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PROBLEMS OF QUANTUM ELECTRODYNAMICS IN HEAVY-ION COLLISIONS

By

Berndt Müller

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PROBLEMS OF QUANTUM ELECTRODYNAMICS IN HEAVY-ION COLLISIONS*

Berndt Müller

Physics Division, Argonne National Laboratory

Argonne, Illinois 60439

Atomic physics has long been an important source for gaining fundamental insights. In the beginning of this century the measurements of the structure and fine structure of atomic spectra led to the invention of quantum mechanics, and around 1950 the Lamb-Shift and the (g-2) experiments started the development of quantum electrodynamics (QED) and field theory in general. Today the perturbative formulation of QED has been tested to several GeV/c momentum transfer in e^+e^- collisions, and also in binding energies of electrons and muons in heavy atoms. The agreement between experiment and theory is excellent. However, all these are cases where the coupling constant e^2 or Ze^2 is well below unity and it must be kept in mind that there is still no satisfactory theory of the strong interactions. We simply do not know what is the right way to handle coupling constants $g^2 \sim 10$. Moreover, there is the problem that the quark hypothesis explains many features of particle dynamics, but the quarks themselves seem to evade every effort of investigation. This has led many physicists to the conjecture of "quark confinement." It immediately confronts one with questions like: Can particles be bound so strongly they cannot escape? Can a system of ultrastrongly interacting particles shield itself so that it interacts only relatively weakly with its surroundings?

Most of these questions boil down to what happens when a particle is bound so strongly that the binding energy equals its rest mass (in case of a boson) or twice its rest mass (for fermions which can only be produced in pairs). It would be very helpful if this problem could be examined in a case where all the interactions are known, i.e., in atomic physics. Therefore, let us have a look at the binding energy of K-shell electrons as a function of nuclear charge Z which is shown in fig. 1. Hartree-Fock calculations predict that the binding curve reaches the negative Dirac continuum ($E_B = 2m_ec^2$) at Z = 172. Unfortunately, such an element does not exist in nature and most likely, it will never exist in the laboratory. The alternative would be to take two very heavy nuclei, e.g., U + U, and make a close collision between them, so that a system of Z > 172 is created for a short instant of time. One can calculate the energy states in such a superheavy quasi-molecule as a function of internuclear distance R. Figure 2 shows that the situation of critical binding is reached at R ~ 35 fm in a U-U collision.

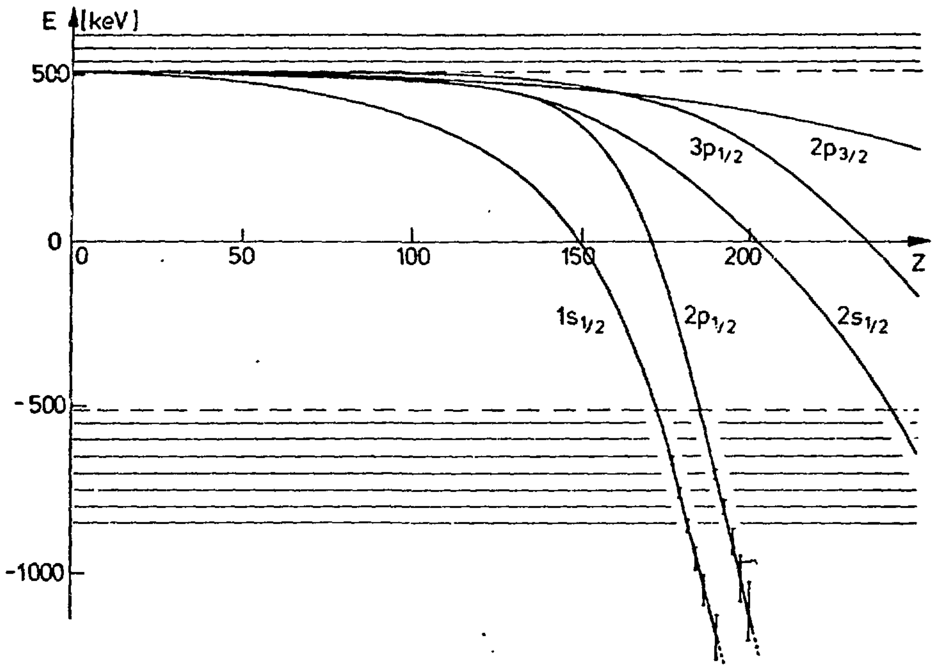

Fig. 1. Atomic binding energies as a function of Z.

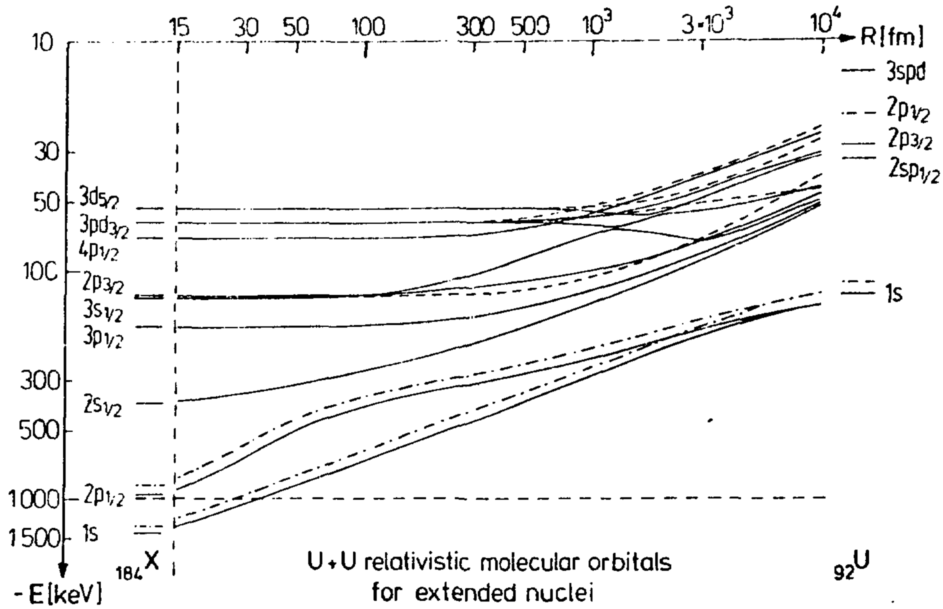

Fig. 2. Molecular orbital diagram of U + U.

Before I come to the experiment, let me first outline our ideas on what will happen in the supercritical region.[1—3] If any bound state is vacant, it can in principle be occupied by an electron, if at the same time a positron is created into a continuum state. The energy difference between these two situations is:

$$\Delta E = 2m_e c^2 - E_B. \tag{1}$$

Obviously, $\Delta E > 0$ if the binding energy is smaller than $2m_ec^2$, i.e., the vacant bound state is energetically more favorable. On the other hand, if $E_B > 2m_ec^2$, the second situation is favored and we speak of a supercritical bound state. The state, in which a positron is spontaneously produced, then is the true ground state of the system. In the language of field theory it is the vacuum state, which now is charged since it contains an electron (the positron eventually will escape).

This process may be viewed intuitively in a variety of ways. In terms of solid state physics there is a gap of $\Delta E = 2m_ec^2$ between unoccupied and occupied states, and when it is deformed by a potential so that there are vacant (bound) states and occupied states of the same energy, tunneling may occur. The hole in the otherwise filled electron sea is a positron. Or, from an atomic physicist's point of view we have a bound state which is embedded into a continuum. The only difference to the usual situation is that now the bound state is vacant and the continuum has to be viewed as occupied. If one does so, the whole formalism of auto-ionization becomes appplicable (see fig. 3). The bound state mixes with the continuum states and acquires a certain energy spreading. Starting from the critical situation $E(Z_{cr}) = -m_e c^2$ and adding charge to the nucleus, the energy of the bound state is lowered to

$$E(Z) = -m_e c^2 + (Z - Z_{cr})\langle\varphi|v(r)|\varphi\rangle. \tag{2}$$

The corresponding width of the state is given by

$$\Gamma = 2\pi(Z - Z_{cr})^2 |\langle\varphi|v(r)|\psi\rangle|^2, \tag{3}$$

where φ is the bound state and ψ the continuum wave function of equal energy, normalized to a delta function $\langle\psi_E|\psi_{E'}\rangle = \delta(E - E')$.

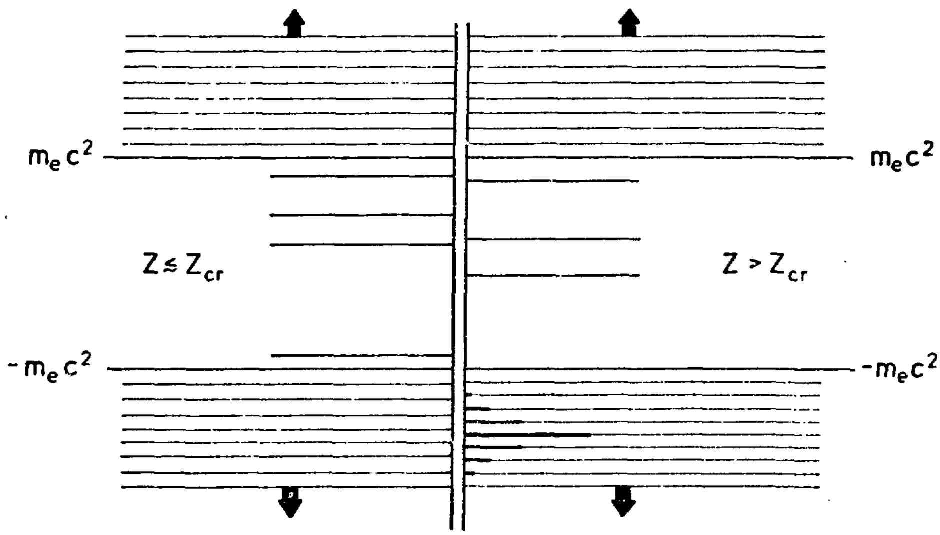

Fig. 3. Schematic view of the change from 1s bound state to continuum resonance.

The numerical way to obtain accurate results is to make a phase shift analysis of the continuum wave functions for $Z > Z_{cr}$. Figure 4 shows the energy of the 1s and $2p_{1/2}$ states as a function of Z, compared with the linear approximation eq. (2) (dashed lines). Similarly, fig. 5 shows the Breit-Wigner distribution of the 1s state in the hypothetical element ${}_{184}X$ (U + U) which exhibits a width of about 5 keV. This number should be kept in mind for the discussion of the experiment.

Let me come to a subtle point in the argument. When the bound state has joined the continuum, the vacuum becomes charged twice (because of the two spin states), but how can one see this afterwards? There is always one continuum state for every energy, how can there be one more? To see this, one has to enclose the superheavy atom in a large box of radius R. Then the continuum solutions become discrete and they are determined by the boundary condition

$$(pR + \delta + \delta_R) = 0 \tag{4}$$

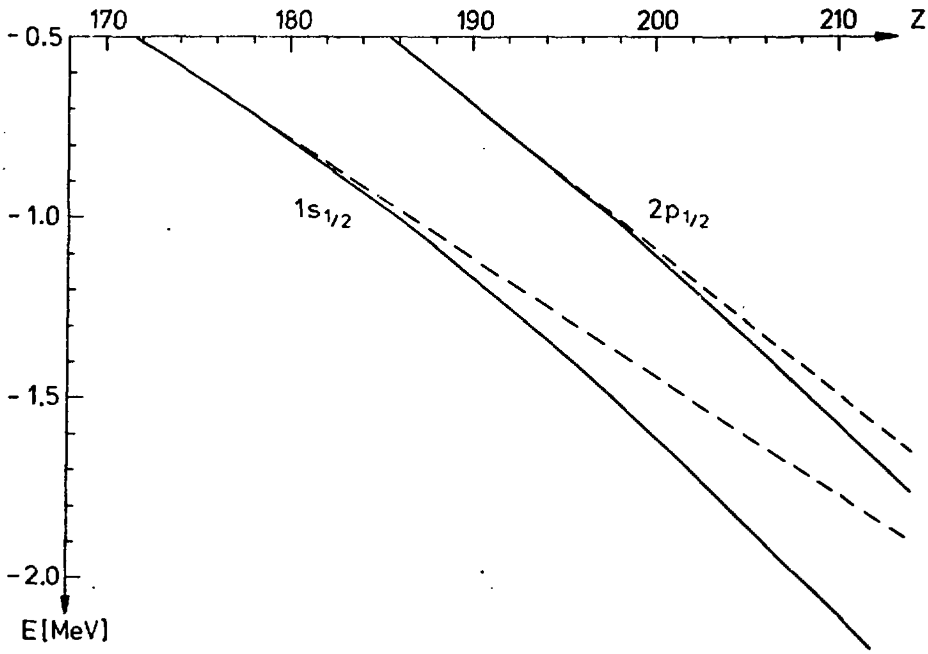

Fig. 4. 1s- and $2p_{1/2}$-resonance energies as a function of Z. Exact calculation (full lines), linear approximation (dashed lines).

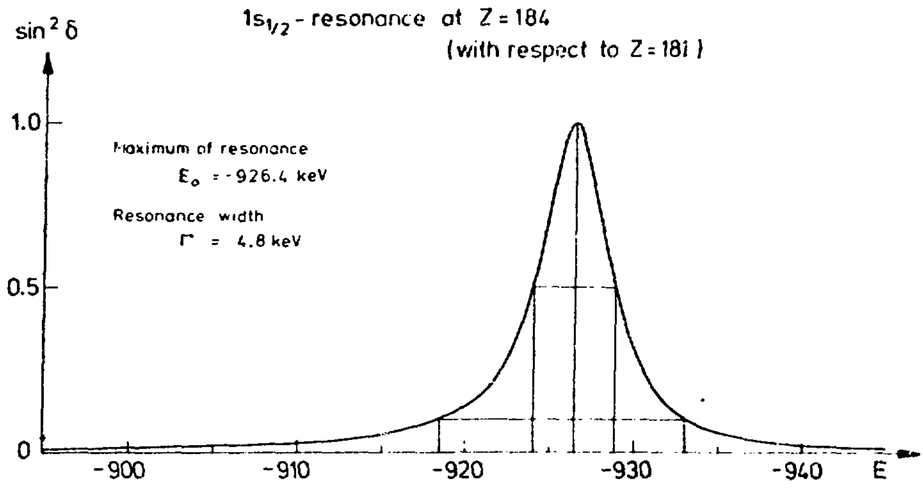

Fig. 5. Resonance shape of the 1s state in $_{184}X$.

where I have split the phase shift into a smooth part and the resonance from the bound state. Obviously, we have

$$n\pi = pR + \delta + \delta_R \tag{5}$$

and

$$\frac{dn}{dE} \approx \frac{1}{\pi}\left[R\,\frac{dp}{dE} + \frac{d\delta_R}{dE}\right]. \tag{6}$$

In eq. (6) the derivative of the smooth part of the phase shift has been neglected. (This is only a very crude argument, but it can be shown exactly to give no contribution to the number of states.) If we integrate over the second part of the density of states, we find indeed:

$$n_R = \frac{1}{\pi}\int dE\,\frac{d\delta_R}{dE} = \frac{1}{\pi}\Delta(\delta_R) = 1. \tag{7}$$

Similarly, the charge distribution of the bound resonance can be extracted from the continuum. Figure 6 shows that it is (for Z = 184) the natural continuation of the charge distribution of the critical ($Z_{cr} \sim 172$) bound 1s state.

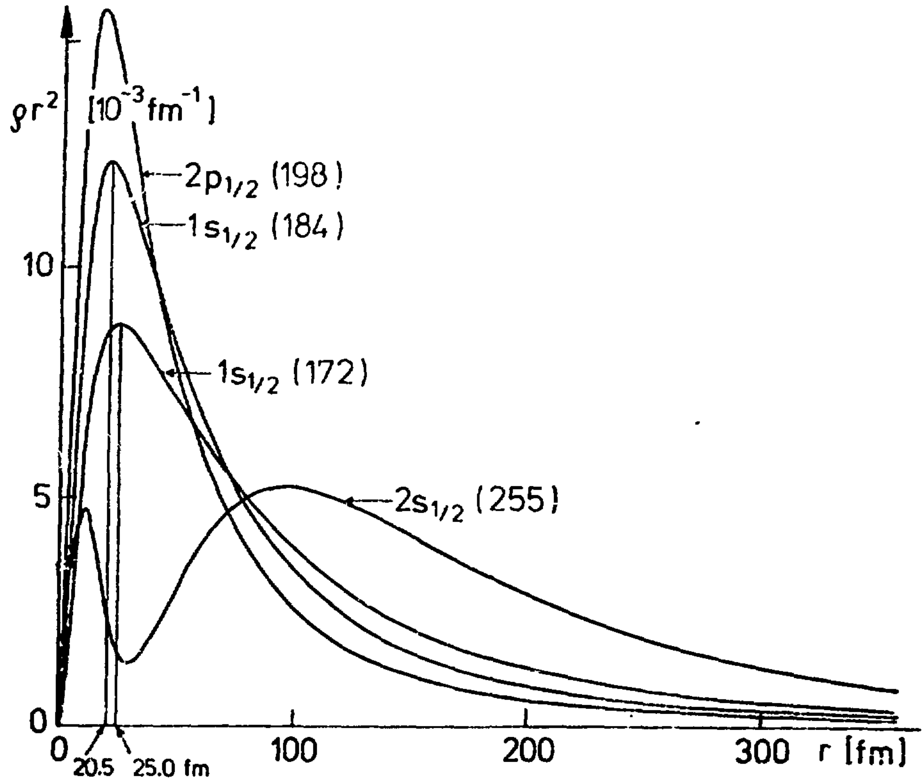

Fig. 6. Charge distribution of various supercritical states.

After this theoretical introduction let us have a look at the possibilities for experiments. Figure 7 shows a schematic view of the collision between two uranium atoms. As I already have mentioned, it is necessary to have a vacancy in the molecular 1s state at the point of closest approach for the positron creation to occur. These vacancies may be created at large distances where the K-shell ionization energy is relatively small. The predictions for the probability of this ionization vary at present over a wide range, from 5×10^{-6} to 10^{-1}. The reason for this ignorance is that the impact parameter dependence of direct K-shell ionization is not yet well understood (the above range only corresponds to an uncertainty of a factor 5 in the average ionization impact parameter). Evidently, it is a crucial number for every experimental effort, and it should be measured in collisions between medium heavy

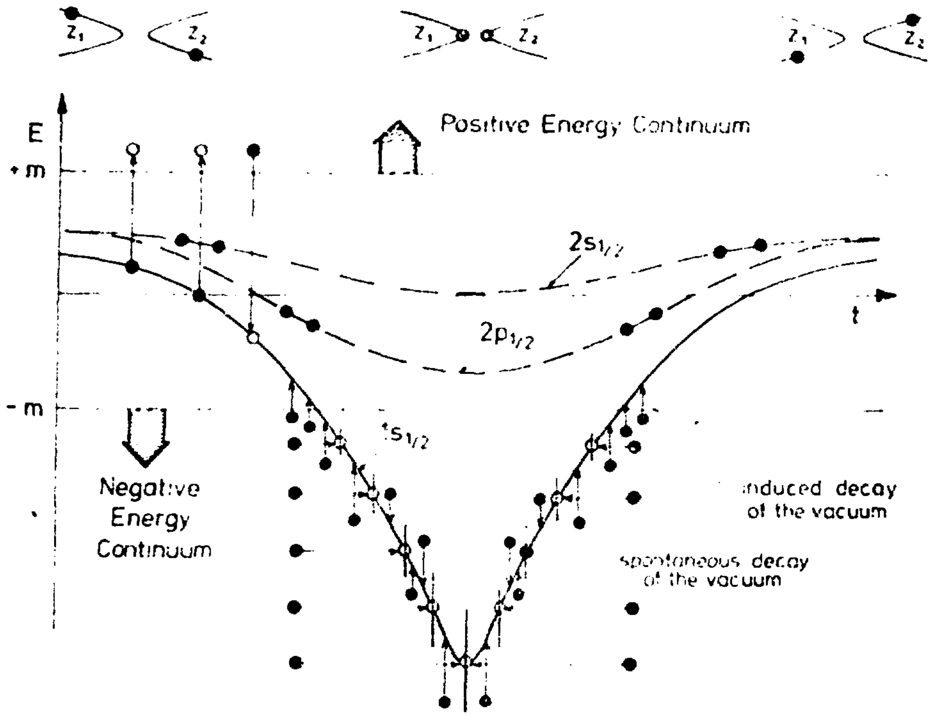

Fig. 7. Schematic view of atomic processes in a U + U collision at ca. 5 MeV/nucl.

atoms. In our calculations we have assumed (!) a probability of 10^{-2}. (It should be mentioned at this point that there is a process which could best be described as virtual vacancy excitation during the collision, which does not finally give rise to real ionization. In adiabatic collisions it can be many times larger than the ionization probability. How much it can contribute for our purposes remains to be seen.)

Then there follows the region—at $R \leq 50$ fm—where the bound $1s\sigma$ state becomes a member of the continuum and spontaneous positron production will occur. A rough estimate of the efficiency of this process is obtained from the collision time $\tau_c \sim 4 \times 10^{-21}$ sec in comparison with the vacancy decay time $\tau_{pos} \sim (5\text{ keV})^{-1} \sim 2 \times 10^{-19}$ sec. Thus one can expect to see one out of fifty vacancies escape as a positron. Figures 8 and 9 show the results of more rigorous calculations, taking into account the full dynamics of the collision process.[4] The e^+ distribution will be broadly peaked around 400 keV which should make the positrons easy to detect. The cross section falls off steeply when the impact parameter grows beyond 50 fm or the collision energy falls below 300 MeV (in the CM system). This is reflected in the energy dependence of the total cross sections. However, because of the collision dynamics, no sharp cutoff can be expected.

The background for the experiment can originate from mainly two sources: (1) the usual e^+e^-pair production in collision

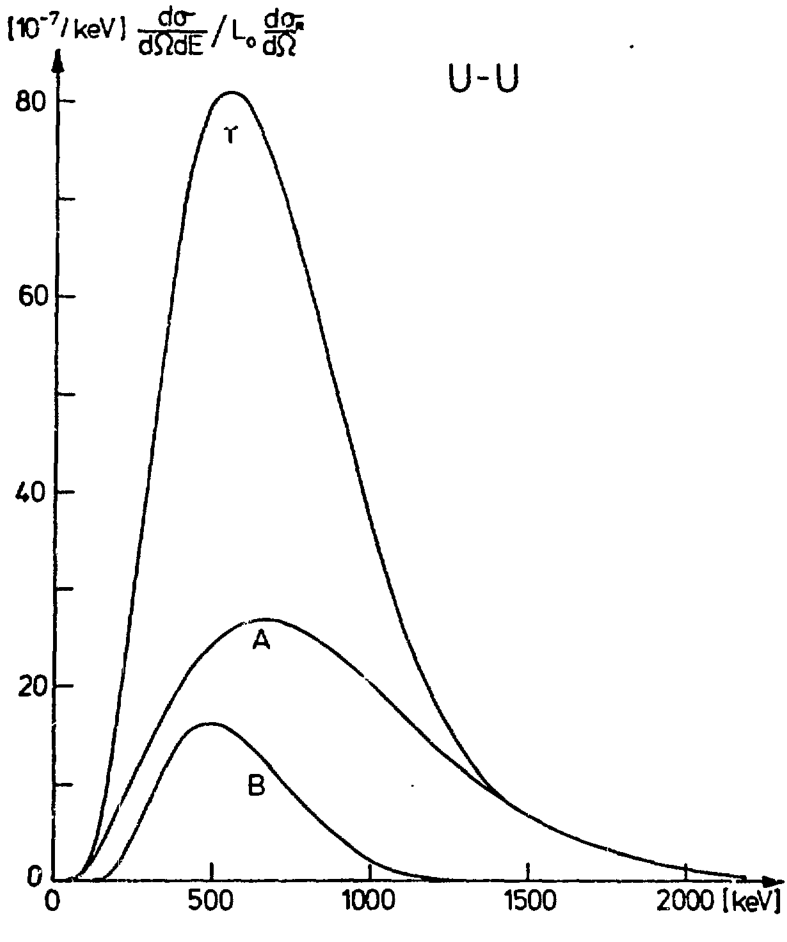

Fig. 8. Differential positron cross section in 1600 MeV U + U backward scattering (T = total).

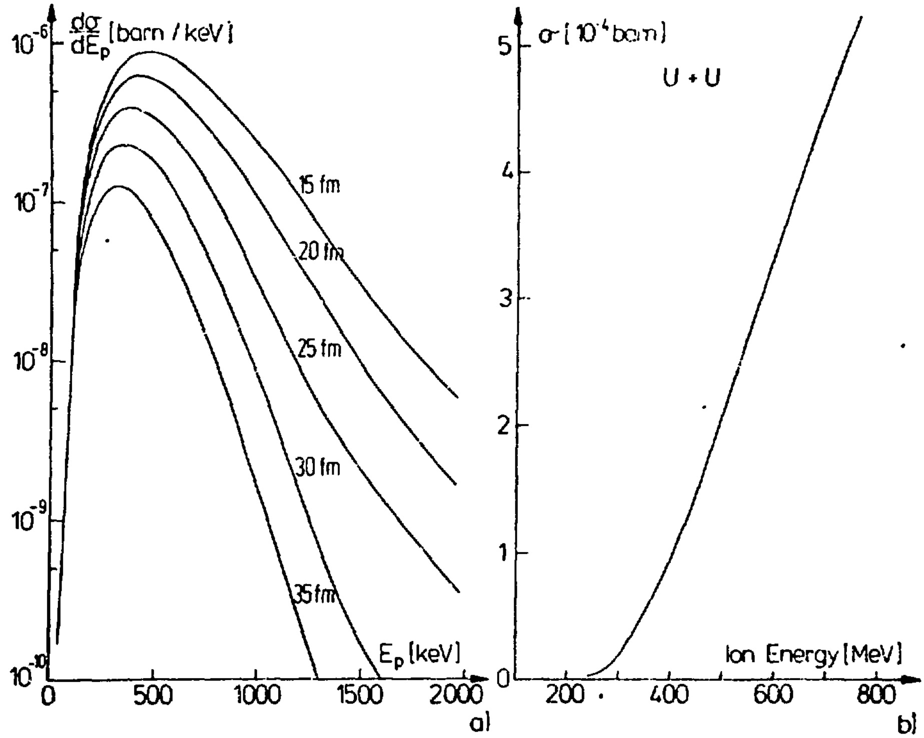

Fig. 9. Positron cross sections in U + U collisions as a function of projectile energy (CM system).

between charged particles. It must be of much smaller cross section since the matrix element are rather small and the energy necessary is at least 1 MeV, whereas the spontaneous process is almost energyless. Computations show that it should be smaller by about 3 orders of magnitude. (2) Coulomb excited nuclear states may decay via pair production if the transition energy is above 1 MeV. Fortunately, uranium is a rather soft nucleus and has very low excitation energies for the collective states and only a very small portion of Coulomb excited states will decay in that way. G. Soff and V. Oberacker[5] have calculated that this background effect would become serious if the K-shell ionization probability is smaller than 10^{-3}.

If the experiment is successful and supports our ideas about positron emission in U + U collisions, it will also be a piece of evidence for a physical situation of supercritical coupling. In the meantime it may be permitted for a theorist to apply the same ideas to more unfamiliar situations. If one could increase the charge of a nucleus more and more, other electronic states ($2p_{1/2}$, $2s$, etc.) will join the continuum and be spontaneously filled (fig. 10). Thus the vacuum will become highly charged, until it really begins to neutralize the bare nuclear charge distribution.[6] One may view this purely hypothetically, but it could be a real occurrence if such speculative things like collapsed nuclear states do exist.

Figure 11 shows how the apparent screeened charge γ of such an object would be connected to the number Z of protons it contains. It is seen that the asymptotic coupling constant $\gamma\alpha$ reaches a value of ca. 15 even for $Z = 10^6$. That this is of the order of magnitude of the strong interactions coupling constant, may be just an accidental fact. But it provides a model for a situation where an extremely strongly interacting object screens itself, so that its apparent coupling to other particles becomes of moderate

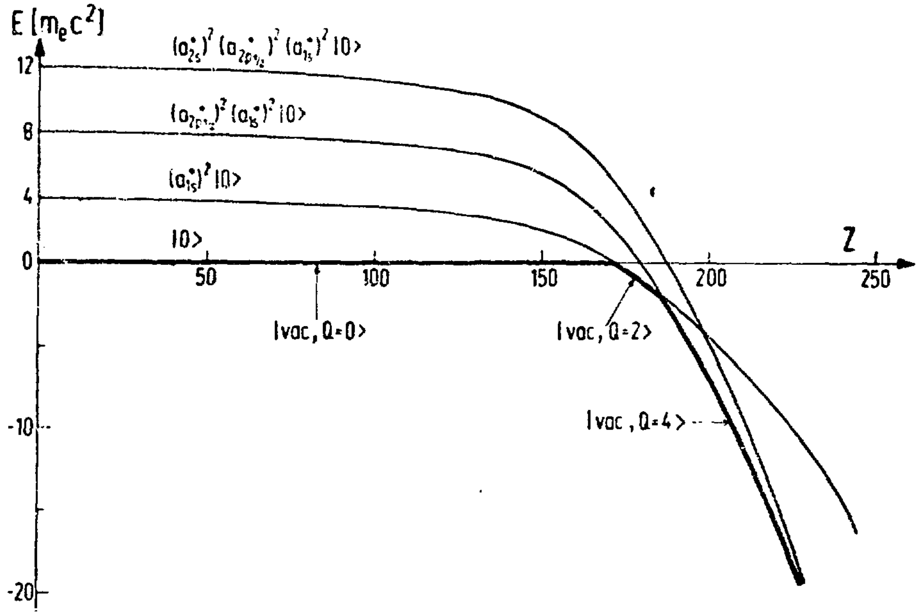

Fig. 10. Higher charged ground states (vacua) are developing as Z increases.

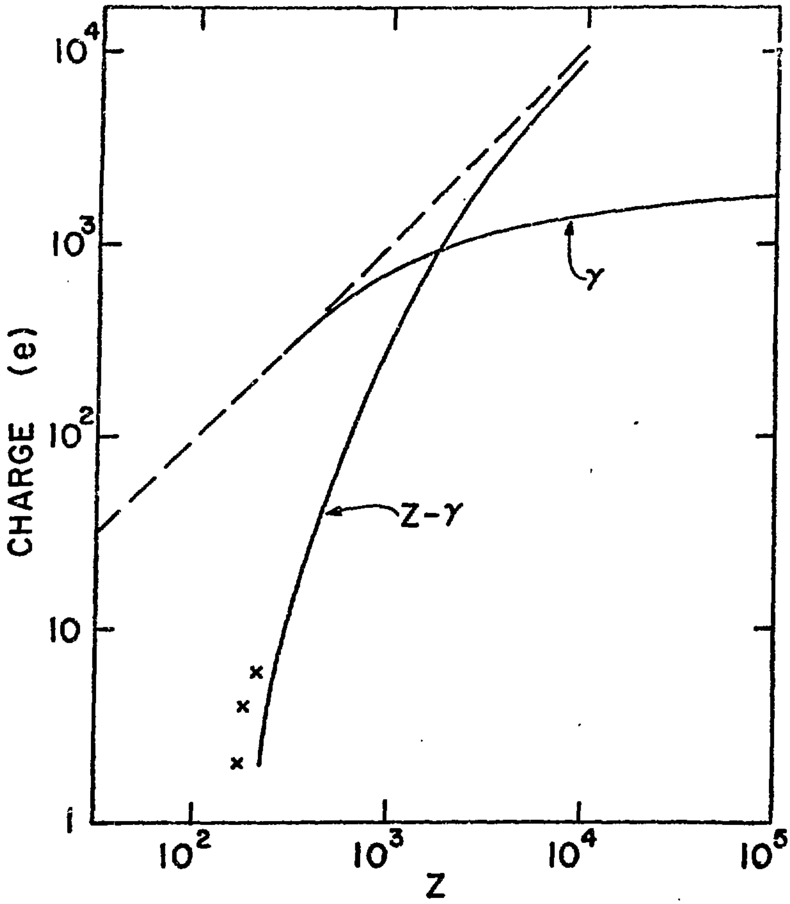

Fig. 11. Charge (Z-γ) of the vacuum and apparent charge γ of the nucleus as a function of Z.

strength. Moreover, if we look at the potential of this hypernucleus (fig. 12), we encounter a square well, in which the electrons can move almost freely yet they are strongly bound. If one would ionize one of the screening electrons its place would immediately be reoccupied spontaneously under positron emission. Thus, we may also be on the way to a model for the confinement of the constituents of elementary particles while they are almost freely moving within their boundaries. This closes the circle to the questions I have raised at the beginning of my talk. And, maybe, atomic physics can provide some of the necessary evidence in this way.

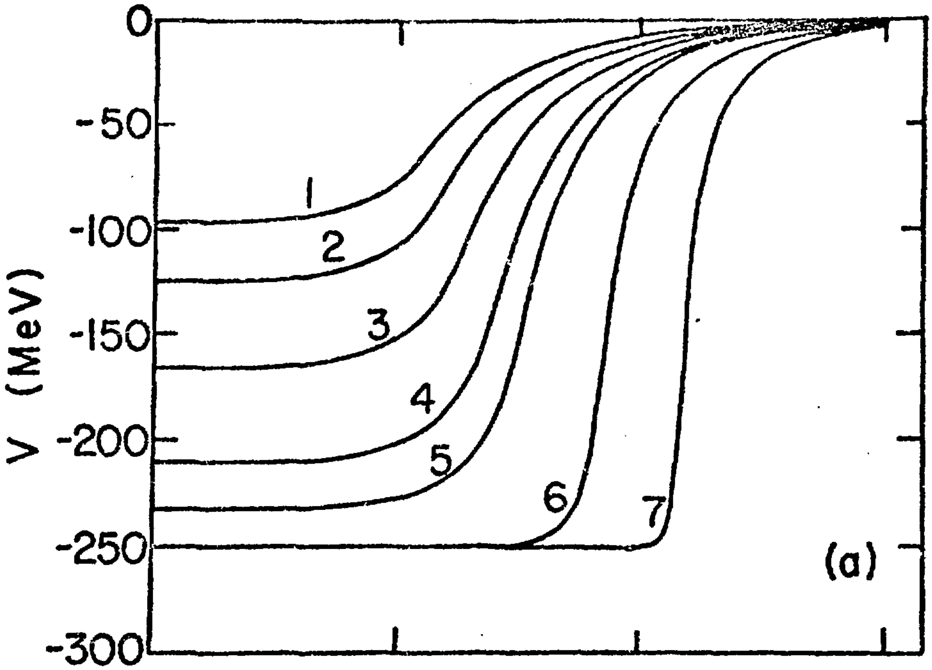

Fig. 12. The potential inside an abnormal nucleus (numbers correspond to increasing Z).

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