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Survey of Electric Field Shear Driven by Radio Frequency Waves in Tokamak Plasmas

E.F. Jaeger, L.A. Berry, and D. B. Batchelor
Oak Ridge National Laboratory, Oak Ridge, Tennessee 37831-8071

1. Introduction

The stabilization of plasma turbulence by sheared poloidal rotation [1] is thought to explain enhanced confinement [2] in tokamak plasmas. One method proposed for controlling sheared flow is the use of externally driven radio-frequency (RF) waves [3]. A number of calculations [4–6] and some experiments [7–8] have suggested that a modest amount of power in the ion cyclotron range of frequencies (ICRF) can drive the needed flows.

Previous calculations [4–6] have relied on incompressible fluid models which balance RF forces in the poloidal direction against neoclassical viscosity. But the incompressible assumption is not always valid, particularly for ion Bernstein waves (IBW). Also, since the IBW is a kinetic wave by nature, a fully consistent model should include kinetic effects. In this paper, RF driven flows are calculated from both compressible fluid and kinetic points of view.

2. Fluid and Kinetic Models for RF Induced Momentum Transport

The starting point is the Vlasov equation for a plasma in the presence of perturbing RF electric and magnetic fields,

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f + \frac{q}{m}[\mathbf{E}_1 + \mathbf{v} \times (\mathbf{B}_0 + \mathbf{B}_1)] \cdot \nabla_v f = 0 \tag{1}$$

where $\mathbf{B}_0(\mathbf{r})$ is the static magnetic field and $\mathbf{E}_1(\mathbf{r}, t)$ and $\mathbf{B}_1(\mathbf{r}, t)$ are perturbing RF fields with time harmonic dependence $\exp[i(\mathbf{k}\cdot\mathbf{r}-\omega t)]$ where ω is the wave frequency. The distribution function f can be expanded in powers of the electric field as

$$f(\mathbf{r},\mathbf{v},t) = f_0(\mathbf{r},\mathbf{v}) + f_1(\mathbf{r},\mathbf{v},t) + F_2(\mathbf{r},\mathbf{v},t) \tag{2}$$

where f_0 is the equilibrium solution, f_1 is the linear solution $\propto \exp[i(\mathbf{k}\cdot\mathbf{r}-\omega t)]$, and F_2 is the slowly varying, second order response. The lowest order solution for $f_0(\mathbf{r}, \mathbf{v})$ is assumed to be an isotropic Maxwellian. The first order equation can be solved for $f_1(\boldsymbol{r},\mathbf{v},t)$ in terms of $f_0(\mathbf{r}, \mathbf{v})$ by integrating along unperturbed particle orbits [9]. The second order equation is [10]

$$\frac{\partial F_2}{\partial t} + \mathbf{v} \cdot \nabla F_2 + \frac{q}{m}(\mathbf{E}_0 + \mathbf{v} \times \mathbf{B}_0) \cdot \nabla_v F_2 = -\left\langle \frac{q}{m}(\mathbf{E}_1 + \mathbf{v} \times \mathbf{B}_1) \cdot \nabla_v f_1(\mathbf{r},\mathbf{v},t) \right\rangle_t \tag{3}$$

where $\langle\;\rangle_t$ represents a time average. The slowly varying ambipolar electric field $\mathbf{E}_2$ has been neglected, but can be included by superposition of results from a later calculation.

In fluid models [4–6], the poloidal flow is found from the velocity moment of Eq. (3) for each species s,

$$\frac{\partial}{\partial t}(n_s m_s \mathbf{V}_{2,s}) + \nabla \cdot \mathbf{P}_{2,s} - n_s q_s \mathbf{E}_0 - n_s q_s \mathbf{V}_{2,s} \times \mathbf{B}_0 = \langle q_s n_{1,s} \mathbf{E}_1 + \mathbf{J}_{1,s} \times \mathbf{B}_1 \rangle_t - \mu\, n_s m_s \mathbf{V}_{2,s} \tag{4}$$

where $n_s \mathbf{V}_{2,s} = \int \mathbf{v} F_{2,s}\, d^3v$ is the slowly varying flow velocity, and $\mathbf{P}_{2,s} = \int m\mathbf{v}\mathbf{v} F_{2,s}\, d^3v$ is the second order, nonlinear pressure tensor. The last term in Eq. (4) represents momentum lost due

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to collisions, where μ is the neoclassical viscosity. The poloidal (y) component of Eq. (4) yields

$$n_s m_s \left(\frac{\partial V^s_{2,y}}{\partial t} + \Omega_s V^s_{2,x} + \mu V^s_{2,y}\right) = -\left(\nabla \cdot \mathbf{P}_{2,s}\right)_y + \langle q_s n_{1,s} E_{1,y} + (\mathbf{J}_{1,s} \times \mathbf{B}_1)_y \rangle_t \qquad (5)$$

where Ω_s is the cyclotron frequency. Summing over species s, the second term on the left (proportional to $n_s V_{2,x}{}^s$) sums to zero by ambipolarity, and in steady state, we are left with

$$V_{2,y} = \frac{1}{\mu \rho^T_m} \sum_s [-(\nabla \cdot \mathbf{P}_{2,s})_y + \langle q_s n_{1,s} E_{1,y} + (\mathbf{J}_{1,s} \times \mathbf{B}_1)_y \rangle_t] \qquad (6)$$

where $V_{2,y}$ is the mass averaged flow velocity in the poloidal direction, and $\rho^T{}_m$ is the total mass density. A similar result can be derived for the toroidal flow velocity. The first term on the right of Eq. (6) is the "Reynolds stress", and the second term is the "electromagnetic (EM) force". For closure in the fluid model, the second order pressure tensor $\mathbf{P}_{2,s}$ is approximated as the time average of the product of first order oscillating velocities $\mathbf{P}_{2,s} \sim n_s\, m_s \langle \mathbf{V}_{1,s} \mathbf{V}_{1,s} \rangle_t$ in which case, the Reynolds stress becomes

$$(\nabla \cdot \mathbf{P}_{2,s})_y = n_s m_s \langle (\mathbf{V}_{1,s} \cdot \nabla) V^s_{1,y} + V^s_{1,y} (\nabla \cdot \mathbf{V}_{1,s}) \rangle_t \qquad (7)$$

where $\mathbf{V}_{1,s}$ is calculated from the plasma current for each species as $\mathbf{V}_{1,s} = \mathbf{J}_{1,s} / n_s\, q_s$. For incompressible forces as in the case of plasma turbulence, the second term (proportional to $\nabla \cdot \mathbf{V}_{1,s}$) has been neglected [4–6]. But for some RF heating modes (particularly IBW) this assumption is not correct, and this term must be retained.

For waves which are intrinsically kinetic such as IBW, momentum can be carried by particle motion on the scale length of the wave. This is analogous to the "kinetic flux" [11] in energy transport. To treat this flow accurately, and to avoid approximating $\mathbf{P}_2$ for closure in the fluid model, a kinetic treatment is necessary. In this case, Eq. (3) can be solved directly for F_2 in terms of f_1 by integrating along unperturbed orbits:

$$F_2(\mathbf{r}, \mathbf{v}, t) = -\lim_{\gamma \to 0} \int_0^t dt' \left\langle \frac{q}{m} [\mathbf{E}_1(\mathbf{r}', t') + \mathbf{v}' \times \mathbf{B}_1(\mathbf{r}', t')] \cdot \nabla_{v'} f_1(\mathbf{r}', \mathbf{v}', t') \right\rangle_t . \qquad (8)$$

To properly treat singularities at cyclotron resonance, ω is assumed to have a small imaginary part γ. Also, the lower limit on the integral is chosen to be 0 rather than $-\infty$ to avoid the singularity in F_2 at $t = -\infty$ and $\gamma \to 0$. The time averaged power deposition dW/dt can be found by taking the energy moment of Eq. (8) and differentiating with respect to time

$$\frac{\partial W}{\partial t} = \int \frac{mv^2}{2} \frac{\partial F_2}{\partial t} d^3v \geq 0 . \qquad (9)$$

This is equivalent to the power deposition calculated in Refs. [12–14]. These references show that Eq. (9) is positive definite even when significant energy flux is carried by the particles. By analogy with Eq. (9), the total force per unit volume in the y direction is given by

$$n_s m_s \left\langle \frac{dV_{y,s}}{dt} \right\rangle_t = \left\langle \int m_s \left[v_y \frac{\partial F_{2,s}}{\partial t} + (\Omega_s v_x + \gamma v_y) F_{2,s} \right] d^3v \right\rangle_t \qquad (10)$$

where the time average is over a cyclotron period to eliminate fast transients. In the limit $\gamma \to 0$, there is a "resonant" part of Eq. (10) that is proportional to time and contains the increasing velocities and energies associated with the RF fields. In the interest of examining the RF forces before the plasma equilibrium has begun to change, we set $t = 0$ in those terms. As in the fluid model, the steady state poloidal flow velocity can be calculated by balancing Eq. (10) against neoclassical viscosity in the poloidal direction.

3. Numerical Results for the RF Induced Poloidal Flow Velocity

In this section, the RF driven flow velocity in the y (poloidal) direction is calculated using the fluid and kinetic models developed above. In both models, the RF fields are calculated from a full wave numerical solution to Maxwell's equations [15] which resolves the IBW scale length in 1-D. The plasma is modeled as a finite temperature, perpendicularly stratified 1-D slab. The plasma current is expanded to second order in gyroradius. Parameters are modeled after the Alcator C-Mod experiment [16]; i.e. major radius $R_0 = 0.67$ m, minor radius $a = 0.20$ m, and applied magnetic field $B_0 = 4.0$ T on axis. The antenna is located just outside the plasma at $R = 0.88$ m, and is characterized by a toroidal wave number, $k_z = 10\ \mathrm{m}^{-1}$, and poloidal wave number, $k_y = 0\ \mathrm{m}^{-1}$ (i.e. no net input of poloidal momentum). Plasma profiles are shown in Fig. 1.

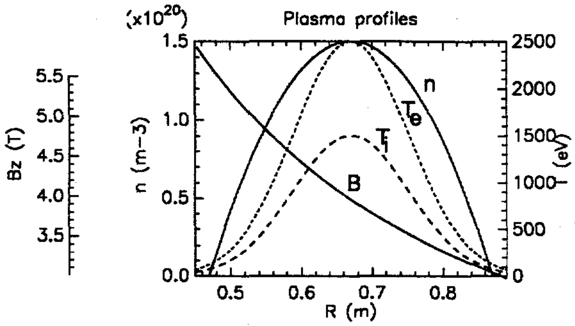

Fig. 1. Density, temperature and magnetic field profiles for a 1-D tokamak plasma with $R = 0.67$ m, $a = 0.20$ m and $B(0) = 4.0$ T.

Figure 2 compares the poloidal flow velocity as calculated from three different models: (a) incompressible fluid, (b) compressible fluid, and (c) kinetic. The plasma consists of a 10% minority of hydrogen (H) in helium-3 (He^3), and the frequency is near the first harmonic of the minority H. Absorbed power is 1 MW for all cases. To suppress mode conversion to IBW near the two ion hybrid resonance layer, k_z has been chosen artificially large. The long dashed line shows the EM force, the short dashed line shows the Reynolds stress, and the solid line shows the sum of both terms, i.e. the total flow velocity. The effect of compressibility in Fig. 2(b) is to slightly reduce the contribution of the Reynolds stress. Kinetic effects, on the other hand, give a slightly larger Reynolds stress than the fluid models. The magnitude of the flow is comparable in all three cases. Similar results are found near the second harmonic resonance.

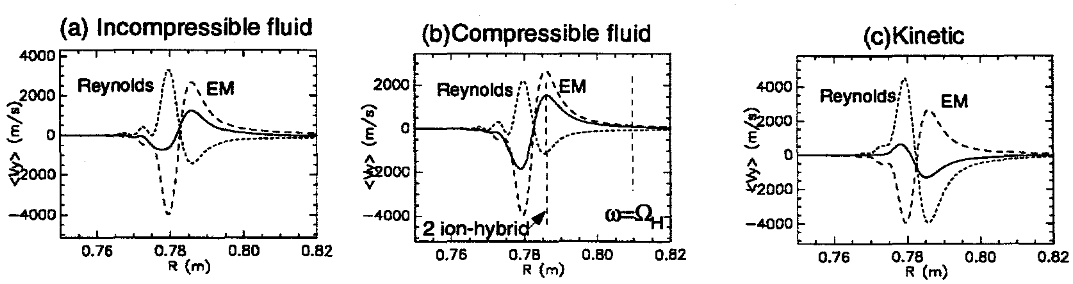

Fig. 2. Poloidal flow velocity for $f = 50$ MHz, $B(0) = 4.0$ T and $kz = 26$ m-1.

The situation is quite different for directly launched IBW as shown in Fig. 3. Here, the plasma consists of a 2% minority of He^3 in deuterium (D). The antenna is just in front of the second harmonic of D, and IBW is launched directly from the plasma edge. Again there is no net input of poloidal momentum ($k_y = 0$). As expected for IBW, compressibility is extremely important in the fluid model, and the flow is reduced by two orders of magnitude in Fig. 3(b). What is perhaps not expected is the additional decrease observed in the kinetic result. In this case, wave damping is due predominantly to Landau damping (LD) and magnetic pumping (TTMP) in the z (toroidal) direction. But for $k_y = 0$, the wave has no poloidal phase velocity, and there can be no analogue to LD or TTMP in the poloidal direction. Therefore, momentum in the y direction can only change due to second order finite Larmor radius terms, and the

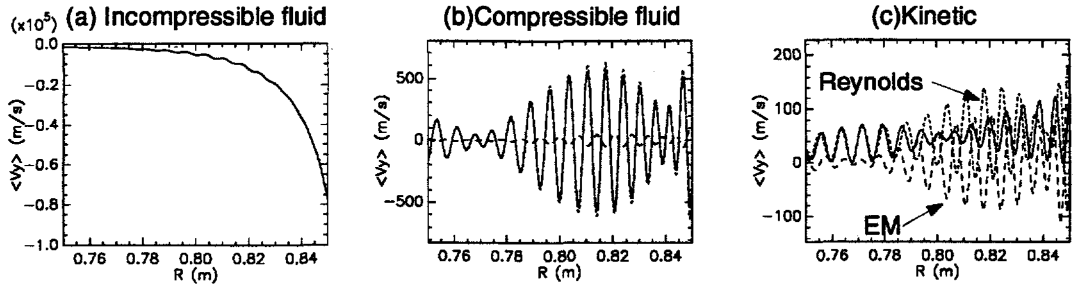

Fig. 3. Poloidal flow velocity for f = 44 MHz, B(0) = 4.0 T and kz = 10 m-1.

resulting flow in Fig. 3(c) is very small. Similar results are found for IBW generated by mode conversion.

We conclude that the choice of an appropriate model depends strongly on the type of wave considered. For electromagnetic waves such as the fast magnetosonic wave in Fig. 2, fluid models give a reasonable approximation to the complete result. But for electrostatic waves such as directly launched IBW in Fig. 3, this is not the case. Here both compressibility and kinetic effects are important, and a kinetic treatment is required. Significant flows are driven at both first and second harmonic ion resonances even for $k_y = 0$. But such flows do not occur when wave damping is predominantly due to LD and TTMP since for $k_y = 0$, there can be no match between thermal speed and the phase velocity of the wave in the poloidal direction.

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