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STATISTICAL APPROACH TO THE VENEZIANO MODEL*

L. N. Chang, P. G. O. Freund, and Y. Nambu

The Enrico Fermi Institute and Department of Physics
The University of Chicago
Chicago, Illinois 60637

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Statistical Approach to the Veneziano Model *

L. N. Chang, P. G. O. Freund and Y. Nambu

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Abstract

A statistical approach to dual (Veneziano) dynamics is proposed. It operates with hadrons of given temperature rather than specifying in detail all hadronic quantum numbers. Heavy resonance production predictions of the statistical approach are found to agree with experiment.

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STATISTICAL APPROACH TO THE VENEZIANO MODEL*

L. N. Chang, P. G. O. Freund and Y. Nambu
The University of Chicago

When factorized,[1] the Veneziano representation requires the density of hadrons to grow exponentially with their mass. This causes divergence problems for closed loop diagrams.[2] Nevertheless, it is an inescapable consequence of duality that a very rich hadron spectrum is needed in order to reproduce the smooth Regge behavior dictated by the crossed channel. If the hadron density does indeed grow so fast, one may question whether distinguishing individual particles at very high masses ($M \gtrsim 5$ GeV) is really an adequate approach to hadron dynamics. At these high masses the number of states available to a hadron is so large and grows so rapidly that a statistical treatment seems more appropriate. Statistical models of high energy hadronic reactions have been considered in the past.[3] These models, while heavily relying on heuristic arguments, were unable to predict detailed reaction cross-sections.

In this paper we formulate a statistical model <u>derived</u> from the Veneziano representation. The crucial element of our treatment will be to abandon the specification of all detailed quantum numbers (such as "the nth of the many spin $\underline{J}$ daughters of...") of heavy reaction products. We shall rather describe such products by indicating a statistical parameter such as their temperature. Our method shall lead us to a particularly simple explanation of the main features of high mass resonance production.

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Let us start with the usual Veneziano four-point function for a process with empty u-channel (like $\pi^+\pi^- \to \pi^+\pi^-$) as depicted in Figs. 1a, 1b:

$$V(s,t) = \int_0^1 dx\, x^{-1-\alpha(s)}(1-x)^{-1-\alpha(t)} = \langle 0|\Gamma(p_3)\Delta\left((p_1+p_2)^2\right)\Gamma(p_2)|0\rangle. \tag{1}$$

Here we have recast $V(s,t)$ in the operational (factorized) form.[4] The Γ's are the vertices

$$\Gamma(p) = \exp\left\{\sum_{r=1}^{\infty} \frac{a^+_{\mu r}p_\mu}{\sqrt{r}}\right\} \exp\left\{-\sum_{r=1}^{\infty} \frac{a_{\mu r}p_\mu}{\sqrt{r}}\right\}, \tag{2}$$

$|0\rangle$ is the ground state of the Regge trajectory and Δ is the propagator[5]

$$\Delta(k^2) = \frac{1}{H-\alpha(k^2)-i\varepsilon}, \quad H = \sum_{r=1}^{\infty} r a^+_{\mu r} a_{\mu r}\,. \tag{3}$$

The absorptive part of $\Delta(k^2)$ is an energy δ-function reminiscent of that characteristic of the microcanonical ensemble of statistical mechanics. We now pass to the canonical ensemble by the replacement

$$\Delta(k^2) = \frac{1}{H-\alpha(k^2)-i\varepsilon} \to \tilde{\Delta}(k^2) = C\beta\, e^{-\beta\left(H-\alpha(k^2)\right)} \tag{4}$$

where

$$\beta = (kT)^{-1} \tag{5}$$

and $C \sim O(1)$ is a complex factor about which we do not worry. To make this replacement meaningful we have to specify what we mean by the temperature T. In the statistical treatment we

have automatically forsaken crossing symmetry as we have selected <u>one</u> channel (say the s-channel) as having a large number of intermediate one particle states or, in short, by going to large s. It is then clear that we expect T to be somehow related to the large kinematical variable s. To see how, insert Eq. (4) into Eq. (2) which then becomes

$$V(s,t) = \beta(1-e^{-\beta})^{-\alpha(t)-1} e^{\beta\alpha(s)} \xrightarrow[|\beta|\to 0]{} \beta^{-\alpha(t)} \; e^{\beta\alpha(s)} \quad . \tag{6a}$$

If we set[6]

$$kT = \beta^{-1} \sim s \tag{6b}$$

then for large s Eq. (6a) is Regge behaved. So far, our argument was heuristic. To make it more precise we observe that the result (6) could have been obtained directly from a steepest descent estimate of the integral in Eq. (1). Substituting $x = e^{-\beta}$ in Eq. (1) we have

$$V(s,t) = \int_0^\infty d\beta \; e^{\beta\alpha(s)} (1-e^{-\beta})^{-\alpha(t)-1} \quad . \tag{7}$$

Actually this representation is valid only for $\alpha(s) < 0$, $\alpha(t) < 0$. Letting therefore Re $s \to -\infty$ a steepest descent estimate of Eq. (7) yields Eq. (6) with $\beta \sim [\alpha(t)+1]/\alpha(s)$. Originally we meant s to be large and positive. In this case our method is still applicable if we first separate a resonance factor $\sim 1/\sin \pi\alpha(s)$ by going to a Pochhammer-type representation[7] of the B-function. This is reasonable since one would expect the transition from micro-canonical to canonical ensemble to be valid at least for the absorptive part of V, which may be smoothed out as in the usual discussion of the finite energy sum rules.[8] (The saddle point

method also fails when the function goes through zero as $\alpha(t)$ is varied.) Thus, not unlike standard statistical mechanics, the statistical limit (6) of the Veneziano amplitude coincides with its steepest descent estimate. Let us emphasize once more that all that is required is that $|\beta| \to 0$, the sign of Re $1/\beta$ does not matter. For brevity we shall call an (internal or external) line of 4-momentum k <u>hot</u>, if $|k^2|$ becomes very large. Our considerations so far are easily generalized to N-point functions.[10] All one has to do is to make the replacement (4) in any <u>hot</u> propagator. Thus the five point function of Fig. 1c is given by (notation $\beta_{ij} \sim 1/s_{ij}$)

$$
\begin{aligned}
F_5(s_{12},s_{23},s_{34},s_{45},s_{51}) &= \langle 0|\Gamma(p_4)\tilde{\Delta}(s_{45})\Gamma(p_3)\tilde{\Delta}(s_{12})\Gamma(p_2)|0\rangle \\
&\sim \beta_{12}\beta_{45}\,[1-e^{-(\beta_{12}+\beta_{45})}]^{-\tilde{\alpha}_{24}}\,(1-e^{-\beta_{45}})^{-\tilde{\alpha}_{34}}\,(1-e^{-\beta_{12}})^{-\tilde{\alpha}_{23}} \\
&\qquad \times e^{(\beta_{12}\tilde{\alpha}_{12}+\beta_{45}\tilde{\alpha}_{45})} \\
&\sim [\beta_{12}e^{\beta_{12}\tilde{\alpha}_{12}}\,(1-e^{-\beta_{12}})^{-\tilde{\alpha}_{23}}\,(1-e^{-(\beta_{12}+\beta_{45})\tilde{\alpha}_{23}})] \\
&\qquad \times [\beta_{45}e^{\beta_{45}\tilde{\alpha}_{45}}\,\{1-e^{(-\beta_{12}+\beta_{45})}\}^{(\tilde{\alpha}_{34}-\tilde{\alpha}_{51})}\,(1-e^{-\beta_{45}})^{-\tilde{\alpha}_{34}}] \,.
\end{aligned}
\tag{8}
$$

We have used here the kinematic relation $\alpha_{24} = \alpha_{51} - \alpha_{23} - \alpha_{34} + \text{const.}$ and as above[5] Regge intercepts have not been carefully adjusted.

The first factor in Eq. (8) depends only on the vertex A in Fig. 1d while the second factor involves p_5 and p_4 and thus must depend also on vertex B in this figure. Theoretically (e.g. for unitarization of the Veneziano amplitude) the more interesting process is that of Fig. 1e in which the decay mode of the heavy reaction product (particle 4) is not specified.

The precise way to calculate diagram 1e is to replace the bra $\langle 0|$ in Eq. (1) by the bra $\langle N|$ describing the excited state (particle 4) in all its gory details. This is extremely complicated and not within the spirit of this paper. More realistic (both theoretically and experimentally) is to calculate the sum of the cross-sections for producing all resonances having a mass within a certain band. This is directly achieved by describing the hot external line 4 of Fig. 5e via a density matrix

$$\rho(p_4) = e^{-\beta_4 H} \tag{9}$$

corresponding to temperature $T_4 \sim p_4^2 = M_4^2$. We can then directly calculate the squared matrix element

$$|M|^2 = \langle 0|\Gamma^+(p_2)\tilde{\Delta}^+(p_1+p_2)\Gamma^+(p_3)\rho(p_4)\Gamma(p_3)\tilde{\Delta}(p_1+p_2)\Gamma(p_2)|0\rangle. \tag{10}$$

Inserting here the statistical propagators $\tilde{\Delta}$ from Eq. (4), the vertices Γ from Eq. (2) and the density matrix ρ from Eq. (9) and performing the standard coherent state manipulations we find

$$|M|^2 = [\beta_{12}{}^2 e^{2\beta_{12}\tilde{\alpha}_{12}} (1-e^{-\beta_{12}})^{-2\tilde{\alpha}_{23}} (1-e^{-(\beta_{12}+\beta_4)})^{2\tilde{\alpha}_{23}}] f(\beta_{12},\beta_4) \tag{11a}$$

with

$$f(\beta_{12},\beta_4) = [(1-e^{-(2\beta_{12}+\beta_4)}) (1-e^{-\beta_4})]^{\text{const.}} . \tag{11b}$$

The bracket in Eq. (11) is just the square of the first bracket in Eq. (8). If we keep in mind that $|M|^2$ is an amplitude squared while F_5 is an amplitude and that the factor f in Eq. (11) is irrelevant as it essentially corresponds only to a specific choice of Regge intercepts,[5] then equations (11) and (8) exhibit a factorization property. This is true in general.

The technique developed above can be applied in numerous other cases. Here we shall limit ourselves to considering the experimental consequences of Eq. (11). As was mentioned above,[5] Eq. (11), as all equations of this paper, is to be trusted only as far as the linear term of Regge trajectories is concerned; intercepts have not been adjusted. Consider now Eq. (11) and expand the exponentials ($\beta_{12} \ll 1$, $\beta_{45} \ll 1$). We then find ($s_4 = M_4^2$)

$$|M|^2 = f(s_{12}^{-1}, s_4^{-1})\ (1 + s_{12}/s_4)^{2\tilde{\alpha}_{23}} = \bar{f}(s_{12}^{-1}, s_4^{-1})\ (1 + s_{12}/M_4^2)^{2\alpha' s_{23}}. \quad (12)$$

Eq. (12) fixes the dependence of the width of the forward (i.e. s_{23} small) peak in reaction ld-e on s_{12} and M_4^2. Using the simplified notation

$$s = s_{12}\ , \quad M^2 = M_4^2\ , \quad t = s_{23} \quad (13)$$

we can write the differential cross-section for reaction ld-e in the form

$$\frac{d^2\sigma}{d\,t\,dM^2} = g(s,M^2)\ e^{A(s,M^2)t} \quad (14)$$

where g is obtained from $\bar{f}$ of Eq. (12) after adjusting the intercepts and

$$A(s,M^2) = 2\alpha' \ln\ (1 + s/M^2) \quad (15)$$

In particular for $s \gg M^2 \gg (2\alpha')^{-1}$

$$A(s,M^2) = 2\alpha' \ln\ s/M^2. \quad (16)$$

Thus <u>at fixed energy (s) the width of the forward peak expands logarithmically with the resonance (mass)^2</u>. This agrees nicely with experiment.[9]

To sum up, we have seen that for large values of the kinematical variables, the behavior of Veneziano amplitudes due to the large number of states available can be simply described in statistical terms. This statistical description agrees with experiment as evidenced by Eq. (14). The major conclusion of this paper is that for hadron masses $M \gtrsim 5$ GeV, in view of the very large density of resonances, a detailed spectroscopic analysis becomes unfeasible and is most judiciously replaced by a statistical analysis. At large masses, it is not the detailed quantum numbers of the many resonances, but rather the temperature of the hadron that matters.

References

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3. H. Koppe, Z. Naturforschung 3A, 251 (1948); E. Fermi, Progr. Theor. Phys. 5, 570 (1950); R. Hagedorn, Nuovo Cim. 56A, 1027 (1969).
4. Y. Nambu, University of Chicago Preprint EFI 69-64; S. Fubini, D. Gordon and G. Veneziano, Phys. Lett. 29B (1969); L. Susskind, "Structures of Hadrons Implied by Duality," Yeshiva University preprint.
5. An extra mode has to be added in order to adjust the Regge intercept $\alpha(0)$. For simplicity we omit this mode. Correspondingly, throughout this paper only results concerning shrinkage and expansion of forward or backward peaks are to be taken seriously. Overall energy dependences depend on $\alpha(0)$ and therefore are sensitive to the omitted mode. We shall occasionally emphasize this point by using the notation $\tilde{\alpha}(s)$ for $\alpha(s)$ + some constant.
6. The concept of temperature as introduced here is derived from the Veneziano representation, including the interaction. It is therefore different from the temperature T of the free Bose gas that characterizes the spectrum. T_f as was shown by Nambu (Ref. 4) is proportional to $\sqrt{s}$ while our $T \sim s$. There is no connection between the temperature T of Eq. (6b) and the temperature used in Ref. 3, either.

7. See, e.g., E. T. Whittaker and G. N. Watson, "Modern Analysis" 4th ed., Cambridge Univ. Press, 1950. pp. 256-257. For our purposes the weaker result of Graf (in which only one variable is allowed to become arbitrary) is sufficient (see N. Nielsen, Die Gammafunktion, Vol. I, Chelsea Publishing Co., Bronx, New York, 1965, pp. 140-142.)

8. If the Veneziano amplitude were appropriately unitarized the large oscillation of the dispersive part would be damped, and the above argument would become stronger.

9. That the Veneziano model leads at fixed s to a decrease of A with increasing M^2 was first observed (using numerical evaluations) by R. Waltz, Phys. Lett. 30B, 490 (1969). The specific logarithmic dependence (16) for $s \gg M^2 \gg (2\alpha')^{-1}$ was conjectured by P. G. O. Freund, Phys. Rev. Letters 22, 565 (1969) and by L. Caneschi, Lett. al Nuovo Cim. 2, 122 (1969) where a detailed and successful comparison with experiments is presented. Observe that here we have derived Eqs. (15) and (18).

10. A similar manipulation has been carried out by K. Bardakci and H. Ruegg, Phys. Rev. 181, 1884 (1969). The set of limits they are taking, however, is different from ours. They consider an N-point function in its multi-peripheral configuration, and obtain a multi-Regge formula factorized along the leading trajectory. On the other hand, we start off from a manifestly factorized amplitude in the successive direct-channel resonance configuration, use the richness of the intermediate states as a physical argument for the approximation, and obtain an expression which retains the factorization property.

Figure caption

Fig. 1. Diagrams illustrating the statistical model. Wiggly lines are hot. Dotted lines correspond to the Regge exchanges implied by the statistical model.

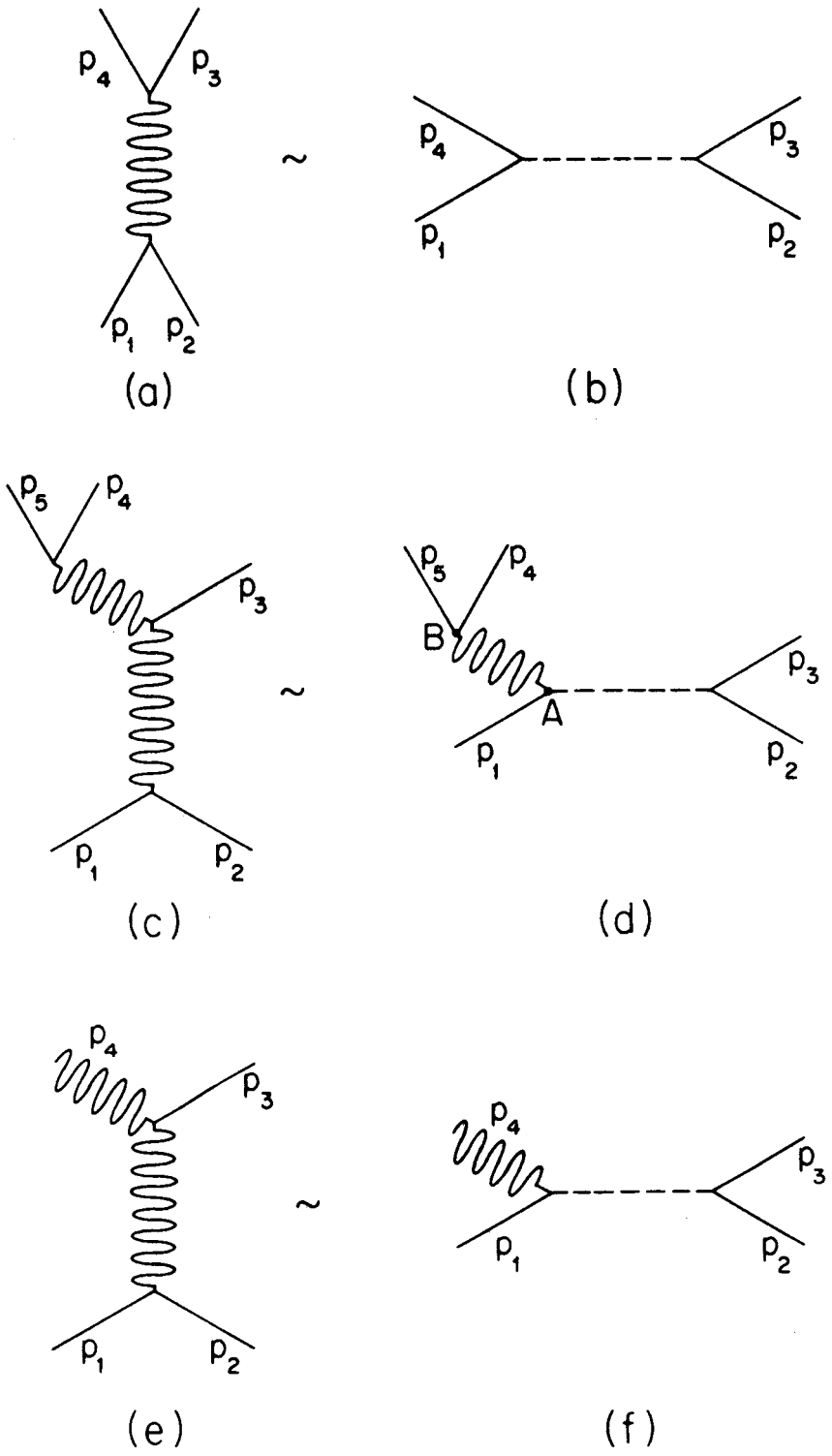
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Fig. 1