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DIFFUSION OF URANIUM AND ALUMINUM-SILICON EUTECTIC ALLOY

By

D. R. Green

Fuels Development
Reactor and Fuels Research and Development Operation

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TABLE OF CONTENTS

	Page
ABSTRACT	6
I. INTRODUCTION	7
II. SUMMARY	8
III. LITERATURE SURVEY	9
IV. EXPERIMENTAL TECHNIQUES	10
A. Preparation of Couples	10
B. Diffusion Anneals	12
C. Identification of the Diffusion Zone	14
D. Measurement of the Diffusion Zone	17
V. DISCUSSION	19
A. Theoretical Considerations	19
B. Statistical Method of Obtaining the Maximum Penetration in Each Couple	20
VI. RESULTS AND EVALUATION	25
A. Analysis of Diffusion Data and Determination of Maximum Penetration Coefficient Values	25
B. Thermal Cycling Effect	27
C. Effect of Silicon Content	27
D. Grain-Boundary Diffusion	27
E. A Comparison of U/AlSi Diffusion to That of U/Al	29
TABLES	30
REFERENCES	35
APPENDIX I	37
APPENDIX II	50

ABSTRACT

The maximum rates of U/AlSi diffusion at temperatures of 200, 250, and 300 C (392, 482, 572 F) have been determined. Maximum overall penetration, x, is approximately related to the time, t, and the absolute temperature, T, by the equation, $x = (K_o e^{-Q/RT} t)^{1/2} = (K_2 t)^{1/2}$, where K_o is a constant, Q the activation energy (assumed constant over the small temperature range involved), and R the gas constant. The values of K_o and Q are $9.7 \times 10^{10} \frac{\text{mils}^2}{\text{hr}}$ and $3.15 \times 10^4 \frac{\text{cal}}{\text{mole}}$, respectively, for the diffusion of uranium into AlSi, and $6.75 \times 10^9 \frac{\text{mils}^2}{\text{hr}}$ and 2.97×10^4 cal/mole, respectively, for the diffusion of AlSi into uranium. A small number of thermal cycles between annealing temperature and room temperature during the anneal decreased the final value of K_2. Variations in silicon content of the AlSi between 11 and 14 per cent had no measurable effect on the value of K_2, possibly because of masking by other variables. Grain boundary diffusion rates were higher than in immediately adjacent metal in some instances, but no general correlation of maximum penetration with visible grain boundaries existed. UAl_3 was the major phase identified in the diffusion layer. Another phase was found by microhardness methods, but could not be identified.

DIFFUSION OF URANIUM AND ALUMINUM-SILICON EUTECTIC ALLOY

I. INTRODUCTION

The diffusion of uranium through the aluminum-silicon eutectic (AlSi) is a significant factor affecting use of AlSi bonded aluminum or aluminum alloy jacketed fuel elements. Obviously, this type of bonding is worthy of consideration because of its relatively low cost, excellent adherence to both uranium and aluminum, low thermal neutron cross-section, and high thermal conductivity.

Commercially pure aluminum is limited to temperatures below 200 C (392 F) in water because of intergranular corrosion; however, aluminum alloys that resist intergranular attack to about 300 C (572 F) have been developed. The development of these alloys has increased the potential range of application of aluminum in water insofar as corrosion is concerned. Another factor equally as limiting as corrosion is possible penetration of the jacket by uranium. The AlSi bond could act as a barrier to diffusion, provided the penetration rate was less than that in aluminum.

Three basic mechanisms have been advanced to explain solid state diffusion. These are vacancy diffusion, interstitial diffusion, and ring diffusion.[1] All of these mechanisms assume that an atom must acquire sufficient energy to overcome a potential barrier in order to move from one position to another. Although satisfactory quantitative applications of these theories to complex systems such as U/AlSi are not possible, one may consider qualitatively the effect of irradiation on diffusion. Theory and experimental evidence indicate that intense radiation can be expected to furnish more than enough energy to displace the atoms from their normal lattice positions.[2] This means that one would expect irradiation to affect the rate of solid state diffusion. All experiments conducted to determine

the rate of solid state diffusion between various metals during irradiation indicate, however, that irradiation did not greatly change the diffusion rates.[3,4,5,6,7,8] In view of this disagreement between theory and experiment, one cannot predict the effect of irradiation on U/AlSi diffusion. The main purpose of the study outlined in this paper was to determine the rate of diffusion of U/AlSi and to furnish a basis for interpreting the results of U/AlSi couples irradiated in the Materials Testing Reactor during diffusion anneals of approximately the same durations and temperatures as the non-irradiated couples described here. The maximum rate of diffusion of uranium into AlSi from the original U/AlSi interface was determined at nominal temperatures of 200, 250, and 300 C for times of 15, 30, and 45 days. In addition, the diffusion of AlSi into uranium was measured.

II. SUMMARY

The maximum penetration coefficient for the diffusion of uranium into AlSi and AlSi into uranium was determined by statistically analyzing data from couples annealed at temperatures between 200 and 300 C (392 and 572 F). The rate of U/AlSi diffusion can be described by an equation, $x^2 = K_2 t$, where x is the maximum distance of the diffusion product/AlSi or diffusion product/U boundary from the original U/AlSi interface, t is the time of anneal (units of x and t conform to units of K in results), and K_2 is the maximum penetration coefficient. This coefficient determines the temperature dependence of the diffusion rate according to the relation $K_2 = K_o e^{-Q/RT}$, where K_o is a constant, Q is the activation energy (considered constant over the small range of temperature involved), R is the gas constant, and T the absolute temperature. The values of K_o and Q determined were 9.7×10^{10} mils^2/hr and 3.15×10^4 cal/mole, respectively, for the diffusion of uranium into AlSi; and 6.75×10^9 mils^2/hr and 2.97×10^4 cal/mole, respectively, for the diffusion of AlSi into uranium. A more exact, but much less convenient,

equation describing the diffusion of uranium into AlSi is $x^{3.3} = K_3 t$ where the variables have the same significance as before. This equation yields about the same results as the more conventient parabolic relationship when annealing times and conditions comparable to those used to obtain the data are considered; extrapolation to much longer times using each equation would yield different answers as can be readily demonstrated. Values of K_3 for the diffusion of uranium into AlSi are given by the equation $K_3 = 1.27 \times 10^{20} e^{-5.24 \times 10^4/RT}$ mils$^{3.3}$/hr for temperatures within the range covered. The rate of penetration was less in couples thermally cycled during the anneal than in similar couples that were not cycled. This could be due to cracking of the brittle diffusion product because of the differences in thermal expansion of uranium, AlSi, and the diffusion products. Variations in silicon content of the AlSi between 11 and 14 per cent did not measurably affect the value of K_2, possibly because the effect was masked by those due to other variables. The rate of diffusion observed at some grain boundaries was higher than in immediately adjacent metal, but there was no general correlation of maximum penetration with visible grain boundaries. The major constituent of the diffusion product was identified as UAl_3 by X-ray diffraction. Other phases in the diffusion zone could not be detected by this method, but small amounts of a second phase were detected by metallographic and microhardness techniques.

Information gained and techniques of analysis developed during this study should minimize many of the problems in the interpretation of results from couples irradiated in the Materials Testing Reactor during their diffusion anneals.

III. LITERATURE SURVEY

An extensive literature search disclosed only one previous quantitative U/AlSi diffusion study.[9, 10, 11, 12, 13, 14] The annealing temperatures used in this study ranged from 325 C to 550 C (615 to 1020 F); unfortunately, attempts to determine diffusion rates at temperatures lower than 325 C (615 F) failed.[14]

IV. EXPERIMENTAL TECHNIQUES

A. Preparation of Couples

For reasons explained in Section V, under Theoretical Considerations, the experimental conditions were carefully controlled to insure maximum diffusion rates. Obviously this meant that the diffusion interfaces had to be as free as possible from contaminants and held in close contact during the diffusion anneal. A previous study has indicated that a process called vacuum cathodic etching is superior to chemical etching for removing surface contaminants prior to the diffusion anneal.[15]

The diffusion specimens consisted of beta heat treated uranium disks and eutectic AlSi disks 1/2-inch in diameter by 1/4-inch thick. One surface of each disk was polished to a mirror finish, flat to within 0.0002-inch except near the edges. The diffusion interfaces were then degreased with xylene, dried with acetone, and vacuum cathodically etched in a dynamic krypton atmosphere at a pressure of 75 microns. A potential of 900-3500 volts, and a current density of approximately three milliamperes per square inch were maintained for 1-1/2 hours. After being etched, the specimens were removed from the etching bulb into a gloved box containing an atmosphere of helium which had been purified by passing it over zirconium turnings at 600 C (1110 F). Equipment used in etching the specimens and the gloved box in which they were removed from the bulb and assembled are shown in Figure 1. Mass spectrographic analysis of the atmosphere in the box showed it to be 99.96 per cent helium; the remaining 0.04 per cent was air. The couples were assembled three at a time into stainless steel clamps. Axial clamping pressure was about five tons per square inch at room temperature. During heating, however, the specimens expand more than the clamp. This causes a slight yielding of the AlSi, which insures uniform contact at the U/AlSi interface and limits the pressure exerted by the clamp to some low value approximating the elastic limit of AlSi at the annealing temperature.

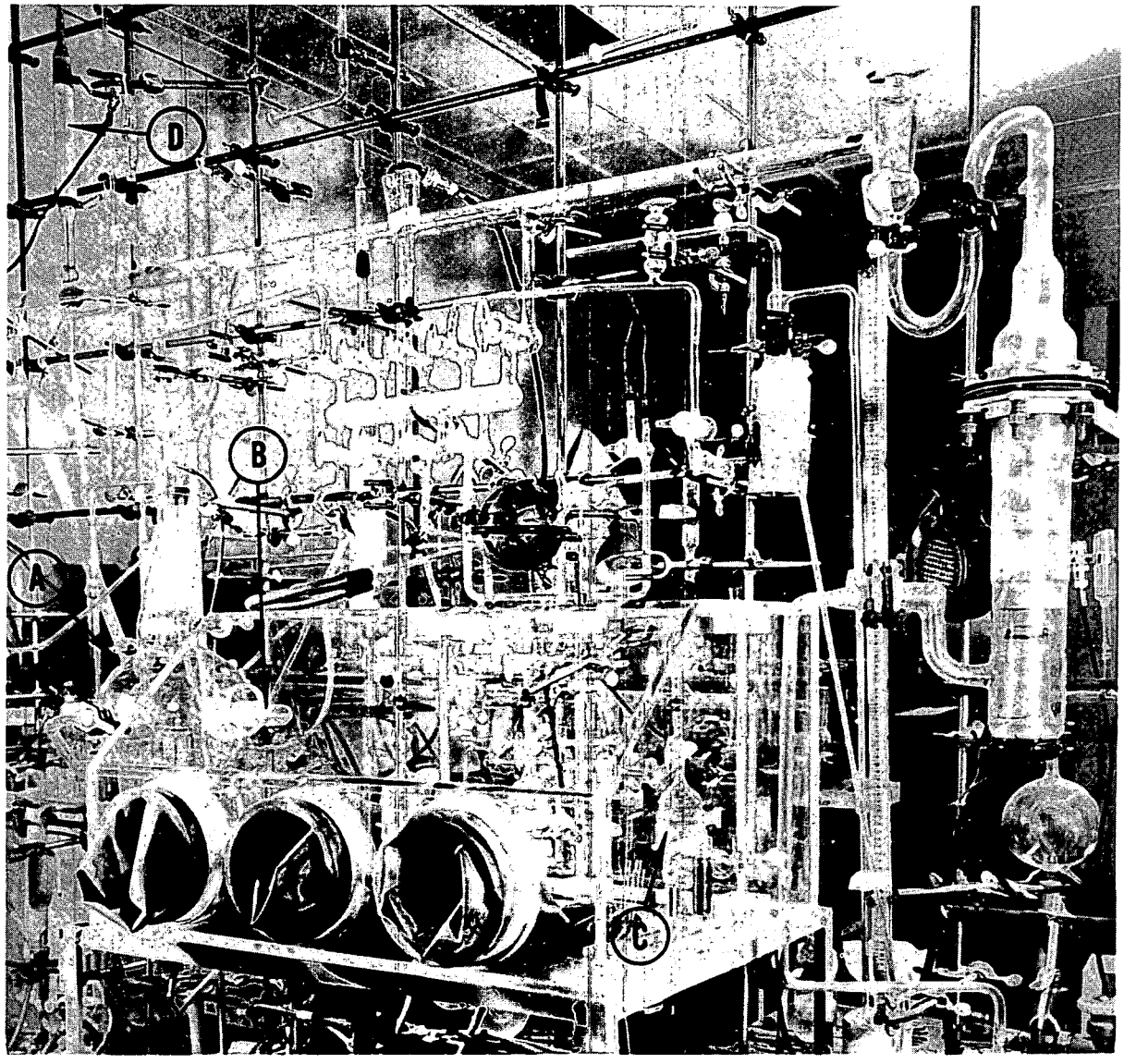

FIGURE 1

Equipment Used in Cleaning Diffusion Interfaces and Assembling Couples

(A) Vacuum Cathodic Etching Bulb
(B) Fitting which Allows Direct Transfer of Specimens from Etching Bulb to Gloved Box after Filling the Bulb with Helium
(C) Vacuum Storage Capsules within Gloved Box
(D) Furnace Containing Zirconium Turnings to Getter the Helium

AEC-GE RICHLAND, WASH.

B. Diffusion Anneals

The diffusion anneals were carried out in a vacuum of less than 0.1 micron pressure. Quartz furnace tubes were used in conjunction with tube furnaces, as shown in Figure 2. Temperature at the diffusion interfaces was controlled to ± 2 C and recorded with an accuracy of ± 5 C throughout the anneals; variation along the furnace length used was ± 2 C at any instant. Temperatures were measured by means of two thermocouples placed beside the specimens in each tube. The first thermocouple of each pair was chromel-alumel and the other was platinum-platinum 13 per cent rhodium calibrated to ± 1/2 C. One of these was used in recording the temperature and the other was periodically used to cross check the reading of the first in order to minimize errors due to thermocouple drift. Temperatures measured by these thermocouples were initially correlated to the actual temperatures at the diffusion interfaces by placing a thin chromel-alumel thermocouple at the interface of a dummy diffusion couple. It was assumed that the temperature correlation was the same in subsequent anneals at the same temperature. Thirty-three diffusion couples were annealed for 15, 30, and 45 days at nominal temperatures of 200, 250, and 300 C, three couples being annealed at each possible time-temperature combination. This was accomplished in some cases by placing three clamps, containing three diffusion couples each, in a vacuum furnace at the appropriate temperature. After 15 days at temperature one of the clamps was removed and the other two annealed for another 15-day period. The second clamp, having been annealed for 30 days, was then removed and the third annealed for the final 15-day period. The thermal cycles involved in this operation could have caused cracking of the diffusion zone as discussed in Section VI, A, under Analysis of Diffusion Data and Determination of Maximum Penetration Coefficient Values. The number of thermal cycles each couple received does not always correspond to the annealing time. This number is shown in Table III for each couple.

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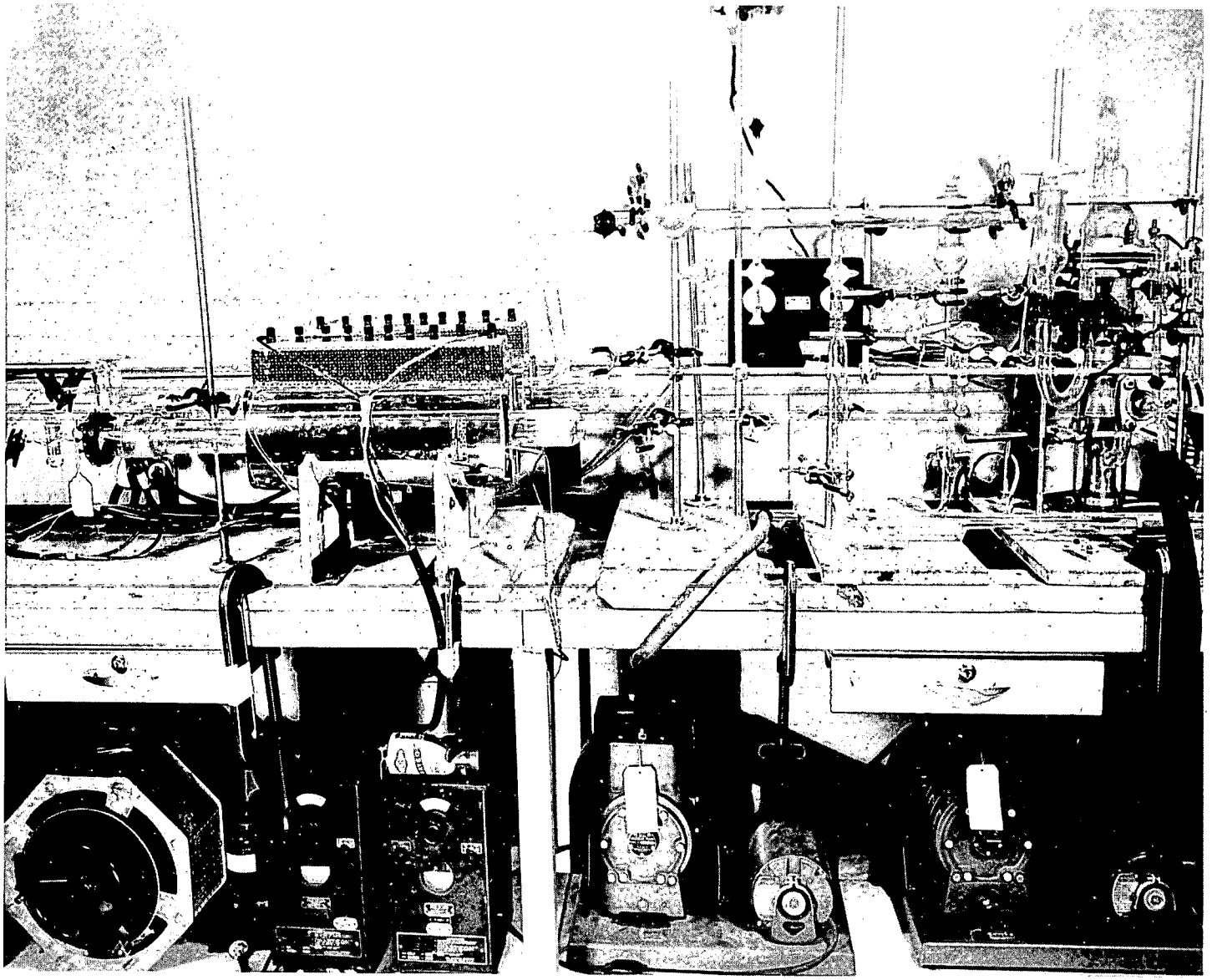

FIGURE 2

Vacuum Annealing Furnaces and Associated Equipment

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C. Identification of the Diffusion Zone

A basic problem in any solid state diffusion study utilizing microscopic measurement of the diffusion zone is the verification that the metallographically observed extent of the diffusion zone is the true diffusion depth, and that the material observed is actually the diffusion product. This problem becomes increasingly difficult at lower annealing temperatures because the diffusion zones are so small, even after long annealing times, as comparison of Figures 3A, B, and C will show. The diffusion product in couples annealed at 200 C appears as small peaks projecting into the AlSi along the interface as illustrated in Figure 3A.

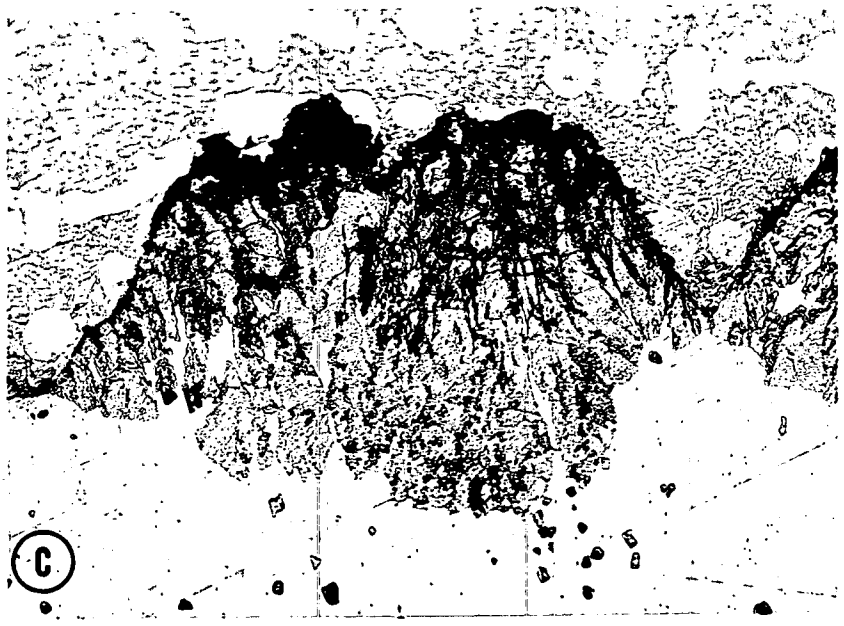
Many small peaks similar to those of diffusion product appeared to project approximately 0.0005-inch into the AlSi in the first few 200 C couples. A close comparison of appearance showed that these projections were not the result of diffusion, but were surface uranium which was dragged into the AlSi under scratches during post-anneal grinding preparatory to metallographic examination. It was found that grinding parallel to the U/AlSi interface and removing more metal on the 600 grit paper after grinding minimized this problem. The diffusion products in couples annealed at all three temperatures covered in this study are similar in appearance as can be seen by further comparison of Figures 3A, B, and C. Composition of the diffusion zone was determined by means of microhardness measurements and X-ray diffraction. The microhardness measurements indicated that there were at least two phases present as shown in Figure 4. Examination of Figures 3A, B, and C shows that the second phase is not always distributed uniformly and parallel to the diffusion interface as is sometimes the case in solid state diffusion, but occurs in various shapes and locations within the diffusion zone. Indentification of the diffusion product by X-ray diffraction indicated that the main phase was UAl_3. Additional phases could not be detected by this method, possibly because such relatively small quantities were present.

AlSi

U

AlSi

U



AlSi

U

FIGURE 3

The Effect of Temperature on Diffusion in U/AlSi Couples Annealed 15 Days at (A) 200 C, (B) 250 C, (C) 300 C.
Lightly Oxidized after Polishing. Magnification 250X

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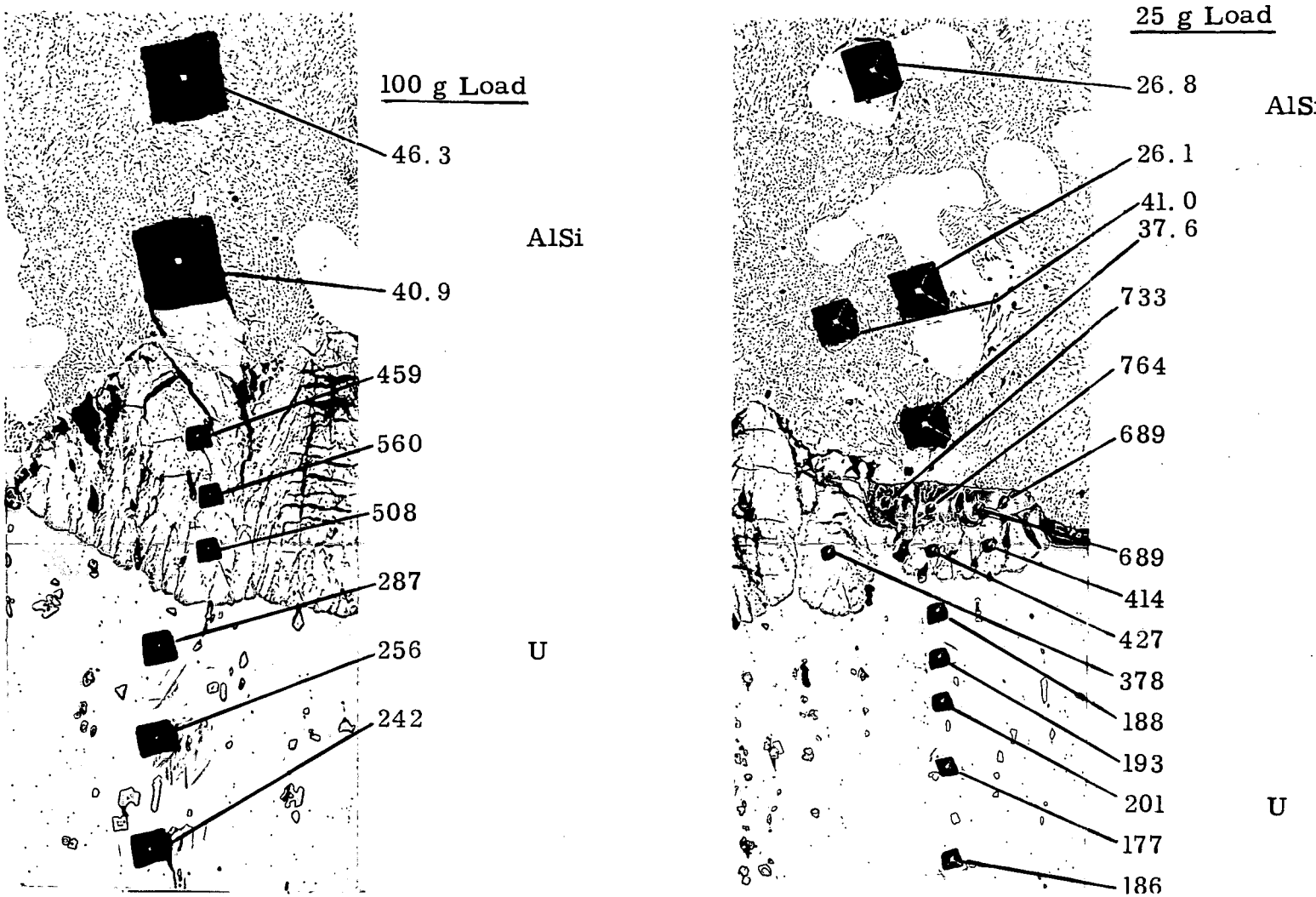

FIGURE 4

Diamond Pyramid Hardness Traverses Indicating the Presence of at Least Two Phases in the Diffusion Zone of Couple No. 34

D. Measurement of Diffusion Zone

The shape of the U/AlSi diffusion zone is very irregular as illustrated by Figure 3. This makes more than one measurement necessary to determine the maximum extent of diffusion. Two methods of measurement were tried. In the first method, the couple was broken apart at the diffusion interface in order to measure the maximum height of the diffusion product above the original uranium interface. A previous investigation has shown that this method works well on low temperature U/Al diffusion couples. The hard diffusion product adheres entirely to the uranium and pulls away from the softer aluminum at the aluminum/diffusion zone interface.[15] Unfortunately, the U/AlSi diffusion product does not pull away from the AlSi in the same manner, but often fractures through the diffusion zone. In addition, it could not be applied to irradiated couples with certainty so the method was discarded. A second, more conventional, method was used to obtain the results that appear in this paper. The couples were cross-sectioned so that six successive planes perpendicular to the diffusion interface were obtained, as illustrated by Figure 5. The first plane was produced at a distance of 0.092-inch from the outer edge and successive planes were produced at 0.063-inch intervals, as discussed under Section V, B, Statistical Method of Obtaining the Maximum Penetration in Each Couple. The final polish on each plane was eliminated to minimize relief due to the difference in hardness between the uranium and the AlSi and to save polishing time; about 200 per cent more time is required for a final polish. Scratches that appear on the surfaces shown in some of the illustrations are due to two-micorn abrasive and do not affect the results. The maximum extent of diffusion in each plane was measured with a filar eyepiece at magnifications of either 250 or 500 diameters, and the resulting data were analyzed by a statistical method described later.

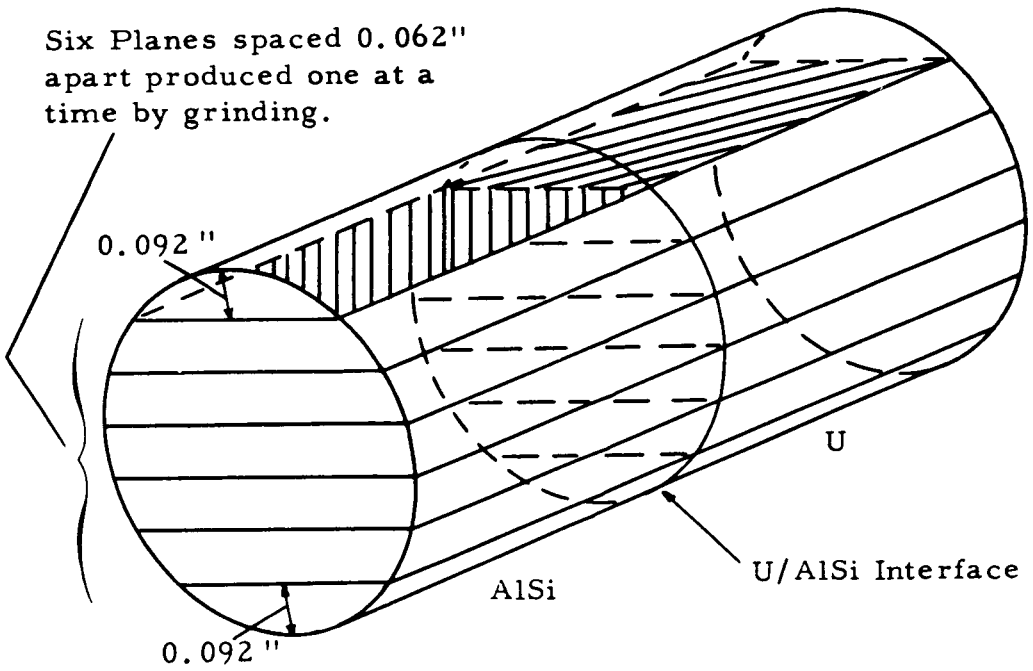

FIGURE 5

Six Successive Planes Provided in Each Couple by Cross Sectioning

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V. DISCUSSION

A. Theoretical Considerations

Well-known mathematical treatments of diffusion, such as the Matano or the Grube method, do not apply to systems that form diffusion zones having discontinuities in composition due to formation of intermetallic compounds.[16] Systems forming such compounds are not readily evaluated as to the relative magnitudes of ring, vacancy, and interstitial diffusion. Bückle[17] has shown that the displacement of the boundary of a particular phase from the original interface of a couple, in diffusion zones of continuous composition, but are discontinuous at phase boundaries, is given approximately by

$$x^2 = K_1 Dt = K_2 t \qquad (1)$$

where x is the distance of the phase boundary from the original interface, K_1 is a constant, D is the diffusion coefficient, K_2 a constant containing D, and t is the time. This equation has been verified by many experimenters and is now widely used to describe diffusion rates. Equation (1) is sometimes applied in a more general form,[18]

$$x^n = K_3 t \qquad (2)$$

The temperature dependence of the diffusion coefficient may be represented as

$$D = D_o e^{-Q/RT} \qquad (3)$$

where D_o is a constant, Q is the activation energy (assumed constant over the small range of temperature investigated), R the gas constant, and T the absolute temperature.[19] Obviously,

$$K_2 = K_o e^{-Q/RT} \qquad (4)$$

where K_o is a constant containing K_1 and D_o.

The maximum penetration which would occur under ideal conditions for a given time, temperature, pressure, and composition should be comparatively reproducible since it represents the condition where such limiting variables as surface contamination and lack of contact between the uranium and AlSi interface approach a minimum. These variables inhibit penetration by a non-reporducible amount and quantitative expressions for the magnitude of their effects are unknown. Also, there is a small finite probability that a maximum rate of diffusion will occur under all reasonable conditions. Therefore, the experimental conditions were maintained as ideally as practicable and the data were statistically analyzed to yield the maximum K values for each couple as described under Section V, B, and in Appendix I.

A large temperature gradient in the vicinity of the diffusion zone will affect the rate of diffusion; however, it was assumed on the basis of high thermal conductivity of AlSi, allowing that the temperature gradient due to normal heat fluxes would be small enough compared to the average temperature, that the effect on diffusion would be negligible. The pressure effects should not be a significant factor since the load applied was realtively small during the diffusion anneals. It was limited by the elastic limit throughout the study (see Section IV, A, Preparation of Couples).

B. Statistical Method of Determining the Maximum Penetration in Each Couple

The irregular shape of the U/AlSi diffusion zone, as shown in Figure 3A, B, and C, presents a severe problem in estimating the maximum extent of diffusion. In addition, the amount of penetration produced in a couple is very sensitive to experimental conditions which, although carefully controlled, could cause erroneous results. To overcome these problems, statistical methods of analyzing the data were used.

The couples were cross-sectioned to provide six successive planes perpendicular to the diffusion interface. The first plane was produced 0.093-inch from the outer edge, as shown in Figure 6, to escape the effect of edge roundness of the diffusion interfaces. This roundness, which existed prior to the diffusion anneal as a natural consequence of polishing, prevented normal diffusion in that region. Each successive plane was ground to permit examination at 0.063-inch intervals, and the maximum penetration in each plane was measured as described in Section IV, D, Measurement of the Diffusion Zone. These values are shown in Table I. The collection of maxima for each couple was analyzed statistically, by J. L. Jaech of Statistics Operation, to determine the over-all maximum penetration in the couples. Two slightly different methods of analysis were used; one for the 300 and 250 C (572 and 482 F) couples, and the other for the 200 C (392 F) couples. Theory and assumptions used in applying these methods are explained in detail in Appendix I. This theory will, however, be described in less detail in the following text.

The form of analysis of the 250 and 300 C (572 and 482 F) couples was determined from couple No. 1, which was annealed 30 days at 250 C. The couple was cross-sectioned after annealing to provide ten planes rather than the six determined later to be a sufficient number. The extent of diffusion across each of the ten planes was systematically observed at 0.015-inch intervals giving a total of 261 observations, about 80 per cent of which were non-zero values. These non-zero values, which are shown in Table IX of Appendix I, were found to be normally distributed and were used in conjunction with a normal distribution curve to determine the over-all maximum penetration in the couple. The maximum penetration observed in each plane was also recorded. From these data the following relationship was derived:

$$c = 3.46\ M \qquad (5)$$

where c is the theoretical over-all maximum and M is the over-all average penetration of either uranium into AlSi, or vice versa in couples annealed at 250 C.

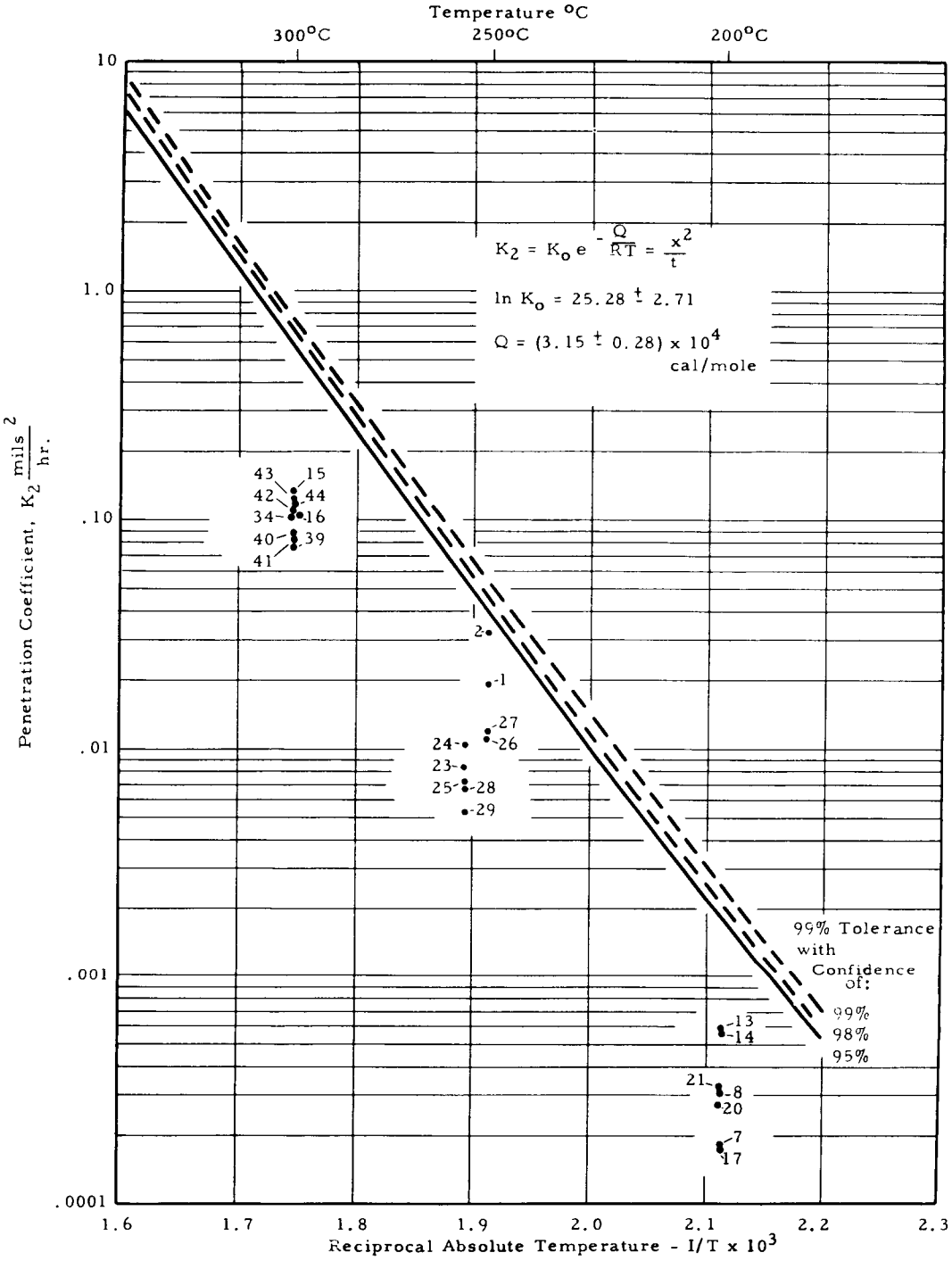

FIGURE 6

Maximum Anticipated Penetration Coefficient, K_2, for the Diffusion of Uranium into AlSi from the Original U/AlSi Interface, Based on 95 Per Cent Confidence Limits

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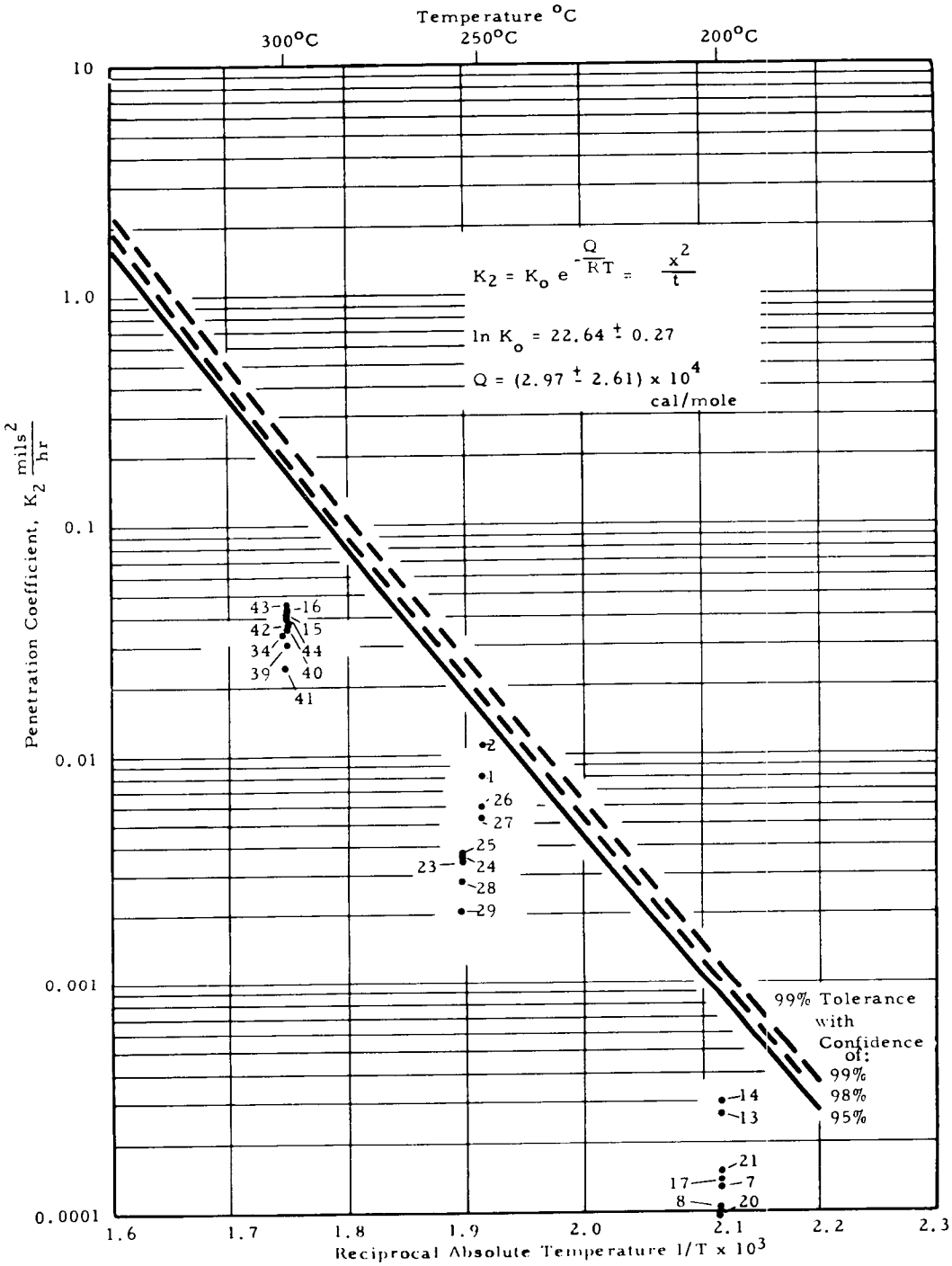

FIGURE 7

Maximum Anticipated Penetration Coefficient, K_2, for the Diffusion of AlSi into Uranium from the Original U/AlSi Interface, Based on 95 Per Cent Confidence Limits

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To use equation (5) one must first determine the average penetration M_p in each plane by applying the data of Table I in Appendix I to the observed maxima, X_L, in each plane. The over-all average, M, is then the median of these values and can be substituted into equation (5). Equation (5) does not apply to couples annealed at 300 C. Analysis of the data appearing in Table X of Appendix I showed that some of the constants involved in the equation change and it becomes

$$c = 4.41 M \qquad (6)$$

where M is determined from Table II of Appendix I. The new value for the coefficient of M was determined from the data in Table X in the same manner as it was obtained from Table I for the 250 C couples.

Analysis of the data from couples annealed at 200 C was more difficult since only a small amount of diffusion product forms after a reasonable annealing time. These couples were cross-sectioned to provide six planes, and the maximum penetration in each plane was measured in the same manner as described for the higher temperature couples. In addition, the magnitudes of all points of diffusion occurring in couples Nos. 17, 20, and 21 were recorded. These values are shown in Table XI, Appendix I. Utilization of these data yielded the following expression for the theoretical over-all maximum:

$$c = M (1 + 0.35 c^*) \qquad (7)$$

where c^* is the normalized median of the maxima from the six planes, and the other terms have the same significance as before. This time, however, M is determined by multiplying X_L for each of the six planes by the factor shown in Table VII of Appendix I and finding the median. The factor, $(1 + 0.35 c^*)$ is given in Table VIII as a function of the number of planes in which diffusion actually occurred. It is interesting to note that a relatively large change in the factors such as shown in Tables I, II, and VII (Appendix I) or in the assumed confidence limits are required to produce a small change in the final results as illustrated by the dashed

lines in Figures 6 and 7. These lines represent a change in confidence limits at a given over-all tolerance level.

VI. RESULTS AND EVALUATION

A. Analysis of Diffusion Data and Determination of Maximum Penetration Coefficient Values

The observed maximum penetrations for the individual planes in each couple are shown in Table I, page 30. From these data the statistical over-all maximum penetration shown in columns 5 and 6 of Table II, page 32, was computed for each couple by the methods outlined in Section V, B. Equation (1) of Section V, A, Theoretical Considerations, was used to determine the maximum penetration coefficient, K_2, from the statistical over-all maximum penetrations of Table II. The values of K_2 for each couple are shown in columns 7 and 8 of Table II. Values of the constants, K_o, and the activation energies, Q, of Equation (4) are shown in Figures 6 and 7. This equation relates values of the penetration coefficient, K_2, to the absolute temperature T. The values of K_2, in turn, relate the maximum over-all penetration, x, which is the distance that the diffusion zone/AlSi or diffusion zone/uranium phase boundary has moved from the original U/AlSi interface, to the time, t, by means of equation (1). Obviously, equations (1) and (4) can be combined as:

$$x = (K_2 t)^{1/2} = (K_o e^{-Q/RT} t)^{1/2}. \tag{8}$$

Consider now, for a moment, equation (2). The values of n and K_3 are, for the penetration of uranium into AlSi,

$$n = 3.3$$
$$K_3 = 1.27 \times 10^{20}\ e^{-5.24 \times 10^4/RT}\ \frac{\text{mils}^{3.3}}{\text{hr}};$$

so that

$$x = (1.27 \times 10^{20}\ e^{-5.24 \times 10^4/RT}\ t)^{\frac{1}{3.3}} \tag{9}$$

The curve represented by equation (9) fits the data slightly better than that represented by equation (8), but the difference is insignificant when annealing times comparable to those used to obtain the data are considered; equation (8) is more convenient to use. Values of ln K_2 versus the inverse of the absolute temperature are shown in Figures 6 and 7.

Note that two of the K_2 values (in Figures 6 and 7) at 250 C stand out above the others. The care with which the temperature was measured indicates that the temperature at the diffusion interface was known within ± 5 C; the error would have been about 25 C to account for the variation of these two points. All other variables such as surface contamination, lack of contact, etc., neglecting pressure, would have caused the value of K_2 for these points to be lower rather than higher. Obviously, these points represent more nearly perfect conditions for diffusion than any of the others. Statistical estimation based on the data showed that a very large number of observations would be necessary to yield a fair probability of observing maxima at 200 and 300 C corresponding to the two high points at 250 C. Therefore, rather than obtaining a large amount of data at these temperatures, estimation of maximum K_2 was based on an over-all 99 per cent tolerance limit as shown in Figures 6 and 7. This means that 99 out of 100 values should fall under the 99 per cent line with the confidence indicated, the latter being determined by the value selected for $Pr(x_i < c)^{19,400}$ in equation (5) of Appendix I. Final results were not very sensitive to the choice of confidence limits, as a comparison of the three confidence limits illustrated in Figures 6 and 7 will show. The true maximum of K_2 should be close to the 0.95 confidence line which is a good compromise between statistical certainty and and physical reality. The difficulty in obtaining maximum diffusion rates indicates that some of the limiting variables have not been entirely eliminated. Note that the maximum penetrations shown in columns 5 and 6 of Table II for couples Nos. 32, 33, 34, 36, 37, 38 were 40 per cent low; values of K_2 for these couples were not included in the results plotted in Figures 6 and 7. Many

factors such as cracking of the diffusion zone due to thermal cycling, or interface contamination during etching and handling, could have been responsible for these low values. Although all of the spurious variables have not been eliminated, they surely have been minimized by the methods used in both preparation of the couples and analysis of the data.

B. Thermal Cycling Effect

As mentioned in Section IV, B, Diffusion Anneals, some of the diffusion couples were thermally cycled between the annealing temperature and room temperature once every 15 days during the anneal. It is interesting to note that without exception, highest values for K_2 occurred in couples which were not cycled during the anneal. Also, in most cases, the second highest values for K_2 occurred in couples that were cycled once, and the lowest in couples cycled twice during the anneal. The rank of K_2 for each diffusion couple and the number of thermal cycles the couple was subjected to during the anneal are shown in Table III, page 34.

C. Effect of Silicon Content

The silicon content of the AlSi was considered as a possible variable affecting K_2, but chemical analysis showed no detectable correlation between small variations in silicon content and the diffusion rate, possibly because of masking by other variables. Results of this analysis are shown in Appendix II.

D. Grain-Boundary Diffusion

Diffusion at some uranium grain boundaries was slightly greater than in the adjacent material, as shown in Figure 8. The rate of diffusion did not appear to be grain boundary dependent, however, since the points of maximum penetration in a particular plane did not always correspond to visible grain boundaries.

FIGURE 8

Increased Diffusion Rate at Some Grain Boundaries
Electrolytic Etch; Polarized Light 500X

E. A Comparison of U/AlSi Diffusion to That of U/Al

A comparison of U/Al[15] to U/AlSi diffusion rates shows that about 10 times as much diffusion would occur in a U/Al couple annealed at 200 C (392 F) as in a U/AlSi couple annealed under the same conditions. The diffusion rates of U/AlSi and U/Al appear to be equal, however, at about 350 C (662 F) since the activation energy for the diffusion of uranium into aluminum is lower than that for the diffusion of uranium into AlSi.

TABLE I

Maximum Penetration of AlSi into Uranium and Uranium into AlSi Observed in Each Plane*

Plane	1	2	3	4	5	6	7	8	9	10
Couple Number 1	A**2.50	2.30	2.50	2.27	2.08	2.71	3.18	2.16	2.43	0
	U 1.40	1.30	1.10	1.85	1.30	1.80	1.72	1.51	1.69	0

	Plane Number 1	2	3	4	5	6
Couple Number 2	A2.96	3.40	3.82	3.84	2.66	2.46
	U1.76	1.85	2.06	2.04	1.51	1.99
7	A0.20	0	0.25	0.12	0	0
	U0.20	0	0.17	0.09	0	0
8	A0.32	0.18	0	0	0.23	0
	U0.19	0.09	0	0	0.15	0
13	A0.18	0	0.23	0.18	0.25	0.44
	U0.11	0	0.16	0.18	0.16	0.26
14	A 0	0	0.28	0.25	0.18	0.27
	U 0	0	0.19	0.21	0.21	0.11
15	A5.51	4.36	2.01	5.34	4.96	5.37
	U3.52	2.64	1.58	3.12	2.59	3.04
16	A4.85	4.75	4.32	4.57	4.35	4.43
	U2.83	2.60	3.24	3.15	3.15	2.85
17	A 0	0	0.10	0.15	0.15	0
	U 0	0	0.10	0.12	0.11	0
20	A0.16	0.32	0.42	0.26	0.26	0.40
	U0.11	0.26	0.19	0.11	0.19	0.23
21	A0.28	0.30	0.40	0	0	0
	U0.25	0.16	0.30	0	0	0
23	A1.57	2.06	1.69	1.88	1.65	1.18
	U0.74	1.23	0.93	1.18	1.37	1.00
24	A0.95	1.76	2.01	1.90	1.60	2.20
	U0.74	1.14	1.32	1.09	1.13	0.92
25	A1.80	1.90	1.37	1.76	1.28	0
	U1.30	1.27	0.97	1.14	1.13	0

*Planes referred to consist of sections normal to the diffusion interface which were examined metallographically for penetration. Plane numbers refer to each section cut at intervals of 0.063 inch in six-plane couples as described in text.

**A means penetration into AlSi, and U means penetration into uranium, from the original U/AlSi interface in mils (0.001 inch).

TABLE I (Contd.)

	Plane Number 1	2	3	4	5	6
Couple Number 26	A1.55	1.42	1.32	1.00	1.46	1.16
	U1.20	1.23	1.14	0.86	0.90	0.69
27	A1.46	1.46	1.48	1.72	1.30	1.18
	U1.06	0.93	1.30	0.95	0.92	0.74
28	A1.85	2.16	1.89	1.48	1.95	0.74
	U1.23	1.37	0.88	1.23	1.25	0.62
29	A2.20	1.44	1.50	2.43	1.76	1.37
	U1.37	1.00	0.95	1.44	1.09	0.92
31	A4.27	5.79	4.32	4.14	5.28	4.32
	U2.71	3.42	2.31	2.73	2.84	2.80
32	A4.79	3.73	4.22	3.04	4.21	3.26
	U2.43	2.08	2.55	1.85	2.24	2.08
33	A4.31	3.77	3.77	4.84	4.63	3.96
	U2.90	2.53	2.36	2.69	3.10	2.96
34	A4-42	4.72	3.52	5.10	4.09	4.67
	U2.64	2.73	1.94	3.52	2.41	2.60
36	A4.26	4.54	5.20	3.78	5.37	2.55
	U2.95	2.52	3.34	2.29	2.85	1.62
37	A4.50	4.00	5.28	4.68	4.15	4.22
	U2.96	2.57	2.92	3.06	2.50	2.62
38	A4.05	3.13	4.93	5.10	5.29	5.10
	U2.47	2.36	3.04	3.13	2.01	3.48
39	A2.99	7.28	8.05	7.61	6.22	4.47
	U2.06	3.98	4.68	5.16	4.49	2.80
40	A7.94	6.30	5.58	8.80	6.62	8.03
	U3.52	4.30	3.52	5.46	4.08	5.01
41	A4.76	4.83	7.58	7.38	8.55	4.14
	U3.06	3.70	4.58	3.89	4.92	2.24
42	A6.60	6.94	5.48	4.79	6.77	7.13
	U3.96	3.66	3.31	3.04	4.22	4.01
43	A2.83	6.24	7.72	7.05	7.55	6.77
	U2.18	3.86	5.03	4.42	4.75	3.87
44	A6.26	8.62	6.61	6.46	7.75	6.36
	U3.78	5.01	3.31	3.96	4.92	3.87

32

TABLE II

Maximum Penetration Values Determined Statistically, Based on 95 Per Cent Confidence Limits, Together with Penetration Coefficients for Each Couple

Couple Number	Time (Days)	Temperature C	Temperature F	Penetration of U into AlSi from Original U/AlSi Interface (mils)	Penetration of AlSi into Uranium from Original U/AlSi Interface (mils)	$K_2 = \frac{x_A^2}{t}$ mils ^{2}/hr for U into AlSi from Interface	$K_2 = \frac{x_U^2}{t}$ mils ^{2}/hr for AlSi into U from Interface
13	15	200	392	0.46	0.31	5.94×10^{-4}	2.70×10^{-4}
14	15	200	392	0.45	0.33	5.69×10^{-4}	3.06×10^{-4}
7	29	200	392	0.36	0.30	1.86×10^{-4}	1.29×10^{-4}
8	29	200	392	0.46	0.27	3.04×10^{-4}	1.05×10^{-4}
20	44	200	392	0.54	0.32	2.76×10^{-4}	9.68×10^{-5}
21	44	200	392	0.58	0.40	3.19×10^{-4}	1.51×10^{-4}
17	15	201	394	0.25	0.21	1.76×10^{-4}	1.24×10^{-4}
26	15	250	482	2.00	1.48	1.11×10^{-2}	6.08×10^{-3}
27	15	250	482	2.08	1.40	1.20×10^{-2}	5.44×10^{-3}
1	28	250	482	3.64	2.35	1.97×10^{-2}	8.20×10^{-3}
2	28	250	482	4.68	2.74	3.25×10^{-2}	1.11×10^{-2}
23	30	255	491	2.46	1.59	8.41×10^{-3}	3.51×10^{-3}
24	30	255	491	2.77	1.63	1.07×10^{-2}	3.69×10^{-3}
25	30	255	491	2.28	1.66	7.22×10^{-3}	3.83×10^{-3}
28	45	255	491	2.72	1.75	6.85×10^{-3}	2.83×10^{-3}
29	45	255	491	2.39	1.52	5.29×10^{-3}	2.14×10^{-3}
15	15	300	572	6.86	3.88	1.32×10^{-1}	4.23×10^{-2}
16	15	300	572	6.15	3.97	1.06×10^{-1}	4.43×10^{-2}

33

TABLE II (Continued)

Couple Number	Time (Days)	Temperature C	Temperature F	Penetration of U into AlSi from Original U/AlSi Interface (mils)	Penetration of AlSi into Uranium from Original U/AlSi Interface (mils)	$K_2 = \frac{x_A^2}{t}$ mils ^{2}/hr for U into AlSi from Interface	$K_2 = \frac{x_U^2}{t}$ mils ^{2}/hr for AlSi into U from Interface
34	15	300	572	6.13	3.57	1.06×10^{-1}	3.57×10^{-2}
31	30	300	572	6.61	4.05	5.18×10^{-2}	2.28×10^{-2}
32	30	300	572	5.71	3.22	3.92×10^{-2}	1.44×10^{-2}
33	30	300	572	6.12	4.05	4.47×10^{-2}	2.28×10^{-2}
42	30	300	572	8.95	5.14	1.11×10^{-1}	3.67×10^{-2}
43	30	300	572	9.44	5.73	1.24×10^{-1}	4.56×10^{-2}
44	30	300	572	9.15	5.38	1.16×10^{-1}	4.02×10^{-2}
36	45	300	572	6.47	3.94	3.53×10^{-2}	1.44×10^{-2}
37	45	300	572	6.47	4.01	3.51×10^{-2}	1.49×10^{-2}
38	45	300	572	7.02	4.01	4.26×10^{-2}	1.49×10^{-2}
39	45	300	572	9.19	5.78	7.82×10^{-2}	3.09×10^{-2}
40	45	300	572	9.81	6.24	8.91×10^{-2}	3.60×10^{-2}
41	45	300	572	9.75	5.18	8.80×10^{-2}	2.48×10^{-2}

TABLE III

Rank of Values of K_2 Corresponding to the Number of Thermal Cycles from Annealing Temperature to Room Temperature During the Diffusion Anneal

Couple Number	Temperature °C	Temperature °F	Time (Days)	Number of Cycles	Rank of K_2 for the Penetration of U into AlSi from the Original U/AlSi Interface*	$K_2 = \frac{x^2}{t}$ (mils^2/hr)
13	200	392	15	0	1	5.94×10^{-4}
14	200	392	15	0	2	5.69×10^{-4}
21	200	392	44	0	3	3.19×10^{-4}
8	200	392	29	0	4	3.04×10^{-4}
20	200	392	44	0	5	2.76×10^{-4}
7	200	392	29	0	6	1.86×10^{-4}
17	201	394	15	0	7	1.76×10^{-4}
18	200	392	30	1	8**	0
19	200	392	30	1	8**	0
2	250	482	28	0	1	3.25×10^{-2}
1	250	482	28	0	2	1.97×10^{-2}
27	250	482	15	0	3	1.20×10^{-2}
26	250	482	15	0	4	1.11×10^{-2}
24	255	491	30	1	5	1.07×10^{-2}
23	255	491	30	1	6	8.41×10^{-3}
25	255	491	30	1	7	7.22×10^{-3}
28	255	491	45	2	8	6.85×10^{-3}
29	255	491	45	2	9	5.29×10^{-3}
15	300	572	15	0	1	1.32×10^{-1}
43	300	572	30	0	2	1.24×10^{-1}
44	300	572	30	0	3	1.16×10^{-1}
42	300	572	30	0	4	1.11×10^{-1}
16	300	572	15	0	5	1.06×10^{-1}
34	300	572	15	0	5	1.06×10^{-1}
40	300	572	45	0	6	8.91×10^{-2}
41	300	572	45	0	7	8.80×10^{-2}
39	300	572	45	0	8	7.82×10^{-2}
31	300	572	30	1	9	5.18×10^{-2}
33	300	572	30	1	10	4.47×10^{-2}
38	300	572	45	2	11	4.26×10^{-2}
32	300	572	30	1	12	3.92×10^{-2}
36	300	572	45	2	13	3.53×10^{-2}
37	300	572	45	2	14	3.51×10^{-2}

* Ranked with highest values first.

** These couples did not adhere.

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APPENDIX I

Statistical Method Used to Estimate the Overall Maximum Penetration in Each Couple

Couple No. 1, which was annealed 30 days at 250 C, was cross sectioned to provide ten planes perpendicular to the diffusion interface. These planes were spaced 0.046-inch apart and the penetration was systematically measured at points spaced at 0.15-inch intervals across the diffusion interface in each plane, giving a total of 261 observations which are shown in Table IX of this Appendix. In addition to these systematic observations, the maximum penetration in each plane was measured; they appear in Table I of the text. From these extensive observations the following facts were determined:

(1) Only the observations within planes 2-9 at points 6-27 were useful; i.e., points within a 0.312-inch square in the diffusion interface. The data indicated that a maximum was not likely to occur outside this region. It was found, therefore, that the number of planes could be reduced to six without changing the results appreciably.

(2) Approximately 20 per cent of the observations were zeros for both the case of the diffusion of uranium into AlSi and that of AlSi into uranium.

(3) The non-zero observations approximate a normal distribution.

(4) The average penetration M_A, of uranium into AlSi was 1.04 mils and the standard deviation, σ_A, was 0.58 mils. Average penetration, M_U, of AlSi into the uranium was 0.69 mils and the standard deviation, σ_U was 0.36 mils.

$$\frac{\sigma A}{M_A} \approx \frac{\sigma U}{M_U} \approx 0.54 = \frac{\sigma}{M}$$

where M is the over-all average penetration for the couple and σ is the standard deviation. The following statistical procedure based on the data from Couple No. 1 was applied to estimate the maximum extent of diffusion in subsequent couples annealed at 250 C.

Assuming that if all points spaced 2 mils apart in each plane within the 312 mil region mentioned in fact (1) above could be observed, the resulting maximum of the 156 points observed in each plane would be the same as the true maximum in that plane. Also, it is assumed from fact (2) above, that 20 per cent of these points would be zero, the remaining 125 non-zero points being normally distributed. Calling X_L the largest penetration of these 125 points, and X_i the penetration at any point, we can write:

$$P = Pr(X_L < c) = Pr\,(\text{all 125 points} < c)$$
$$= Pr(X_i < c)^{125} = P_1^{125} \qquad (1)$$

if we assume statistical independence of the points involved. Suppose we select a value c_1 for c. We assume that this value is a median value. Based on this assumption, P in the above formulation has the value 0.50. Knowing P, we can find $P_1 = Pr(X_i < c)$. Normalizing this we have

$$P_1 = Pr(X_i < c_1) = Pr\left(\frac{X_1 - M}{\sigma} < \frac{c_1 - M}{\sigma}\right) \qquad (2)$$

where M is the overall average for the couple and σ the standard deviation. Since P_1 is known, the quantity $\frac{c_1 - M}{\sigma}$ can be found from a table of normal distribution. We can then solve for M in terms of c_1 by using the result of fact (4) above. Thus, since $P_1^{125} = 0.50$, $P_1 = 0.99446$. From a table of normal distribution we have

$$\frac{c_1 - M}{\sigma} = 2.54 \qquad (3)$$

Solving this equation for M we have, since $\frac{\sigma}{M} = 0.54$,

$$M = 0.42\, c_1 \qquad (4)$$

which means that the over-all average for the couple can be estimated by multiplying the median of the maxima by 0.42. This is in excellent agreement with the average of actual experimental observed ratios of M_P, the average for a plane, to X_L which appear below.

$\frac{M_P}{X_L}$ for uranium into AlSi	$\frac{M_P}{X_L}$ for AlSi into uranium
0.38	0.38
0.36	0.53
0.53	0.43
0.43	0.51
0.51	0.49
0.35	0.38
0.31	0.39
0.44	0.40
Avg. $= \frac{M}{c_1} = 0.41$	Avg. $= \frac{M}{c_1} = 0.44$

Further examination of the data from couple No. 1 at this time indicated that refinement in the calculation of this ratio is possible. It may be seen that the $\frac{M_P}{X_L}$ ratio for a plane decreases as the size of the maximum penetration increases relative to the other maxima observed in the couple. Rather than taking $P = 0.50$ for each plane, we can let P equal the ratio of the rank of the maximum observed in the plane to one plus the number of observations (six planes in later couples). Values for these refined $\frac{M_P}{X_L}$ ratios appear in Table I.

TABLE I

Ratio of $\frac{M_P}{X_L}$ for Six Planes in Couple No. 1.

Rank of maxima (smallest = 1)	$\frac{M_P}{X_L}$
1	0.464
2	0.444
3	0.431
4	0.416
5	0.401
6	0.380

In practice, it was found that using Table I to determine M_P from observed X_L values,and finding M from these, gave only slightly different results than those obtained by multiplying X_L from each plane by 0.42 and using the mean value.

Assuming that a 312 mil square area of the diffusion interface is used, and that the diffusion is measured at points located on the corners of all 2 mil squares contained in this area, we have a total of 19,400 non-zero points. It was previously assumed that one of these observations would be the true over-all maximum. If we change our assumption of this spacing from 2 mils to 4 mils the final result is not affected appreciably; therefore, the final result is not very sensitive to this assumption. Given an over-all median of the average penetration, M, for the couple as determined by applying Table I to the observed X_L's, and σ from the relationship $\sigma = 0.54$ M, we can determine the value c such that the probability of exceeding c in the 19,400 points is 0.05 assuming a confidence of 0.95. It was found that the final result was not extremely sensitive to the value chosen for the confidence limits as illustrated in Figure 6 and 7. The following equations resulted from these considerations:

$$\Pr(X_L < c) = 0.95$$

$$\text{or} \quad \Pr(X_i < c)^{19,400} = 0.95 \qquad (5)$$

Normalizing this, it becomes

$$\left[\Pr\left(\frac{X_i - M}{\sigma} < \frac{c - M}{\sigma}\right)\right]^{19,400} \tag{6}$$

Solving for c we get

$$c^* = \frac{c-M}{\sigma} = 4.55$$

$$\text{or} \quad c = 4.55\sigma + M \tag{7}$$

$$c = 3.46\ M$$

The over-all maximum penetration, c, expected in the couple can now be calculated by determining the over-all median of M with Table I and using this value in equation (7).

The theory used in the analysis of couples annealed at 300 C was the same as that described above. The values of $\frac{M_P}{X_L}$ ratios were changed, however, since the ratio of $\frac{\sigma}{M}$ was found to be 0.75. Also $\frac{M_P}{X_L}$ was observed to be 0.31 on the average for two extensively measured planes in couples 30 and 31. Data used to determine these ratios are shown in Table X. This information was used to determine Table II in the same manner that Table I was obtained for 250 C couples.

TABLE II

Rank of maxima (1 = smallest)	$\frac{M_P}{X_L}$
1	0.335
2	0.323
3	0.314
4	0.305
5	0.295
6	0.282

Table II may be used to find the median value of M_P, which is the value for M, and this result used in the equation

$$c = 4.41\ M \tag{8}$$

to obtain the over-all maximum penetration for the couple.

The method used to analyze the 200 C couples was slightly different from the method applied to couples annealed at higher temperatures: the assumption that 80 per cent of the points in a 2 mil x 2 mil lattice would be non-zero was not valid. Actual penetrations at all points along the diffusion interface in each plane of couples Nos. 17, 20, and 21 were recorded, as shown in Table XI, in addition to the maxima from each plane, since diffusion in these low temperature couples was found only at a few isolated points. A relation between the total number of planes containing non-zero points and the total number of non-zero points found is shown in Table III.

TABLE III

Couple Number	Observed Number of Planes With Non-Zero Points	Observed Total Number of Non-Zero Points
17	3	7
20	6	27
21	6	36

Note that the total number of non-zero points increases approximately as the square of the number of planes having non-zero points. This is reasonable since if a number of non-zero points expected in one direction along the interface increased, they should be expected to increase an equal amount in the perpendicular direction. This relation between column two and three in Table III gives rise to Table IV.

TABLE IV

Observed Number of Planes with Non-Zero Points	Theoretical Total Number of Non-Zero Points (average)
3	8
4	14
5	21
6	31

Observations indicate that the penetration in the couples annealed at 200 C would have to be measured at the corners of 0.5 mil squares within the diffusion interface similar to the 2 mil squares mentioned previously in order to find the over-all maximum penetration. Assuming this, there would be 625 theoretical observation positions in each plane and 625 theoretical planes. The total number of points at which non-zero values of penetration would be observed was determined from Table IV by multiplying the total number of non-zero points per plane by the theoretical number of planes with non-zero points. These values are shown in Table V.

TABLE V

Actual Observed Number of Planes Having Non-Zero Points	Theoretical Total Number of Non-Zero Points in Planes Spaced 1/2 mil Apart
3	830
4	1460
5	2180
6	3220

The last column of Table V supplies the figures corresponding to the 19,400 figure found for the higher temperature couples. Corresponding to the c* value of 4.55 found in that case, we get c* values shown in Table VI for the 200 C couples.

TABLE VI

Observed Number of Planes With Non-Zero Points	c*
3	3.84
4	3.98
5	4.07
6	4.16

Again, as before, the theoretical over-all maximum penetration is $c = c^*\sigma + M$. Observations on couples Nos. 17, 20, and 21 show that on the average, $\frac{\sigma}{M} = 0.35$. Therefore, the estimate of c becomes

$$c = 0.35\, c^* M + M = M(1 + 0.35\, c^*)$$

The ratio of $\frac{M_P}{X_L}$ in each plane was observed to average 0.732, and as before, the theoretical computation of this ratio agrees with the empirical value. To obtain this value theoretically, we proceed in the same manner as before, but replace the exponent of P_1 in equation (1) by the average number of points per plane as taken from Table IV, and get Table VII.

TABLE VII

Number of Planes Containing Non-Zero Points	Non-Zero Points per Plane	$\frac{M_P}{X_L}$
3	2.67	0.794
4	3.50	0.759
5	4.20	0.735
6	5.17	0.713

The factor, $(1 + 0.35\, c^*)$ depends upon the number of planes observed to have non-zero points; it is tabulated in Table VIII.

TABLE VIII

Number of Planes Having Non-Zero Points	(1 + 0.35 c*)
3	2.34
4	2.39
5	2.43
6	2.46

The over-all maximum penetration of the uranium into AlSi and AlSi into uranium can be calculated from the observed maxima in six planes by merely finding M and multiplying by the appropriate factor from Table VIII.

46

APPENDIX I

TABLE IX

Observations of Penetrations at Points Spaced 0.015-inch Apart Along the Diffusion Interface in Each Plant Produced in Couple No. 1*

Observation No.	Plane No. 1		2		3		4		5		6		7		8		9		10	
	A	U	A	U	A	U	A	U	A	U	A	U	A	U	A	U	A	U	A	U
1	----	----	----	----	----	----	0	0	0	0	0	0	0	0	----	----	----	----		
2	----	----	----	----	0	0	0	0	0	0	0.72	0.30	0	0	----	----	----	----	0	0
3	----	----	----	----	0	0	0	0	0.09	0.07	0.90	0.72	1.06	0.70	0	0	----	----	0	0
4	----	----	0	0	0	0	0.12	0.06	1.25	0.84	0	0	0	0	0	0	----	----	0	0
5	----	----	0	0	0.95	0.49	0.18	0.09	0.33	0.19	0.58	0.53	0.44	0.28	0.70	0.65	0	0	0	0
6	----	----	0	0	0.35	0.14	1.07	0.70	0.30	0.37	0.39	0.23	0.18	0.12	0.81	0.56	0	0	0	0
7	----	----	0.50	0.37	0	0	0.19	0.09	0.03	0.02	0.25	0.23	0.99	0.46	1.32	1.11	0	0	0	0
8	0	0	0	0	0	0	0.19	0.09	1.35	0.90	0.40	0.48	0.72	0.48	1.32	1.37	0	0	0	0
9	0	0	0	0	1.14	0.83	0.65	0.44	1.18	0.81	1.94	0.88	0.88	0.40	0.28	0.48	0	0	0	0
10	1.79	1.06	1.50	0.87	1.32	0.90	0.62	0.46	1.35	0.90	1.58	1.44	1.18	0.70	0.48	0.46	0.62	0.32	0	0
11	0	0	0	0	1.20	0.60	0.32	0.14	0.42	0.39	1.81	1.23	0.86	0.63	0.14	0.12	2.11	1.27	0	0
12	1.67	1.06	0	0	0	0	1.76	1.02	0.74	0.58	1.46	0.93	0.28	0.69	0.49	0.23	0.92	0.37	0	0
13	0	0	0	0	0	0	1.58	0.93	1.34	1.05	1.11	0.63	1.64	1.07	0.44	0.30	0.46	0.53	0	0
14	0	0	0	0	0.65	0.49	1.60	1.12	1.76	0.97	2.27	1.48	1.41	0.91	1.11	0.95	0.39	0.39	0	0
15	0	0	0	0	0.40	0.28	1.36	0.85	1.76	0.93	1.50	1.18	0.62	0.65	1.21	0.93	0.09	0.05	0	0

47

TABLE IX (Continued)

Observation No.	Plane No. 1 A	1 U	2 A	2 U	3 A	3 U	4 A	4 U	5 A	5 U	6 A	6 U	7 A	7 U	8 A	8 U	9 A	9 U	10 A	10 U
16	0.26	0.12	0	0	0	0	1.93	1.30	0.72	0.56	1.20	0.97	1.62	1.13	1.50	0.92	0.95	1.55	0	0
17	0	0	0	0	0	0	1.32	1.25	1.50	0.88	1.97	1.05	0.33	0.11	0.32	0.25	0.11	0.05	0	0
18	0	0	0.08	0.03	0	0	1.62	1.02	0.74	0.40	1.88	1.00	0.86	0.55	0.21	0.21	2.13	1.43	0	0
19	0	0	0	0	0	0	0.99	1.05	0.90	0.77	1.02	0.48	1.58	0.86	0.21	0.07	0.77	0.56	0	0
20	0.79	0.53	0	0	0	0	2.20	0.90	0.81	0.62	2.02	0.97	0.18	0.16	0.12	0.37	0.74	0.53	0	0
21	0	0	0.32	0.19	0.32	0.19	2.04	1.11	1.37	0.93	1.36	0.65	0.92	0.51	0.76	0.44	1.72	0.99	0	0
22	0	0	0	0	0	0	1.55	1.07	0.65	0.62	1.21	0.83	1.21	0.83	0.16	0.35	1.72	1.14	0	0
23	0	0	0	0	0	0	1.67	0.95	0.76	0.53	1.51	0.88	2.46	1.21	1.06	0.93	0.49	0.19	0	0
24	0	0	0.99	0.56	1.53	0.95	1.25	0.74	0.76	0.70	1.50	1.07	0.51	0.39	0.90	0.70	0	0	0	0
25	0	0	1.36	0.90	1.21	0.95	0.70	0.56	0.35	0.32	1.35	1.06	1.87	1.11	1.14	0.93	0	0	0	0
26	----	----	1.09	0.46	0.54	0.40	0	0	0	0	1.57	0.81	1.57	0.69	2.01	0.79	0	0	0	0
27	----	----	0.53	0.28	1.18	0.63	0.55	0.35	0	0	1.25	0.77	1.95	1.14	0.69	0.49	0	0	0	0
28	----	----	0	0	0.90	0.67	0.09	0.11	1.28	0.69	0.83	0.21	0.99	0.65	0	0	----	----	----	----
29	----	----	0	0	0.76	0.42	0.04	0.05	1.63	0.95	0.56	0.42	0	0	0	0	----	----	----	----
30	----	----	----	----	0	0	0.18	0.04	0.88	0.63	0	0	0	0	0	0	----	----	----	----
31	----	----	----	----	0	0	1.16	0.37	0	0	1.55	0.69	1.11	0.11	----	----	----	----	----	----
32	----	----	----	----	----	----	0.32	0.14	0	0	0	0	0	0	----	----	----	----	----	----

* A indicates the penetration of uranium into AlSi from the original U/AlSi interface and U indicates the reverse.

TABLE X, APPENDIX I

Penetration at Thirty-Two Equally Spaced Points Along the Diffusion Interface in Plane No. 3 or (A), Couple No. 39, and (B), Couple No. 40

Observation Number	Penetration of U Into AlSi from Original U/AlSi Interface (mils)		Penetration of AlSi Into U from Original U/AlSi Interface (mils)	
	A	B	A	B
1	2.02	0	1.92	0
2	0.53	0	0.69	0
3	1.87	0	1.06	0
4	3.59	4.76	2.08	3.48
5	6.26	5.50	4.75	3.29
6	0.93	3.31	1.00	1.90
7	0	1.13	0	0.70
8	2.90	0.18	1.39	0.30
9	0	2.16	0	1.13
10	0	3.66	0	1.58
11	0	0	0	0
12	0.30	0	0.33	0
13	0	2.82	0	1.13
14	0	4.48	0	2.29
15	0	2.71	0	2.20
16	7.25	0.23	4.70	0.25
17	1.48	0.60	1.02	0.26
18	1.41	1.46	1.41	0.74
19	0.30	0.79	0.44	0.58
20	0.07	0.65	0.07	0.40
21	0	1.58	0	1.09
22	0	4.58	0	2.68
23	0	5.88	0	3.17
24	1.27	3.13	0.83	1.14
25	4.10	0.25	2.60	0.21
26	4.44	2.15	2.08	1.00
27	5.26	1.74	2.75	0.93
28	1.69	0.47	0.92	0.25
29	0	0	0	0
30	0.30	0	0	0
31	0.35	0	0.25	0
32	0.37	0	0.28	0

49

TABLE XI, APPENDIX I

All Penetrations Along the Interface in Each Plane of Couples Nos. 17, 20, and 21. Numbers in Body of Table are Penetrations (A), into AlSi, and (U) into Uranium, from the Original U/AlSi Interface in Thousandths of an Inch.

Couple No.	Observation No.	Plane Number* 1		2		3		4		5		6	
		A	U	A	U	A	U	A	U	A	U	A	U
17	1	--	--	--	--	0.1	0.1	0.15	0.12	0.15	0.11	--	--
	2	--	--	--	--	0.07	0.09	0.14	0.08	0.11	0.08	--	--
	3	--	--	--	--	--	--	--	--	0.08	0.09	--	--
20	1	0.16	0.11	0.32	0.26	0.42	0.19	0.26	0.11	0.26	0.19	0.40	0.23
	2	0.12	0.09	0.26	0.18	0.26	0.18	0.25	0.09	0.14	0.12	0.30	0.09
	3	--	--	0.30	0.19	0.07	0.12	--	--	0.19	0.18	0.16	0.12
	4	--	--	0.16	0.12	0.11	0.05	--	--	0.16	0.18	0.14	0.16
	5	--	--	--	--	--	--	--	--	0.19	0.14	--	--
	6	--	--	--	--	--	--	--	--	0.11	0.14	--	--
	7	--	--	--	--	--	--	--	--	0.14	0.11	--	--
	8	--	--	--	--	--	--	--	--	0.09	0.16	--	--
	9	--	--	--	--	--	--	--	--	0.14	0.14	--	--
	10	--	--	--	--	--	--	--	--	0.14	0.12	--	--
	11	--	--	--	--	--	--	--	--	0.19	0.12	--	--
21	1	0.28	0.25	0.30	0.16	0.40	0.30	--	--	--	--	--	--
	2	0.25	0.25	0.14	0.12	0.12	0.11	--	--	--	--	--	--
	3	0.23	0.16	0.12	0.09	0.19	0.12	--	--	--	--	--	--
	4	0.19	0.14	0.26	0.16	0.16	0.14	--	--	--	--	--	--
	5	--	--	0.16	0.16	0.16	0.12	--	--	--	--	--	--
	6	--	--	0.18	0.14	0.14	0.11	--	--	--	--	--	--
	7	--	--	0.23	0.14	0.11	0.09	--	--	--	--	--	--

* The maximum extent of all points of diffusion in each plane were recorded.

APPENDIX II

Silicon Content of the AlSi Used in Some of the Couples

Couple Number	Silicon Content
1	13.4
2	12.7
8	12.3
13	13.7
14	12.2
15	12.1
16	12.2
17	12.9
21	12.3
25	13.0
28	12.7
29	14.2
33	12.3
36	12.4
37	11.0

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