THEORY OF NEUTRAL BEAM INJECTION INTO A TOKAMAK

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Abstract: Plasma perturbations caused by injection of a neutral beam into a Tokamak plasma are examined to determine if there are any limitations to this method of heating and replenishing the plasma.

Introduction: In addition to the desirable heat and particle inputs, neutral injection may have deleterious effects on plasma equilibrium and confinement in a Tokamak. Plasma perturbations arising from neutral injection are caused primarily by the presence of a fast ion group and by the momentum imparted to the plasma by beam absorption. The various perturbations which occur are discussed below. The numbers used in estimating the effects are for a standard ORMAK plasma discharge (major radius R_o = 79.5 cm, minor radius r_p = 23 cm, peak density $n_e(0) \sim 3 \times 10^{13}/cm^3$, peak electron temperature $T_e \sim 800$ eV, plasma current $I_p \sim 120$ kA, toroidal magnetic field $B_T \sim 18$ kG, safety factor $q(0) \sim 2$) and a standard injector (beam energy $E_b \sim 25$ keV, beam current $I_0 \sim 6$ Amps).

Plasma Perturbations Caused by Tangential Neutral Beam Injection: In present generation Tokamaks injection tangent to a toroidal magnetic field line in the plasma is generally preferred because of the high beam absorption (> 90% in ORMAK). Collisions of injected fast neutrals with the background plasma via ionization (~ 25%) and charge exchange (~ 75%) processes produce low energy electrons (which quickly join the background plasma), and fast ions. While the electrons remain on the flux surface on which they are born, the fast ions follow drift orbits. The poloidal motion of the drift orbits is approximately circular, but centered about the stagnation point x_s where the curvature drift and parallel, rotational transform motions just cancel. This point is located a distance of $q\, V_b/\Omega_i \simeq 2.5$ cm (V_b is the fast ion speed, Ω_i the ion gyroradius) outside (inside) the magnetic axis for injection parallel (antiparallel) to the Ohmic heating current. The total fast ion density accumulated in the plasma after an initial transient is given by

$$n_f(r,\theta) = \frac{I_0 \tau}{(2\pi R_o)(\pi r_p^2)e} H(r,\theta) \sim 3 \times 10^{11}/cm^3 . \qquad (1)$$

Here, $H(r,\theta)$ is an order unity spatial shape factor which for a pencil beam in the equatorial plane is sharply peaked about the stagnation point (where the area of the drift surface vanishes). For the large beam spread in the ORMAK injector H is a smooth function which falls off like 1/r at large minor radii, but is nearly constant at a value slightly greater than unity near the magnetic axis. The remainder of the right side of Eq. (1) is the total number of fast ions produced in the plasma divided by the plasma volume. The time τ is the lifetime of a fast ion in the plasma; i.e., it is the minimum of the slowing down (momentum exchange $\tau_s \sim 10$ msec) and charge exchange ($\tau_{cx} \sim 4$-5 msec if $n_o \sim 2 \times 10^9/\mathrm{cm}^3$) times.

As the fast ions slow down they impart momentum to the background plasma and, in combination with it, produce a current

$$j_{inj} = n_f\, e\, Z_b V_b \left[1 - \gamma \frac{Z_b}{Z_{eff}} \frac{\langle H \rangle}{H}\right] = Z_b \left(\frac{I_0}{\pi r_p^2}\right)\left(\frac{\tau V_b}{2\pi R_o}\right)\left[H - \frac{\gamma Z_b}{Z_{eff}}\langle H \rangle\right] \sim 10\ \mathrm{kA}/(\pi r_p^2) \qquad (2)$$

in which Z_b, Z_{eff} are the mean charge numbers of the beam and background plasma ions, γ is the fraction of the momentum transferred to the flowing (untrapped) ions and electrons, and $\langle\ \rangle$ indicates the average over a flux surface (θ). Note that this current is larger than the neutral beam current by the "stacking factor" $V_b\tau/2\pi R_o \gtrsim 10^3$, which is simply the ratio of the fast ion lifetime to its transit time around the machine.

The fast ion density and hence the induced current do not grow monotonically with time. However, the momentum imparted to the plasma by beam absorption does. The plasma flow velocity so produced is given by

$$U = \frac{m_b \langle n_f \rangle}{m_i\ n_i} \left(\frac{t}{\tau}\right) V_b \sim 2 \times 10^6\ (t/\tau)\ \mathrm{cm/sec}. \qquad (3)$$

Since the time-scale of interest is long compared to both the fast ion lifetime τ, and the time-scale ($\tau_{ii} \sim 100$ μsec) for the relaxation of poloidal $\underline{\mathcal{E}}_r \times \underline{B}$ flows into toroidal $\underline{\mathcal{E}} \times \underline{B}$ flow by parallel ion viscosity,[1] the presence of such a flow implies a radial electric field given by U times the poloidal magnetic field B_p:

$$\mathcal{E}_r = B_p U \sim 2 \times 10^3\ (t/\tau)\ \mathrm{V/cm}\ . \qquad (4)$$

<u>Possible Limitations on Tangential Injection</u>: From the preceding discussion we infer the following potentially deleterious consequences:

(1) <u>Current</u>: If the current density induced by neutral injection is comparable to the Ohmic heating current density ($j_{Ohmic} = I_p/\pi r_I^2$, where r_I is the current channel radius), then the q(r) [inverse rotational transform] profile has a dip (bump) in it for co (counter)-injection. In order to minimize this effect we require $j_{Ohmic} < j_{inj}$, or

$$I_0\tau < I_p\left(\frac{r}{r_I}\right)^2 \frac{2\pi R_o}{V_b(1-\gamma Z_b/Z_{eff})} \frac{r_p^2}{2\int_0^r r dr \langle H(r,\theta)\rangle} \sim 0.5\ \mathrm{Amp.\text{-}sec}$$

(2) Momentum: If the injection induced plasma flow velocity is greater than the sound speed ($V_s = \sqrt{T_e/m_i}$), shocks (or Kelvin-Helmholtz instabilities) may form and be deleterious to confinement. In order to keep $U < V_s$ we require

$$I_0 t < \sqrt{\frac{KT_e}{eE_b}\left(\frac{m_i}{m_b}\right)}^{1/2} \frac{(2\pi R_o)(\pi r_p^2) n_i e}{\langle H \rangle} \sim 0.5 \text{ Amp.-sec.}$$

(3) Radial Electric Field: Since quasi-equilibrium Tokamak plasmas typically have radial potentials on the order of the electron temperature, the radial field produced in conjunction with the toroidal plasma flow may perturb the equilibrium if it implies a radial potential over the plasma greater than T_e. This situation can be avoided if

$$I_0 t < \frac{(KT_e/e)}{B_p\ V_b\ r_p} \left(\frac{m_i}{m_b}\right) \frac{(2\pi R_o)(\pi r_p^2) n_i e}{\langle H \rangle} \sim 0.1 \text{ Amp.-sec.}$$

(4) Net Charge: Since the fast ion drift orbits deviate from the flux surfaces on which the ions are born by an amount $2x_s$(~ 7 cm near the plasma edge) the counter-injected fast ions produced within $2x_s$ of the limiter impinge on the limiter and result in a net negative plasma charge. The fraction of injected beam lost is of order $(2x_s/r_p)^2$. The resultant net charge produces a radial potential over the outer $2x_s$ of the plasma. A crude estimate for keeping this less than KT_e/e is

$$I_0 t < (KT_e/e)(r_p/2x_s)^3\ (4\pi\epsilon_\perp)(2\pi r_o) \sim 0.03 \text{ Amp.-sec.}$$

where $\epsilon_\perp \simeq \epsilon_o\ (1 + \omega_{pi}^2/\Omega_i^2) \sim 10^3\ \epsilon_o$ is the perpendicular dielectric constant appropriate for this outer plasma region.

Since the shape factor $H(r,\theta)$ is approximately the same for co- and counter-injection, effects (1)-(3) can, on the average, be cancelled out by appropriate adjustments of the opposing beam intensities. However, there will remain some localized effects due to the slight (e.g., 20%) difference in the shape factors. Also, the introduction of a counterstreaming beam always causes net charge buildup. Since the dynamics of the limiter-plasma interface are not well understood, it is not clear what effect the net charge and concomitant radial electric field will have.

Perpendicular Injection: Injection of neutral beams vertically or radially inward (as planned in the French TFR) into a Tokamak will eliminate the current and momentum buildups which occur for tangential injection. There are, nonetheless, two main plasma perturbations caused by perpendicular injection. First, as with tangential counter-injection there is a net charge buildup due to fast ions being injected into (trapped) drift orbits which strike the limiter. The resultant effect is similar to that in (4) with x_s replaced by the trapped-particle "banana" width,

$\Delta r_T \sim (2qV_b/\Omega_i)\sqrt{R_o/r}$.

The other major plasma perturbation arises because the trapped ions do not spread themselves uniformly over the poloidal cross-section; the "bananas" are only on the outer side of the torus. Thus, the charge distribution on a flux surface will be nonuniform. The charge distribution and concomitant poloidal electric field is relaxed by the slowing down process for the fast ions since as they undergo pitch-angle diffusion the poloidal position of the banana tips is also diffused. Taking account of the Debye shielding effects along the flux surfaces, we find that the induced poloidal electric field is given by

$$\mathcal{E}_\theta = \frac{I_0 \tau \lambda_D^2}{\epsilon_0 (2\pi R_o)(\pi r_p^2)} \frac{1}{r} \frac{\partial G(r,\theta)}{\partial \theta}$$

where λ_D is the Debye length and $G(r,\theta)$ is the poloidal charge imbalance shape factor which is of order unity if most of the fast ions are confined to a wedge smaller than half the poloidal cross-section. This electric field can cause a radial $\underline{\mathcal{E}}_\theta \times \underline{B}_T$ plasma flow which could convect plasma from the hot center to the cold exterior (i.e., a distance r_p) during the time of the experiment unless

$$I_0 t < \frac{B_T \, \epsilon_0 (2\pi R_o)(\pi r_p^2)}{\tau \, \partial G/\partial \theta} \left(\frac{r \, r_p}{\lambda_D^2}\right) \sim 0.03 \text{ Amp.-sec.}$$

Conclusion: In the ORMAK device tangential neutral beam injection with $I_0 \tau \sim 0.03$ and $I_0 t \sim 0.3$ (a single beam) is expected to be sufficient to cause significant plasma heating, and in particular ion heating. The above estimates indicate that at this beam strength the plasma perturbations are small, except for the buildup of radial electric fields greater than those in quasi-equilibrium Tokamaks without injection [cf. (3), (4) above], the net effect of which is unclear. Perpendicular injection, while having some advantages, appears to cause more serious poloidal electric fields and hence radial $\underline{\mathcal{E}} \times \underline{B}$ plasma flow or convection.

References

[*] On leave of absence from M.I.T.

[+] Operated by Union Carbide Corporation for the U.S.A.E.C.

[1] See for example: M. N. Rosenbluth, P. H. Rutherford, J. B. Taylor, F. A. Frieman, L. M. Kovrizhnikh in Plasma Physics and Controlled Nuclear Fusion Research (IAEA, Vienna, 1971), Vol. I, p. 495.