

July 1974

CONF-740756--1

Electron-Phonon Scattering in
Potassium at High Magnetic Fields*

H. Taub

Brookhaven National Laboratory, Upton, New York 11973

NOTICE

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Energy Research and Development Administration, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

MASTER

* Work performed under the auspices of the U. S. Atomic Energy Commission.

20

Abstract

We have measured the temperature dependence of both the zero-field resistivity and the transverse magnetoresistance of polycrystalline potassium wires ($\rho(300 \text{ K})/\rho(4.2 \text{ K}) = 140$ to 6000) in fields $H \lesssim 35 \text{ kG}$ and at temperatures $T \lesssim 4.2 \text{ K}$. Our principal findings are: 1) The presence of a large magnetic field $H = 35 \text{ kG}$ does not alter the temperature dependence of ρ from that observed at $H = 0$; below 4.2 K the T-dependent part of the resistivities, $\rho_T(H=0)$ and $\rho_T(H=35 \text{ kG})$, fit well to the function $\exp(-\Theta^*/T)$ with the same $\Theta^* = 23 \text{ K}$. 2) Deviations from Matthiessen's rule are significantly reduced in a strong field so that the magnitude of $\rho_T(H=0)$ approaches that of $\rho_T(H=35 \text{ kG})$ as sample purity decreases. 3) The slope of the high-field linear magnetoresistance increases slightly ($\lesssim 8\%$) from 1.5 K to 4.2 K . We attribute the exponential temperature dependence of $\rho_T(H)$ to the freezing out of electron-phonon umklapp processes as has been shown for the zero-field resistivity. The reduction in deviations from Matthiessen's rule at high fields can be understood within semiclassical theory, but the latter cannot explain the failure of $\rho_T(H)$ to saturate at high fields. A proposal by Young that electron-phonon umklapp scattering may contribute a T-dependent high-field linear magnetoresistance in potassium is considered.

I. Introduction

In this paper we shall be concerned with the effect of a strong magnetic field on the contribution of electron-phonon scattering to the electrical resistivity of a free-electron-like metal at low temperatures. The metal which we have chosen for this study is potassium since it is known to have a nearly spherical Fermi surface.¹ We report measurements of the temperature dependent part of the resistivity of potassium in a transverse field H ,

$$\rho_T(H) = \rho(H, T) - \rho(H, T=0)$$

as well as the corresponding zero-field resistivity,

$$\rho_T(H=0) = \rho(H=0, T) - \rho(H=0, T=0),$$

in samples of a wide range in purity. Our interest in this problem has been stimulated by the understanding of the electron-phonon contribution to the zero-field resistivity which has recently been achieved. Independent measurements of $\rho_T(H=0)$ in potassium^{2,3} agree well with each other and are also in good agreement with calculations^{2,4,5} which include umklapp scattering in the electron-phonon interaction. The identification of electron-phonon umklapp processes is simplified in potassium by a nearly exponential T -dependence of the resistivity⁶ at liquid helium temperatures. This is not true of other simple metals such as aluminum and indium where the Fermi sphere contacts the first Brillouin zone boundary, allowing umklapp processes to persist

to the lowest temperatures.

Another reason for investigating electron-phonon scattering in the magnetoresistivity of potassium is the presence of a linear magnetoresistance which has always been observed at high fields.⁷ In contrast to the zero-field resistivity, the linear magnetoresistance is generally not reproducible in measurements of different investigators and conflicts fundamentally with the semiclassical prediction of a saturating magnetoresistance at high fields. By studying the effect of high fields on a scattering mechanism which is relatively well understood at zero-field, one may gain a new perspective on the puzzling magnetoresistance of potassium.

II. Experimental Technique

Experimentally, the problem with the magnetoresistance of potassium has been to obtain reproducible results with this soft and reactive metal. Thus we thought it crucial to measure both $\rho_T(H=0)$ and $\rho_T(H=35 \text{ kG})$ of the same sample. Comparison with previous measurements of $\rho_T(H=0)$ ^{2,3} could then be used as a means of standardizing our samples as well as check on our measurement technique.

The samples used in this investigation are summarized in Table I. They are the same ones used in our previous study⁷ and range in purity from a residual resistance ratio $RRR = \rho(300 \text{ K}) / \rho(4.2 \text{ K})$ of 135 to 6090. The samples were helically-wound (pitch $\leq 30^\circ$) polycrystalline extruded wires with potential contacts ~ 80 cm apart located ~ 6 cm from the current contacts. The wires ranged from 1.5 mm to 2.2 mm diameter in the purer samples. Besides annealing each sample for at least one week at room

temperature,⁷ we also took the precaution of slow-cooling the samples to 77 K overnight. As can be seen from Table I, this can result in a doubling of the RRR in the purest samples as previously noted by Ekin and Maxfield.²

A conventional four-terminal dc technique was used for the resistivity measurements.⁷ We simultaneously measured $\rho(H=0)$ and $\rho(H=35 \text{ kG})$ at each temperature ($T \pm 4.2 \text{ K}$) by first measuring the sample voltage in one field polarity, at $H=0$, and then in the opposite field polarity. In addition, for each sample, $\rho(H,T)$ vs H was measured in both field polarities for at least two temperatures, 4.2 K and either 2.5 K or 1.5 K. After averaging out thermal and Hall voltages, we estimate a relative precision for ρ of $\pm 0.02\%$. Absolute resistivity values are computed using the specimen RRR and the room temperature resistivity $\rho(293 \text{ K}) = 7.19 \times 10^{-6} \text{ } \Omega\text{cm}$ reported by Dugdale and Guban.⁸ To compute ρ_T , the residual resistivity $\rho(T=0)$ was taken to be the resistivity at $T=1.5 \text{ K}$. This introduces negligible error in ρ_T for $T \geq 2.5 \text{ K}$. In the purest samples at 4.2 K, the relative precision of ρ_T is $\sim \pm 0.3\%$ with an accuracy of $\sim \pm 3\%$.

III. Experimental Results and Comparison with Previous Measurements

Our first observation is that the presence of a large transverse magnetic field does not alter the T -dependence of the resistivity of our samples below 4.2 K. Fig. 1 is a semi-log plot of $\rho_T(H=0)$ vs $1/T$ for a high-purity sample (lower curve) together with $\rho_T(H=35 \text{ kG})$ (upper curve). That is, we have repeated the same measurement at a field of 35 kG which is well into the range where the magnetoresistivity is linear in H (see Fig. 3). Although there is clearly a magnetoresistance effect, $\rho_T(H=35 \text{ kG}) > \rho_T(H=0)$,

the most striking feature is that both curves are characterized by the same exponential T-dependence. The lines drawn correspond to the function $\exp(-\Theta^*/T)$ with $\Theta^* = 23$ K. This value of Θ^* is in good agreement with previous measurements of Ekin and Maxfield² and Guban³ in the same temperature range. The magnitude of $\rho_T(H=0) = 0.282$ m Ω cm at 4.2 K also agrees, to within our experimental accuracy, with their values for samples of comparable purity.

The second observation we have made is that deviations from Matthiessen's rule are significantly reduced in a strong field. That is, $\rho_T(H=35$ kG) is less dependent on sample purity than $\rho_T(H=0)$. As shown in Table I, at 4.2 K $\rho_T(H=0)$ increases about 12% as sample RRR decreases from 6000 to 1000 while $\rho_T(H=35$ kG) remains constant to our accuracy of 3%. This uniformity in $\rho_T(H=35$ kG) occurs despite an order of magnitude range in the Kohler slope S of the dominant T-independent linear magnetoresistance. We do not observe a change in the T-dependence of either $\rho_T(H=35$ kG) or $\rho_T(H=0)$ with sample purity so that the upper curve in Fig. 1 remains fixed as sample purity decreases while $\rho_T(H=0)$ rises maintaining its slope. Hence the T-dependent part of the magnetoresistance is quenched as sample purity decreases.

We may regard the addition of impurities to a pure sample as having the same effect on ρ_T as a large magnetic field: both tend to increase ρ_T without altering its temperature dependence. This is well illustrated in Fig. 2 where we have plotted $\rho_T(H=0)$ for the lowest purity sample, RRR = 135, as measured by Ekin and Maxfield² and $\rho_T(H=35$ kG) for one of the highest purity samples, RRR = 5380. We see that the two curves very nearly coincide. The

similarity in the T-dependence of $\rho_T(H=0)$ and $\rho_T(H=35 \text{ kG})$ found here conflicts with a previous measurement of Babiskin and Siebenmann.⁹ They report a larger value of $\Theta = 28 \text{ K}$ for $\rho_T(H)$, $15 \text{ kG} \leq H \leq 35 \text{ kG}$, in measurements on encapsulated wire samples. We shall return below to possible reasons for this discrepancy.

We have also investigated the field dependence of $\rho_T(H)$ in our samples. In Fig. 3 the magnetoresistivity of the purest sample, RRR = 6090, is plotted at both $T = 4.2 \text{ K}$ and $T = 2.5 \text{ K}$. The high-field linear magnetoresistance is evident at both temperatures. To a good approximation, the difference between the curves also plotted in Fig. 3 will be the T-dependent part of the magnetoresistivity at 4.2 K, $\rho_T(H, 4.2 \text{ K}) \approx \rho(H, 4.2 \text{ K}) - \rho(H, 2.5 \text{ K})$. We see that $\rho_T(H)$ rises to a "knee" at $H \approx 10 \text{ kG}$ which is followed by a region linear in H. The failure of $\rho_T(H)$ to saturate by 35 kG implies that the slope of the high-field linear magnetoresistance is increasing slightly as the temperature is raised. This T-dependence of the slope was present in all the samples measured, the largest increase being $\sim 8\%$ for KX 26 in Fig. 3. The T-dependent component of the slope can be defined by

$$\Sigma_T = \frac{\delta \rho_T(H)}{\delta H} ; \quad 15 \text{ kG} \leq H \leq 35 \text{ kG}$$

which has dimensions $\Omega \text{ cm/kG}$ and is denoted by a Greek letter to distinguish it from the dimensionless Kohler slope S. Values of $\Sigma_T(4.2 \text{ K}) = \Sigma(4.2 \text{ K}) - \Sigma(2.5 \text{ K})$ have been listed in Table I. At both 4.2 K and 2.5 K, $\rho(H, T)$ was measured at 5 kG intervals in the range 15 kG to 35 kG, and at each temperature Σ was obtained from a least squares fit over the five field points. The magnitude of Σ_T tends to increase in the higher purity samples.

In the case of sample KX 13 we verified that the T-dependence of the slope persisted to 95 kG, although the value of Σ_T was somewhat smaller than for the other samples.

In the case of sample KX 13 we verified that the T-dependence of the slope persisted to 95 kG, although the value of Σ_T was somewhat smaller than for the other samples. The T-dependent linear term is a small fraction of the high-field linear magnetoresistance and was not observed in our previous study.⁷ In that work we estimated S graphically only to a relative precision of $\sim 3\%$ rather than subtracting the corresponding magnetoresistivities at each temperature point-by-point as done here. Babiskin and Siebenmann⁹ also did not observe a T-dependent component of the linear magnetoresistance of potassium. They found $\rho_T(H)$ to saturate at $H \sim 10$ kG for $T \leq 4.2$ K. As shown by the dashed line in Fig. 3 our value of $\rho(H, 4.5 K) - \rho(H, 2.5 K)$ is still approaching their value from below at 35 kG. Babiskin and Siebenmann do not specify the annealing time or cooling rate of their samples, but the principal difference in sample preparation and handling appears to be the plastic encapsulation which they used. The effect of dilational strain on the zero-field resistivity of potassium sealed in glass capillary tubes has been observed previously by Dugdale and Gufan,¹⁰ and they have cautioned against the use of encapsulated samples in T-dependence measurements. It is unfortunate that Babiskin and Siebenmann did not also measure $\rho_T(H=0)$ to confirm that it agreed with previous results.

IV. Discussion

Some time ago Kohler¹¹ derived the following equality between the ideal resistivities in the high-field and high-

impurity limits for a free electron gas:

Some time ago Kohler¹¹ derived the following equality between (1) the ideal $\rho_{el-ph}(H \rightarrow \infty) = \rho_{el-ph}^{impure}(H = 0)$.

impurity limits for a free electron gas:

(1) $\rho_{el-ph}(H \rightarrow \infty) = \rho_{el-ph}^{impure}(H = 0)$

Here $\rho_{el-ph}(H \rightarrow \infty)$ is the saturating transverse magnetoresistivity at high fields due to normal electron-phonon scattering in a sample of arbitrary purity and $\rho_{el-ph}^{impure}(H=0)$ is the electron-phonon contribution to the zero-field resistivity in the impure limit ($\rho_0 \gg \rho_{el-ph}$). Recently, Wagner¹² has extended Eq. (1) to include umklapp scattering using an approach based on the theory of Lifshitz, Azbel, and Kaganov.¹³

Qualitatively, Eq. (1) expresses the fact that a large magnetic field has the same randomizing effect on the free electron distribution function as isotropic impurity scattering. Wagner's derivation depends on the relation $\rho_{el-ph}(H \rightarrow \infty) = \rho_V\{\vec{k} \cdot \vec{E}\}$ ¹⁴ where $\rho_V\{\vec{k} \cdot \vec{E}\}$ is the variational expression for the electron-phonon contribution to the zero-field resistivity, evaluated for the trial function $\vec{k} \cdot \vec{E}$ (\vec{k} being the wave vector of the electron, \vec{E} the electric field). This trial function is exact in the case of a spherical Fermi surface for impurity scattering describable by a relaxation time, and it appears to be a good approximation for electron-phonon scattering in low purity samples of potassium.¹⁵ We note that Eq. (1) implies the validity of Matthiessen's rule at high fields. The residual and electron-phonon resistivities are additive since $\rho_{el-ph}(H \rightarrow \infty)$ is the same value independent of sample purity. In addition, if $\rho_{el-ph}(H=0)$ is the zero-field electron-phonon resistivity of a sample of arbitrary purity, then Eq. (1) can be rewritten as

$$(2) \rho_{el-ph}(H \rightarrow \infty) - \rho_{el-ph}(H=0) = \rho_{el-ph}^{impure}(H=0) - \rho_{el-ph}(H=0).$$

This states that the enhancement of the electron-phonon resistivity in the high-field limit is equal to the deviation from

Matthiessen's rule at zero field.

There are some features of our results which can be interpreted consistently within this semiclassical theory. First, the constancy of $\rho_T(H=35 \text{ kG})$ for samples of all purities indicates that Matthiessen's rule is obeyed at high fields. The equality of $\rho_T(H=0)$ of the lowest purity sample, KX 20, and $\rho_T(H=35 \text{ kG})$ of KX 27 shown in Fig. 2 also appears to agree with Eq. (1). Referring to Eq. (2), we can say that the magnetoresistance of KX 27, $\rho_T^{\text{KX 27}}(H=35 \text{ kG}) - \rho_T^{\text{KX 27}}(H=0)$ (the difference between the two curves in Fig. 1), is within experimental error equal to the observed deviation from Matthiessen's rule at zero field, $\rho_T^{\text{KX 20}}(H=0) - \rho_T^{\text{KX 27}}(H=0)$. Finally, our observation that a large magnetic field does not alter the T-dependence of ρ_T (see Fig. 1) can be related by Eq. (1) to previous observations^{2,3} that the T-dependence of $\rho_T(H=0)$ does not depend significantly on sample purity.

This semiclassical analysis depends on identifying the lowest purity sample with the impure limit of Eq. (1) and assuming that the high-field limit has been attained by 35 kG. Although at 4.2 K $\rho_0/\rho_T \approx 150$ for KX 20, $\rho_T(H=0)$ could continue to increase in samples of even lower purity. It would be valuable to check experimentally whether $\rho_T(H=0)$ becomes independent of purity in samples of RRR $\lesssim 100$. The failure of $\rho_T(H)$ to saturate by 35 kG raises the question of whether the high-field limit of Eq. (1) has been satisfied. Using the values of \sum_T in Table I, we estimate $\rho_T(H=100 \text{ kG}) = 0.42 \text{ n}\Omega\text{cm}$ and $0.37 \text{ n}\Omega\text{cm}$ for KX 26 and KX 17, respectively. This suggests that at extremely high fields one would begin to observe deviations from Matthiessen's rule

comparable to those found at zero field.

It is possible that the electron-phonon magnetoresistivity does saturate by 35 kG. The onset of the high-field regime could be associated with the "knee" in $\rho_T(H)$ (see curve at the bottom of Fig. 3) which occurs at ~ 10 kG or $\omega_c \tau \gg 10$ where $\tau = m/ne^2\rho(H=0, T)$ is an effective electron relaxation time and ω_c is the cyclotron frequency. However, this leaves open the question of the origin of the high-field linear term in $\rho_T(H)$. The other possibility which we have considered is that the linear term in $\rho_T(H)$ is due to electron-phonon umklapp scattering. The freezing out of electron-phonon umklapp processes is generally thought to be the source of the nearly exponential T-dependence of $\rho_T(H=0)$.²⁻⁶ Therefore, it seems reasonable to assume that umklapp scattering is also dominating $\rho_T(H=35 \text{ kG})$ which has the identical T-dependence. On the basis of a simple model of localized umklapp scattering on a spherical Fermi surface, Young¹⁶ has suggested that electron-phonon umklapp scattering could contribute an observable, T-dependent linear magnetoresistance in potassium. If so, we would expect the T-dependent slope of the linear term, $\bar{\rho}_T$, to have the same exponential T-dependence as $\rho_T(H=0)$ and $\rho_T(H=35 \text{ kG})$.⁶

As a test of Young's proposal we have measured $\bar{\rho}_T$ of one of the purest samples, KX 27, at temperatures below 4.2 K, using the fitting procedure described earlier. In Fig. 4 we have plotted $\bar{\rho}_T = \rho_T(T) - \rho_T(1.5 \text{ K})$ on a semilog scale vs $1/T$. The straight line indicates the functional dependence $\exp(-\Theta^*/T)$ with $\Theta^* = 23 \text{ K}$ as observed for $\rho_T(H=0)$ and $\rho_T(H=35 \text{ kG})$ in the same sample (Fig. 1). In the narrow temperature range near 4 K where the error bars are small enough to permit comparison, we

see that the fit to the line is satisfactory. A log-log plot yields a $\sim T^6$ dependence over the same temperature range. Thus we believe Young's proposal cannot be ruled out at this time. Clearly, it would be desirable to have higher precision measurements of Σ_T on pure samples at fields above 35 kG.

In summary, unlike the dominant T-independent linear term, the T-dependent component of the magnetoresistance of potassium is sufficiently reproducible to make detailed comparison with theory worthwhile. Although $\rho_T(H)$ fails to saturate at high fields, we have found several features of the T-dependent magnetoresistivity to be consistent with a semiclassical analysis. Further theoretical work on a non-saturating component in the magnetoresistance of potassium due to electron-phonon umklapp scattering would be of particular interest.

V. Acknowledgements

The measurements reported here were performed while the author was a research assistant in the Laboratory of Atomic and Solid State Physics, Cornell University. The author wishes to thank R. Bowers for guidance and consultations in the course of this investigation and D.K. Wagner, J.W. Ekin, and B.W. Maxfield for many helpful discussions.

References

1. D. Shoenberg and P.J. Stiles, Proc. Roy. Soc. ^(London) A281, 62 (1964).
2. J.W. Ekin, Phys. Rev. Lett. 26, 1550 (1971); J.W. Ekin and B.W. Maxfield, Phys. Rev. B 4, 4215 (1971).
3. D. Guban, Proc. Roy. Soc. ^(London) A325, 223 (1971).
4. P.N. Trofimenkoff and J.W. Ekin, Phys. Rev. B 4, 2392 (1971) and references cited therein.
5. M. Kaveh and N. Wiser, Phys. Rev. Lett. 29, 1374 (1972).
6. R.E. Peierls, Quantum Theory of Solids, first edition, Oxford University Press, London, 1965, p. 42 ff.
7. For a review of the linear magnetoresistance problem in potassium see H. Taub, R.L. Schmidt, B.W. Maxfield, and R. Bowers, Phys. Rev. B 4, 1134 (1971).
8. J.S. Dugdale and D. Guban, Proc. Roy. Soc. ^(London) A270, 186 (1962).
9. J. Babiskin and P.G. Siebenmann, Phys. Rev. Lett. 27, 1361 (1971).
10. J.S. Dugdale and D. Guban, J. Sci. Instr. 40, 28 (1963).
11. M. Kohler, Ann. Physik 38, 283 (1940).
12. D.K. Wagner, Phys. Rev. B 5, 336 (1972); see also D.K. Wagner, Ph.D. thesis, Cornell University, 1971 (unpublished).
13. I.M. Lifshitz, M. Ya. Azbel, and M.I. Kaganov, Zh. Eksperim. i Teor. Fiz. 30, 220 (1955); 31, 63 (1956) (Sov. Phys. JETP 3, 143 (1956); 4, 41 (1957)).
14. This relation is valid under the following conditions:
(i) the metal is cubic and the magnetic field is directed along a two-fold or higher symmetry axis; (ii) the Fermi surface consists of closed surfaces each with the above symmetry; (iii) the scattering rate is independent of magnetic field.

15. J.W. Ekin and A. Bringer, Phys. Rev. B 7, 4468 (1973).
16. R.A. Young, Phys. Rev. 175, 813 (1968).

Figure Captions

- Fig. 1. Semi-log plot of the T-dependent part of the resistivity of high-purity potassium at $H=0$ (lower curve) and $H=35$ kG (upper curve).
- Fig. 2. Semi-log plot of the T-dependent part of the resistivity of a high purity potassium wire at $H=35$ kG (this work) and a low-purity wire at $H=0$ as measured by Ekin and Maxfield, Ref. 2.
- Fig. 3. Field dependence of the transverse magnetoresistivity of potassium at 4.2 K and 2.5 K. Difference curve (below) approximates $\rho_T(H)$ ^{at 4.2 K.} The dashed line indicates the value of $\rho(H, 4.2 \text{ K}) - \rho(H, 2.5 \text{ K})$ measured by Babiskin and Siebenmann, Ref. 9.
- Fig. 4. Semi-log plot of the T-dependent part of the slope of the linear magnetoresistance of high-purity potassium. The error bars have been estimated assuming a precision of $\pm 0.02\%$ in $\rho(H, T)$.

Table Caption

Table I. Sample properties at 4.2 K.

Table I. Sample properties at 4.2 K.

| Sample | RRR | $\rho_T(H=0)$ ($10^{-10} \Omega \text{cm}$) | $\rho_T(H=35 \text{ kG})$ ($10^{-10} \Omega \text{cm}$) | $10^2 S^a$ | Σ_T ($10^{-13} \Omega \text{cm/kG}$) |
|--------|---------------------|--|--|------------|--|
| KX 20 | 135 | 3.64 ^b | | 0.47 | |
| KX 17 | 1020 | 3.11 | 3.55 | 0.64 | 3.2 |
| KX 7 | 1730 | 3.08 | 3.58 | 2.04 | |
| KX 27 | 2630 ^c | 2.96 | 3.60 | | |
| KX 13 | 2960 ^c | 2.91 | 3.61 | 0.38 | 1.0 ^d |
| KX 25 | 3240 ^c | 2.90 | 3.57 | 0.42 | 5.6 |
| KX 27 | 5260 | 2.82 | 3.63 | 0.41 | |
| KX 27 | 5380 ^e | 2.83 | 3.73 | 0.59 | 8.6 |
| KX 26 | 5470 ^{c,f} | 2.79 | 3.64 | 0.20 | 6.6 |
| KX 26 | 6090 | 2.89 | 3.77 | 0.24 | 8.4 |

^aKohler slope $S = \delta \rho_T / \rho_0 \delta H$ (in Ω)

^bAs measured by Ekin and Maxfield, Ref. 2.

^cSample quenched from 300 K to 77 K.

^d Σ determined for $40 \text{ kG} \leq H \leq 95 \text{ kG}$.

^eSecond run after annealing at 77 K for 12 weeks.

^fSample not annealed.

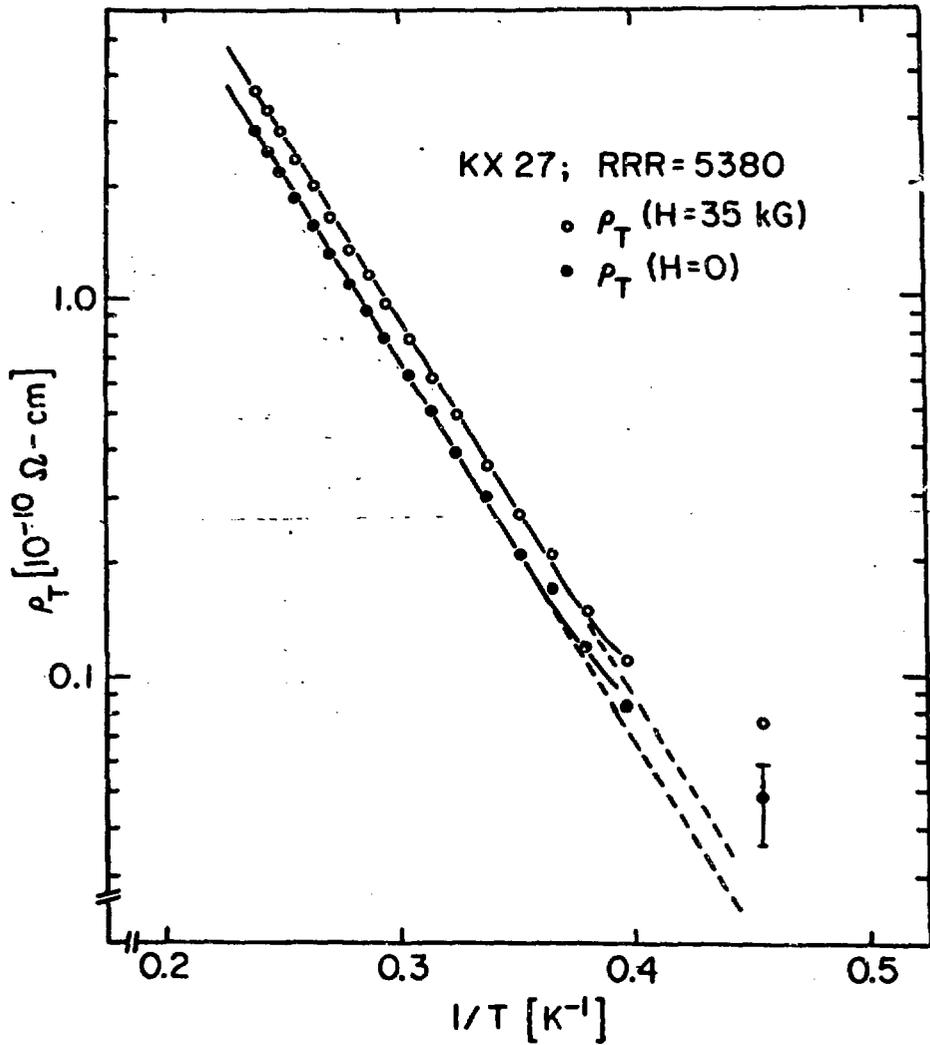


FIGURE 1

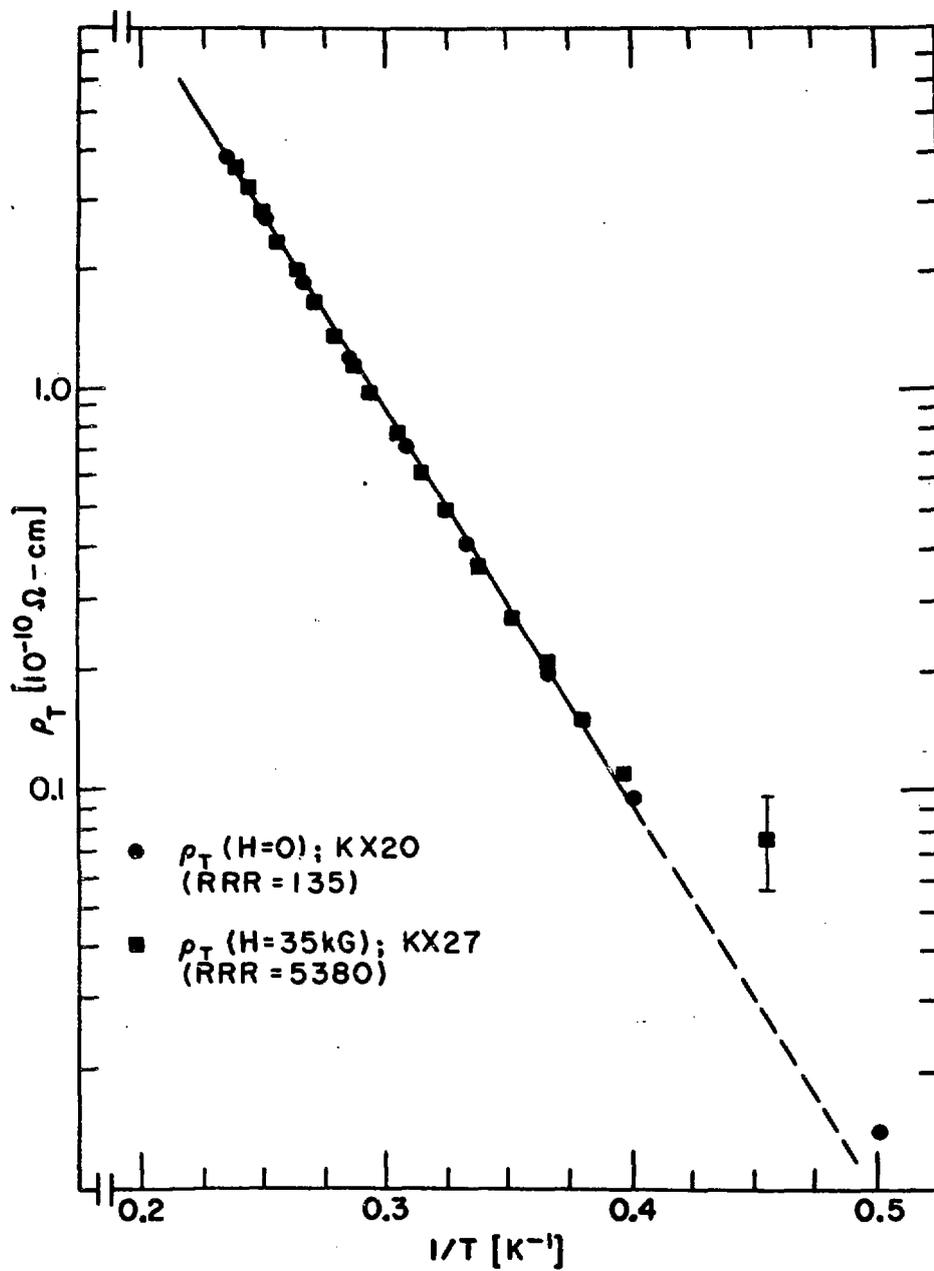


FIGURE 2

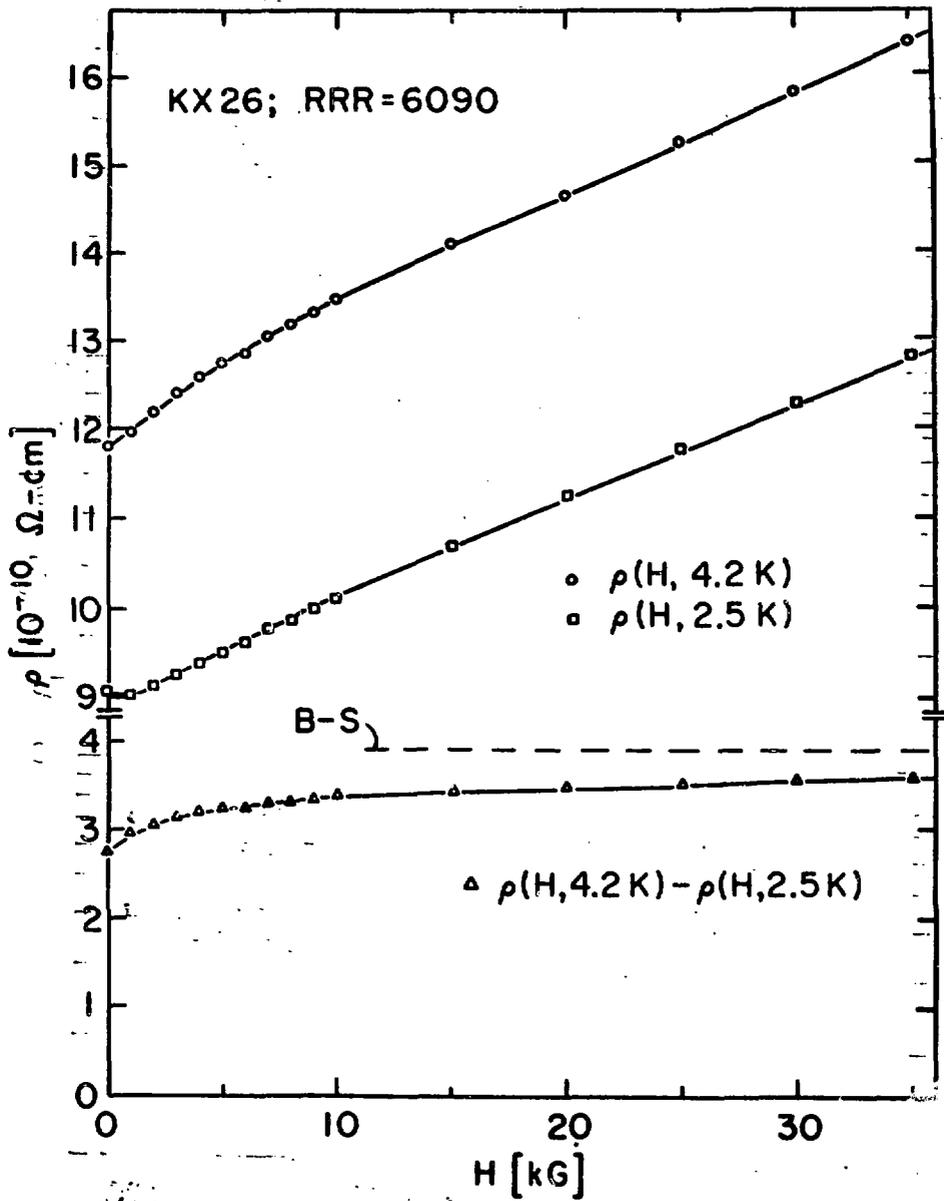


FIGURE 3

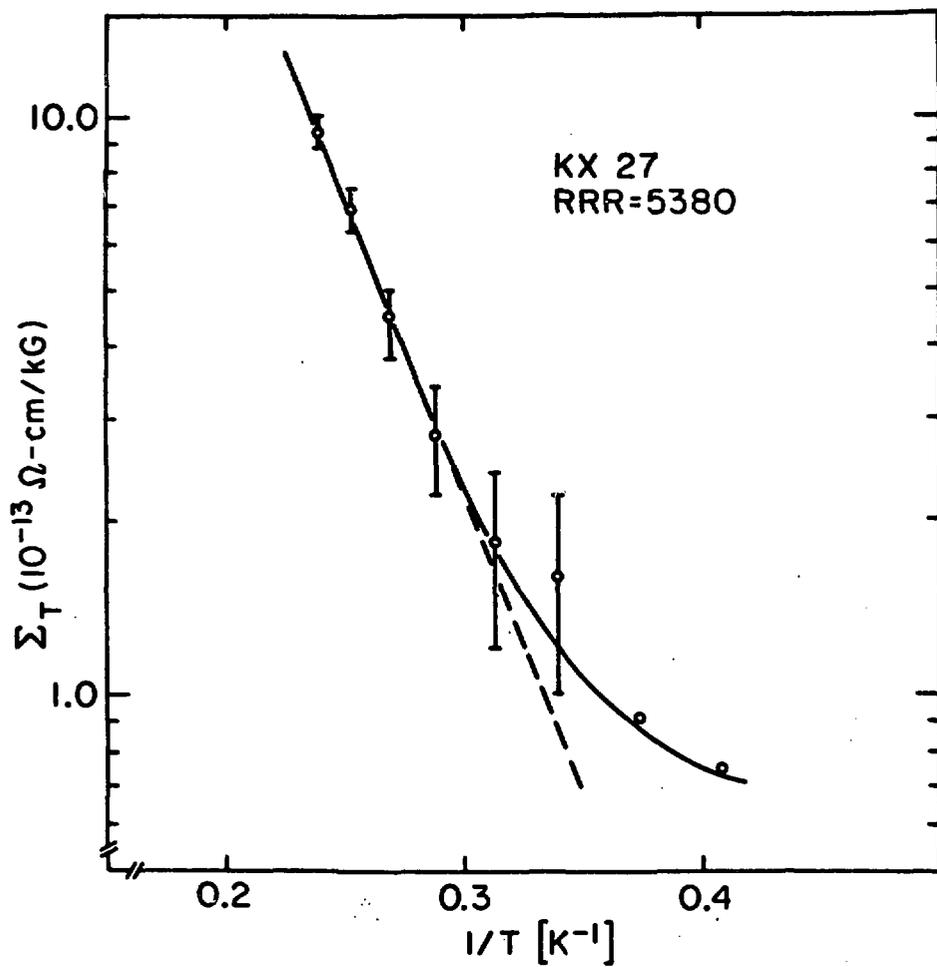


FIGURE 4