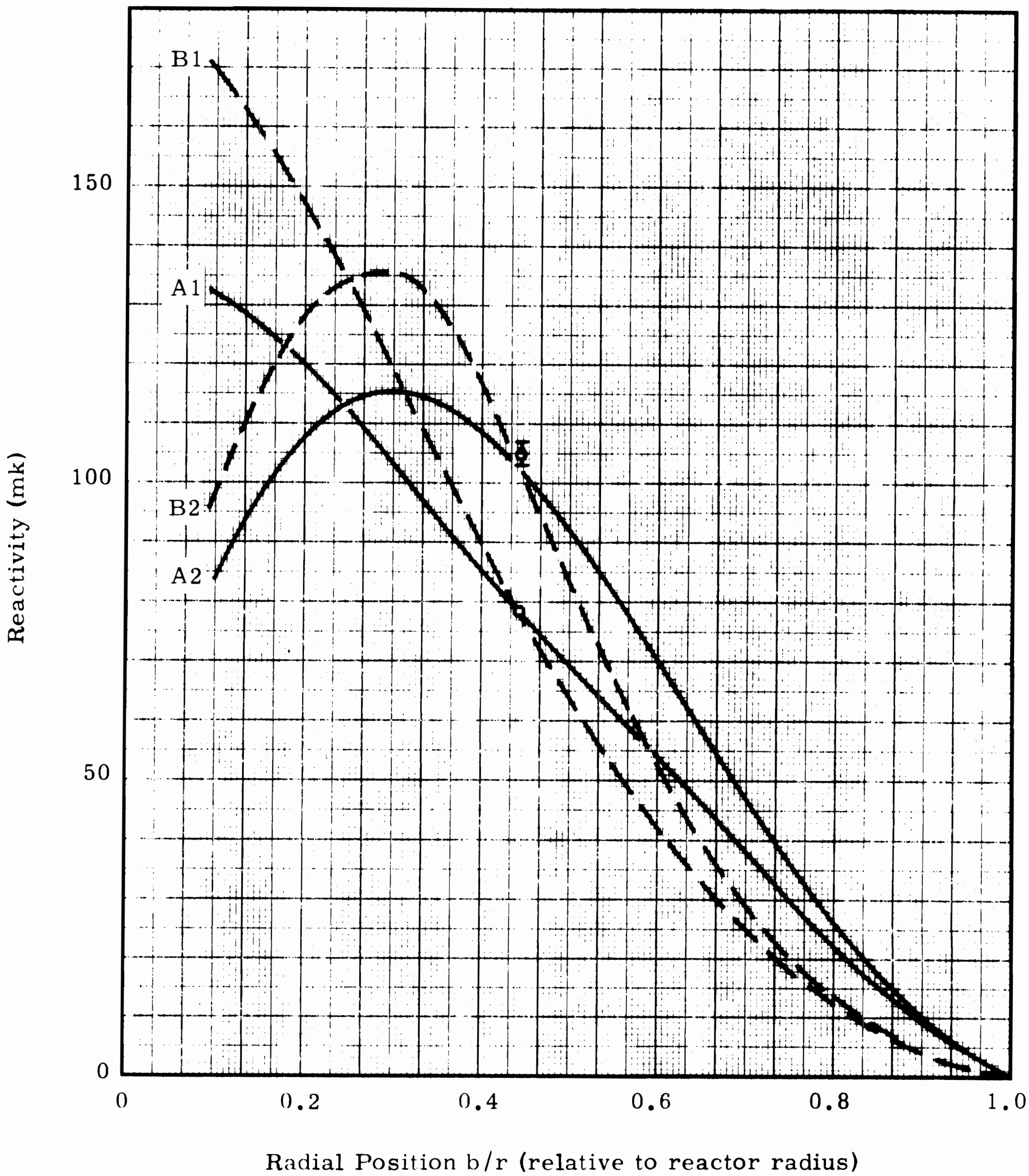

FIGURE 2

Reactivity Worth of Safety Rods as a Function of Position in the Reactor. Curves A-1 and A-2 are, respectively, the worths of three rods inserted separately and simultaneously for an effective radius of 1.67 cm and for the experimental flux distribution. Curves B-1 and B-2 are, respectively, the worths of three rods inserted separately and simultaneously for an effective radius of 3.53 cm and for a J_o flux distribution. Values measured experimentally are shown as points.

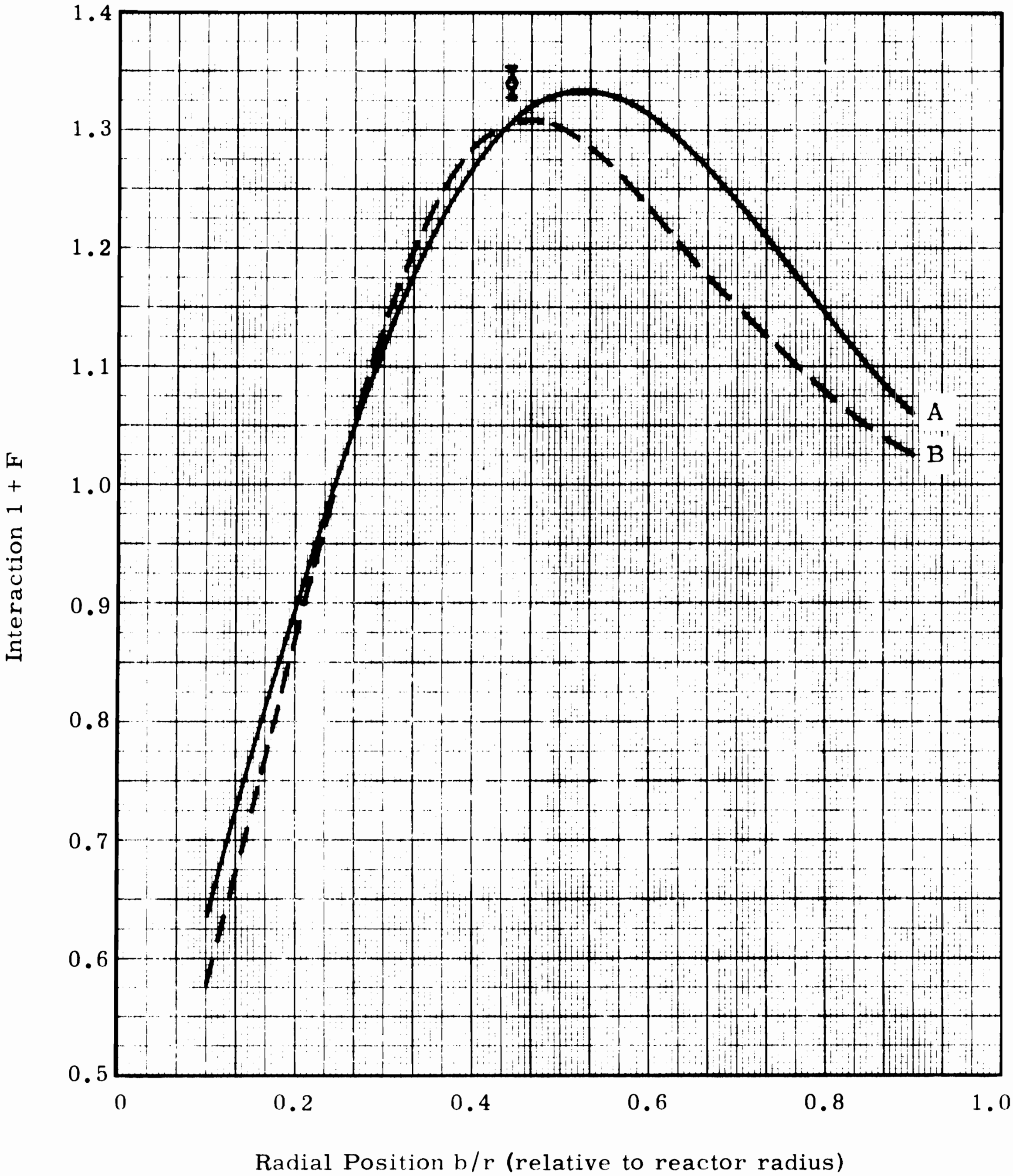

FIGURE 3

Interaction Effect of Safety Rods as a Function of Position in the Reactor. Curves A and B are for the experimental and J_0 flux distributions respectively. An experimental value is shown as a point.

This value of a corresponds to 27.7 mk or 22.3 mk for the worth of a safety rod using the φ_{exp} or J_o distribution respectively. Although the worth of a safety rod is sensitive to the value of a, the interaction results are not.

As shown by Equations (4) and (9), $(\Delta k)_1$ and $(\Delta k)_3$ depend on the effective radius a of a safety rod. The calculation was forced to agree with the experimental value $(\Delta k)_1$ through the choice of a, nevertheless the calculated value $(\Delta k)_3$ shows good agreement with the experimental value. Thus this empirical value for the effective radius appears to be a good value for the calculation.

The effects of the variation of a on the interaction effect have been calculated and are plotted in Figure 4. This variation S of the interaction with changes in the effective radius a of the safety rod is calculated as

$$S = \frac{a \frac{d}{da}\left[1+F(a)\right]}{1+F(a)}$$

$$= \frac{a}{1+F}\left\{ \frac{3G_3 - G_1}{\left[1-3G_3\left(\frac{\Delta B_r}{B_r}\right)_o\right]^2} \cdot \left(\frac{\Delta B_r}{B_r}\right)_o \frac{2.4048}{R} \frac{Y_1\left(\frac{2.4048\, a_o}{R}\right)}{Y_o\left(\frac{2.4048\, a_o}{R}\right)} \right\} \qquad (15)$$

Therefore S is the percent change in (1+F) per percent change in a. Except for the smaller values of b/R, the interaction effect defined as the ratio of $(\Delta k)_3$ to $3(\Delta k)_1$ does not depend strongly on a. The large changes at the smaller radii are not valid because the calculational method does not apply close to the safety rods where the flux gradients are large. The method consists of replacing the safety rod by a point singularity which is not appropriate for the smaller radii (i.e., r must be greater than b).

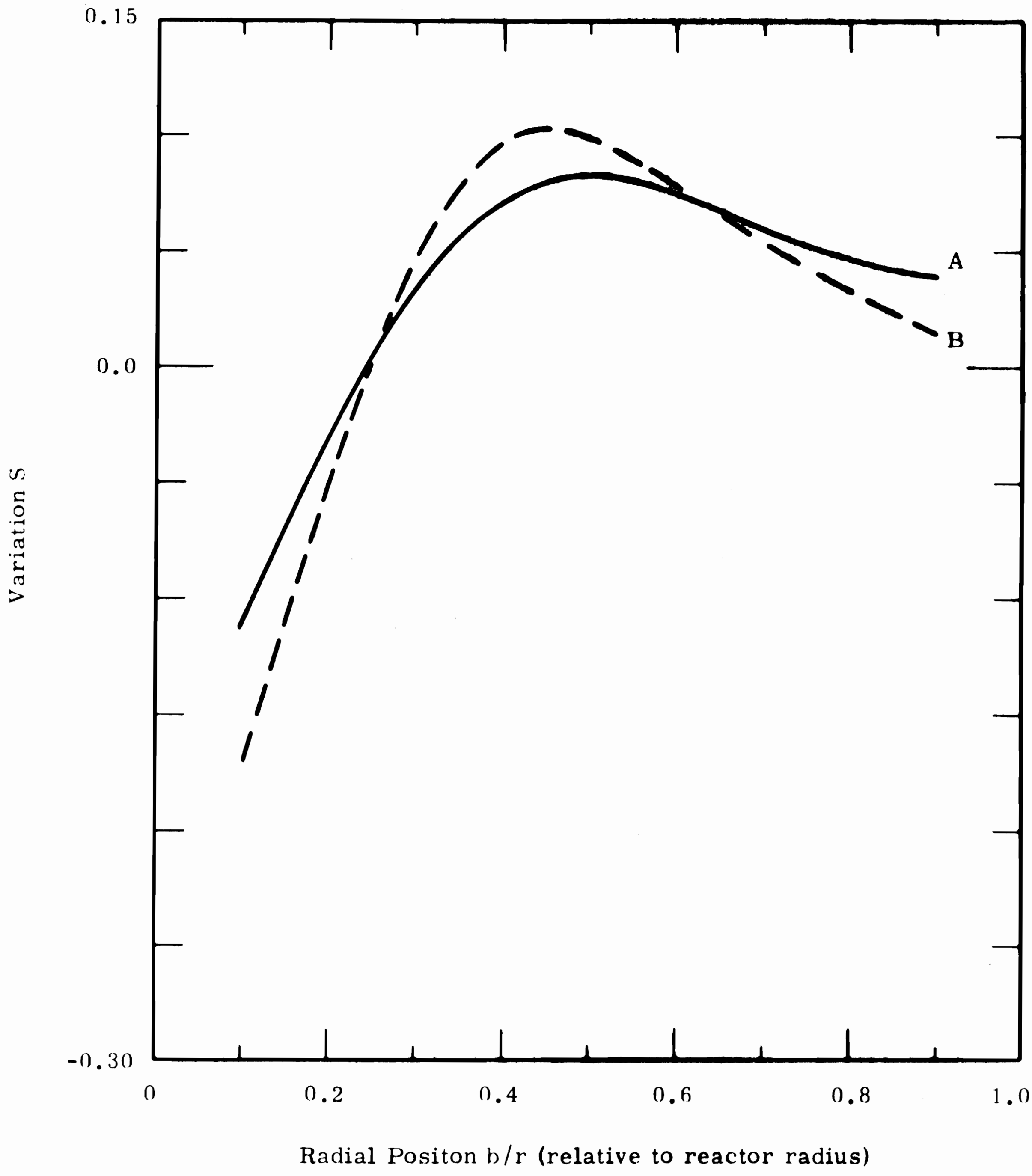

FIGURE 4

Variation of the Interaction Effect with Effective Radius of the Safety Rods. Curves A and B are for the experimental and J_o flux distributions respectively.

As examples of the variation of the interaction effect for large changes in the radius a, two other values 0.95 cm and 2.94 cm were chosen for a and the calculations repeated using the experimental flux distribution. For a = 0.95 cm the worth of a single rod $(\Delta k)_1$ is 22.8 mk and the interaction effect is increased from 1.34 for a = 1.67 cm to 1.26 for a = 0.95 cm. For a = 2.94 cm the worth of a single rod is 30.6 and the interaction effect is increased to 1.38. The effects of the variation of a on the interaction effect at the new radii are shown in Figure 5 as a function of safety rod position in the core.

Experimental values are also plotted in Figures 2 and 3 for comparison with the calculated results. The experimental results are described in Reference 1 and are summarized in Table III. The experimental values of each one of the rods were different from each other, but the average value 26.1 mk was taken as the worth of a single safety rod. This is reasonable since the rods were distributed symmetrically in the core and their physical conditions were the same and the calculated worths would be equal. The combined worth of three rods inserted simultaneously is 34 ± 3% larger than the sum of the three rods inserted alone. The corresponding value of 31% has been calculated using Equation (12).

TABLE III

EXPERIMENTAL VALUES OF REACTIVITY WORTHS OF SAFETY RODS IN PRCF

	Rod	Worth (mk)
Single Rod $(\Delta k)_1$	No. 1	23.7 ± 0.7
	No. 2	30.1 ± 0.7
	No. 3	24.6 ± 0.6
	Average	26.1 ± 0.4
Combined worth of three rods $(\Delta k)_3$		105 ± 2
$(\Delta k)_3/3(\Delta k)_1$		134 ± 3

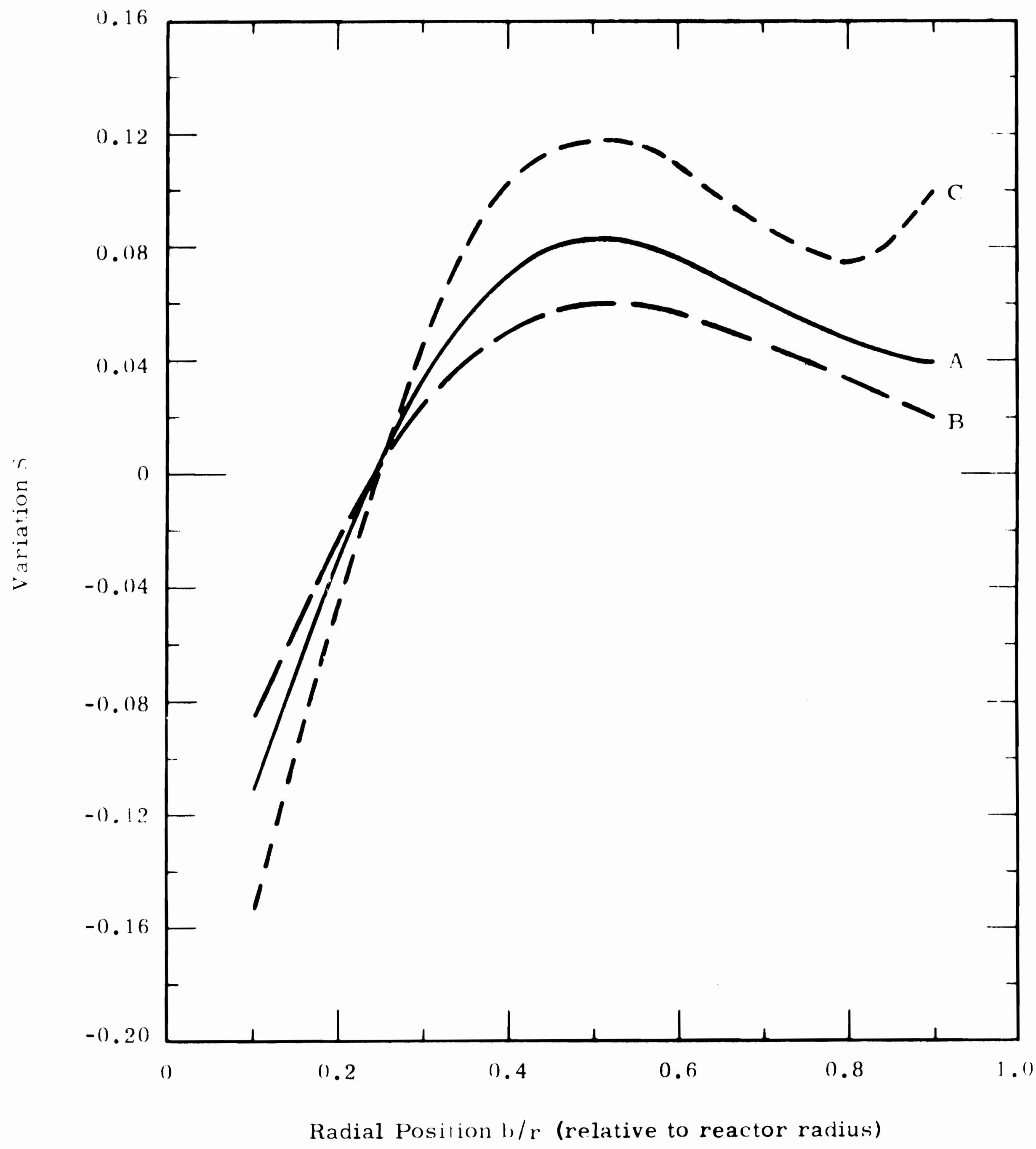

FIGURE 5

Variation of the Interaction Effect with Effective Radius of the Safety Rods. Curves A, B, and C are for variations about effective radii of 1.67, 0.95, and 2.94 cm respectively.

DISCUSSION AND CONCLUSION

The interaction effect of the safety rods in the D_2O-moderated PRCF was calculated on the basis of one group, diffusion theory approximations and assuming a bare homogeneous loading. As shown in Figures 2 and 3, the agreement between the experimental and calculated values for the interaction effect are good for $b/R = 0.444$. At small distances from the center of the reactor the interference factor is negative and the sum of the effects of the safety rods is less than that of the safety rods considered separately. At large distances the factor is positive and the effect of the safety rods together is greater than that of the safety rods considered separately. In addition to the increased absorption from inserting a safety rod in the PRCF, the reactivity is affected because the neutron distribution is changed (different neutron leakage effects) and this changed distribution in turn effects the neutron absorption and production characteristics of the loading. The enhancement of the worth of the safety rods because of the antishodowing effect can be understood qualitatively by considering only two rods. The change in the neutron flux distribution caused by the introduction of the first rod depresses the flux in the region of the rod and as a result the reactivity worth distributions for the core is shifted so the maximum is in the vicinity of the second rod. Therefore the second rod will have a greater effect on the asymmetric reactor than the first rod had on the symmetric reactor.

Only one set of experimental values for safety rod worths is available so the accuracy of the calculations cannot be checked at other radii. Measurements of the interaction effects for the safety rods at other radii are required for evaluation of the accuracy of the calculations. However, until more measurements are available, Figures 2 and 3 can be used to estimate the effect of moving the safety rods radially in the D_2O moderated PRCF.

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