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MODELING OF STATISTICAL TENSILE STRENGTH OF SHORT-FIBER COMPOSITES

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ABSTRACT

This paper develops a statistical strength theory for three-dimensionally (3-D) oriented short-fiber reinforced composites. Short-fiber composites are usually reinforced with glass and ceramic short fibers and whiskers. These reinforcements are brittle and display a range of strength values, which can be statistically characterized by a Weibull distribution. This statistical nature of fiber strength needs to be taken into account in the prediction of composite strength. In this paper, the statistical nature of fiber strength is incorporated into the calculation of direct fiber strengthening, and a maximum-load composite failure criterion is adopted to calculate the composite strength. Other strengthening mechanisms such as residual thermal stress, matrix work hardening, and short-fiber dispersion hardening are also briefly discussed.

INTRODUCTION

Short-fiber composites have several advantages over continuous fiber composites,. They often have improved strength and stiffness over the unreinforced matrix. In addition, they can be adapted to conventional manufacturing techniques, such as powder metallurgy, casting, molding, drawing, extruding, machining, and welding [1-5]. As a result, the part fabrication cost is relatively low [6], which is an important design criterion [7]. Short-fiber composites also can be made with relatively isotropic mechanical properties and can be easily molded into complex shapes [1], as required in some applications. These advantages have led to wide applications of these composites in automobile, sporting goods and cutting tools industries [8,11].

Strength is one of the most important properties of structural short-fiber composites and its prediction is essential for composite design. In a real short-fiber composite, short fibers are usually three dimensionally (3-D) oriented [12], which makes it more difficult to calculate the composite strength. In addition, ceramic or glass short fibers and whiskers are usually used as reinforcements. These reinforcements are brittle and display a range of strength values, which can be statistically characterized by a Weibull distribution [13-16]. Therefore, both the 3-D fiber orientation and statistical nature of fiber strength need to be taken into account in the prediction of composite strength. Unfortunately, this cannot be handled by current available composite strength models.

The models of Chen [1] and Halpin and Kardos [17] approximate the composite as a stack of unidirectional short-fiber reinforced laminae bonded together at different angles, which does not represent the real situation. In addition, these two theories do not provide any clear relationship between the composite strength and the properties of its constituents since they rely on the experimental failure strength and strain data of the unidirectional laminae. Friend has [18, 19] proposed an empirical strength equation for randomly-oriented short-fiber-reinforced metal matrix composites. Due to its empirical nature, his equation can only be used in particular alloy matrix composites. For example, it seems to agree with experimental data for some aluminum alloy matrix composites, but it can not explain the high strength of composites with a pure aluminum matrix.

Zhu *et al* [20-22] have developed models to predict the strength of composites reinforced with randomly or 3-D oriented short fibers. However, the statistical nature of short-fiber strength is not included in these models. Therefore, it is necessary to develop a new strength model to take into account both the 3-D fiber orientation and the statistical nature of fiber strength in the prediction of composite strength.

The objective of this paper is to develop a strength model for composites reinforced with 3-D oriented short fibers. The statistical nature of short fiber strength will be included in the calculation of composite strength. The maximum total load is adopted as the composite failure criterion. Special cases, such as the strengths of composites reinforced with unidirectionally-oriented short fibers, with two dimensionally (2-D) randomly-oriented short fibers, and with 3-D randomly-oriented short fibers, are also presented.

Modeling

The strengthening mechanisms in short-fiber reinforced metal- and polymer-matrix composites include several or all of the followings: direct short-fiber strengthening[20-22], residual thermal stress in fibers [22-24], and matrix work hardening induced by short-fiber dispersion [20, 22] and by thermal stress-induced dislocations [22-27]. The matrix work hardening has been worked out by Zhu *et al* [22], whose results will be used in the present model. A new method to calculate the direct short-fiber strengthening is developed below to take into account both the 3-D fiber orientation and the statistical nature of fiber strength, and the effect of residual thermal stress is also incorporated into the strength calculation.

Direct short-fiber strengthening

For simplicity, we assume an isotropic Poisson's ratio ν for the composite, and perfect bonding between fibers and the matrix. The short fiber strength can be characterized with a Weibull distribution function:

$$f(\sigma_f) = \alpha l \beta \sigma_f^{\beta-1} \exp(-\alpha l \sigma_f^{\beta}), \tag{1}$$

where α and β are parameters of Weibull distribution, l is fiber length and σ_f is fiber strength. The cumulative strength distribution function can be expressed as

$$F(\sigma_f) = \int_0^{\sigma} f(\sigma_f) d\sigma_f = 1 - \exp(-\alpha l \sigma_f^{\beta}). \tag{2}$$

Fibers with lower strength will start to break first during tensile loading. In addition, fibers with smaller inclination angles from the loading direction (see Fig. 1 for the definition of inclination angle) bear larger stresses and break first than fibers with the same strength, but larger inclination angles. Shown in Fig. 1 is a fiber with an inclination angle $0 \le \theta \le \pi/2$ in a composite sample. Under a total load P_c on a composite sample along the x_3 direction, the composite strain, ε_c, is produced in the loading direction:

$$\varepsilon_c = \varepsilon_{33}, \tag{3}$$

and strains in x_1 and x_2 directions can be calculated as:

$$\varepsilon_{11} = \varepsilon_{22} = -\nu\varepsilon_{33}. \tag{4}$$

To calculate the strain in a fiber with an inclination angle θ, let us rotate the coordination system around x_1 axis clockwise by an angle of θ (see Fig. 1). The transformation matrix A is

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}, \tag{5}$$

where $a_{ij} = \cos\alpha_{ij}$, and α_{ij} is the angle between y_i and x_j. The strain in y_3 direction (along the fiber) can be calculated as

$$\varepsilon_{33}^{y}(\theta) = \sum_{i=1}^{3} a_{3i} \sum_{j=1}^{3} a_{3j}\varepsilon_{ij} = \varepsilon_{33}\left(\cos^2\theta - \nu\sin^2\theta\right). \tag{6}$$

Substituting Eq. 3 into Eq. 6 yields

$$\varepsilon_{33}^{y}(\theta) = \varepsilon_c\left(\cos^2\theta - \nu\sin^2\theta\right). \tag{7}$$

$\varepsilon_{33}^{y}(\theta)$ calculated using Eq. 7 is the strain in a fiber with an inclination angle θ. The stress in the fiber can be calculated as

$$\sigma(\theta) = E_f\varepsilon_{33}^{y}(\theta) = E_f\varepsilon_c\left(\cos^2\theta - \nu\sin^2\theta\right), \tag{8}$$

where E_f is the fiber Young's modulus. Due to the Poisson contraction, fibers with inclination angle close to π/2 will be under compressive stress although the composite sample is under tensile load in the x_1 direction. The angle at which σ(θ) changes from positive to negative can be found by setting σ(θ) =0 and solving for θ, which yields

$$\theta_f = \arcsin\left(\frac{1}{\sqrt{1+\upsilon}}\right), \tag{9}$$

From equation (8), it can be seen that $\sigma(\theta)$ is positive if $\theta < \theta_f$, which means tensile stress in the fiber. But, if θ is larger than θ_f, $\sigma(\theta)$ will be negative due to the Poisson contraction. Since fibers usually have higher Young's modulus than the matrix, fibers should always have higher resistance to elastic deformation than the matrix. Therefore, the absolute value of $\sigma(\theta)$ should be used in the calculation of total load carried by fibers toward the loading direction, i.e.

$$\sigma(\theta) = \begin{cases} E_f \varepsilon_c\left(\cos^2\theta - \nu\sin^2\theta\right) & 0 \le \theta < \theta_f \\ -E_f \varepsilon_c\left(\cos^2\theta - \nu\sin^2\theta\right) & \theta_f \le \theta < \pi/2 \end{cases}. \tag{10}$$

Assuming that fibers with a strength $\sigma_f = \sigma_0 = E_f \varepsilon_c$ and parallel to the loading direction begin to break under a total load P, then

$$\varepsilon_c = \sigma_0 / E_f. \tag{11}$$

Substituting Eq. 11 into Eq. 10 yields

$$\sigma(\theta) = \begin{cases} \sigma_0\left(\cos^2\theta - \nu\sin^2\theta\right) & 0 \le \theta < \theta_f \\ -\sigma_0\left(\cos^2\theta - \nu\sin^2\theta\right) & \theta_f \le \theta < \pi/2 \end{cases}. \tag{12}$$

Fibers with a strength $\sigma_f < \sigma_0$ and a small inclination angle will break. The critical inclination angle, within which every fiber with a strength $\sigma_f < \sigma_0$ is broken, can be derived by setting

$$\sigma_f = \sigma_0\left(\cos^2\theta - \nu\sin^2\theta\right),$$

and solving for θ, which yields

$$\theta_c\left(\sigma_f\right) = \arcsin\sqrt{\frac{\sigma_0 - \sigma_f}{(1+\nu)\sigma_0}}. \tag{13}$$

The next step is to derive the total load, $P_f(\sigma_0)$, carried by all the remaining short fibers as a function of σ_0 at a specimen cross-section, and to find the maximum value of the total load. The maximum of $P_f(\sigma_0)$ can be considered as the total load that short fibers carry at composite failure and can be used to calculate the direct fiber strengthening.

To obtain $P_f(\sigma_0)$ at a specimen cross-section perpendicular to the loading direction (hereafter referred as cross-section A as indicated in Fig. 2), the effective orientation-density distribution of fibers intercepted by the cross-section A, $n_c(\theta)$, is needed. Defining the fiber orientation-density distribution in the volume of the specimen as $n_v(\theta)$, we can obtain $n_c(\theta)$ from $n_v(\theta)$ by taking into account the following two factors: first, the probability variation of a fiber being intercepted by the cross-section A with the inclination angle of a fiber; second, the ineffective length of short fibers. $n_v(\theta)$ can be expressed as

$$n_v(\theta) = Ng(\theta), \tag{14}$$

where N is the total number of short fibers in the composite specimen and $g(\theta)$ is the normalized fiber-orientation distribution, which can be determined using image analysis [12, 28].

The effective load-carrying length of a short fiber can be expressed as

$$l_e = \bar{l} - 2\delta, \tag{15}$$

where $\bar{l}$ is the average fiber length, which also can be obtained from image analysis [28], and δ is the equivalent non load-carrying length at each end of the short fiber, which can be obtained by a shear-lag-type analysis [22, 29]:

$$\delta = \frac{\bar{d}}{2}\left[\left(V_f^{-1/2} - 1\right)E_f/G_m\right]^{1/2}, \tag{16}$$

where $\bar{d}$ is the average fiber diameter, which is known before the fabrication of a composite, or can be determined from image analysis [28], G_m is the shear modulus of the matrix, and V_f is the fiber volume fraction.

The projected effective fiber length on the loading direction can be calculated as

$$l_p(\theta) = l_e \cos\theta = \left[\bar{l} - 2\delta\right]\cos\theta. \tag{17}$$

The total number of fibers in a composite specimen, N, can be calculated by the following equation

$$N = \frac{LAV_f}{\bar{l}\bar{a}_f}, \tag{18}$$

where A is the sample cross-section area and $\bar{a}_f = \pi\bar{d}^2/4$. $n_c(\theta)$ can be calculated from $n_v(\theta)$ and $l_p(\theta)$ as

$$n_c(\theta) = n_v(\theta) l_p(\theta)/L, \tag{19}$$

where L is the composite specimen length. Substituting Eqs. 14 and 16-18 into Eq. 19 yields

$$n_c(\theta)=\frac{V_f A}{\bar{a}_f}\left[1-\frac{\bar{d}}{\bar{l}}\sqrt{\left(V_f^{-1/2}-1\right)E_f/G_m}\right]g(\theta)\cos\theta. \tag{20}$$

Knowing $n_c(\theta)$, we are ready to calculate $P_f(\sigma_0)$, which consists of two parts:

$$P_f(\sigma_0)=P_f^1(\sigma_0)+P_f^2(\sigma_0), \tag{21}$$

where $P_f^1(\sigma_0)$ is the total load carried by all unbroken short fibers with $\sigma_f < \sigma_0$, and $P_f^2(\sigma_0)$ is the total load carried by all unbroken short fibers with $\sigma_f \geq \sigma_0$. $P_f^1(\sigma_0)$ can be calculated as

$$P_f^1(\sigma_0)=\int_0^{\sigma_0}\int_{\theta_c(\sigma_f)}^{\pi/2} n_c(\theta)f(\sigma_f)a_f\sigma(\theta)\cos\theta d\theta d\sigma_f, \tag{22}$$

and $P_f^2(\sigma_0)$ can be calculated as

$$P_f^2(\sigma_0)=\left[1-F(\sigma_0)\right]\int_0^{\pi/2} n_c(\theta)a_f\sigma(\theta)\cos\theta d\theta. \tag{23}$$

Substituting Eqs. 22 and 23 into Eq. 21 yields

$$\begin{aligned}P_f(\sigma_0)=&\int_0^{\sigma_0}\int_{\theta_c(\sigma_f)}^{\pi/2} n_c(\theta)f(\sigma_f)a_f\sigma(\theta)\cos\theta d\theta d\sigma_f\\&+\left[1-F(\sigma_0)\right]\int_0^{\pi/2} n_c(\theta)a_f\sigma(\theta)\cos\theta d\theta\end{aligned}. \tag{24}$$

Substituting Eqs. 12 and 20 into Eq. 24, integrating and rearranging yields

$$P_f(\sigma_0)=V_f A\eta\sigma_0\left\{\int_0^{\sigma_0} f(\sigma_f)d\sigma_f\int_{\theta_c(\sigma_f)}^{\theta_f} h(\theta)d\theta+\left[1-F(\sigma_0)\right]\int_0^{\theta_f} h(\theta)d\theta+\int_{\pi/2}^{\theta_f} h(\theta)d\theta\right\}, \tag{25}$$

where

$$\eta = 1 - \frac{\bar{d}}{\bar{l}}\sqrt{\left(V_f^{-1/2} - 1\right)E_f/G_m}\,, \tag{26}$$

and

$$h(\theta) = g(\theta)\left[(1+\nu)\cos^2\theta - \nu\right]\cos^2\theta. \tag{27}$$

Substituting Eqs. 1 and 2 into Eq. 25 yields

$$\begin{aligned} P_f(\sigma_0) = V_f A\eta\sigma_0 \Big[& \int_0^{\sigma_0} \alpha l\beta\sigma_f^{\beta-1}\exp\left(-\alpha l\sigma_f^\beta\right)d\sigma_f \int_{\theta_c(\sigma_f)}^{\theta_f} h(\theta)d\theta \\ & + \exp\left(-\alpha l\sigma_f^\beta\right)\int_0^{\theta_f} h(\theta)d\theta + \int_{\pi/2}^{\theta_f} h(\theta)d\theta \Big] \end{aligned}. \tag{28}$$

The direct short-fiber strengthening can be calculated as

$$\begin{aligned} \sigma_c^f = \frac{\left[P_f(\sigma_0)\right]_{\max}}{A} = V_f\eta\Big[& \sigma_0\int_0^{\sigma_0} \alpha l\beta\sigma_f^{\beta-1}\exp\left(-\alpha l\sigma_f^\beta\right)d\sigma_f \int_{\theta_c(\sigma_f)}^{\theta_f} h(\theta)d\theta \\ & +\sigma_0\exp\left(-\alpha l\sigma_f^\beta\right)\int_0^{\theta_f} h(\theta)d\theta + \sigma_0\int_{\pi/2}^{\theta_f} h(\theta)d\theta \Big]_{\max} \end{aligned}. \tag{29}$$

For composites reinforced with unidirectional short fibers, g(θ) is a delta function at θ = 0:

$$g(\theta) = \begin{cases} 1 & \theta = 0 \\ 0 & \theta \neq 0 \end{cases}. \tag{30}$$

Substituting Eq. 30 into Eq. 28 and integrating yields

$$P_f(\sigma_0) = V_f A \eta \sigma_0 \exp\left(-\alpha l \sigma_0^{\beta}\right). \tag{31}$$

Setting $dP_f(\sigma_0)/d\sigma_0 = 0$ and solve it for σ_0, which is the value at which $P_f(\sigma_0)$ reaches its maximum, yields:

$$\sigma_0 = (\alpha l \beta)^{-1/\beta}. \tag{32}$$

Substituting Eq. 32 into Eq. 31 yields

$$\left[P_f(\sigma_0)\right]_{\max} = A V_f \eta (\alpha \beta l e)^{-1/\beta}. \tag{33}$$

The total direct short-fiber strengthening can be calculated by substituting Eq. 33 into Eq. 29, which yields

$$\sigma_c^f = V_f \eta (\alpha \beta l e)^{-1/\beta} \tag{34}$$

where e is the base of the natural logarithm.

For composites reinforced with 2-D randomly-oriented short fibers, the fiber-orientation distribution can be expressed as

$$g(\theta) = \frac{2}{\pi}. \tag{35}$$

Substituting Eq. 35 into Eq. 29, integrating and rearranging yield the total direct short-fiber strengthening as

$$\sigma_c^f = \frac{V_f \eta}{8\pi}\left[B(\nu)\sigma_0 - 16\sigma_0 \int_0^{\sigma_0} f(\sigma_f) C(\sigma_0,\ \sigma_f) d\sigma_f\right]_{\max}, \tag{36}$$

where

$$B(\nu) = \frac{4\left(3\sqrt{\nu} + \nu^{3/2}\right)}{1+\nu} + (3-\nu)\left(4\arcsin\frac{1}{\sqrt{1+\nu}} - \pi\right), \tag{37}$$

and

$$C(\sigma_0,\ \sigma_f) = \frac{3-\nu}{8}\arcsin\sqrt{\frac{\sigma_0 - \sigma_f}{(1+\nu)\sigma_0}} + \frac{\sqrt{(\sigma_0 - \sigma_f)(\nu\sigma_0 + \sigma_f)}\left[(3+\nu)\sigma_0 + 2\sigma_f\right]}{8(1+\nu)\sigma_0^2}. \tag{38}$$

For composites reinforced with 3-D randomly-oriented short fibers, the fiber-orientation distribution can be expressed as [20]:

$$g(\theta) = \sin\theta. \tag{39}$$

Substituting Eq. 39 into Eq. 29, integrating and rearranging yield the total direct short-fiber strengthening as

$$\begin{aligned} \sigma_c^f = \frac{V_f \eta}{15}\Bigg[&(3-2\nu)\sigma_0 \exp\left(-\alpha l \sigma_0^{\beta}\right) + \frac{4\nu^{5/2}\sigma_0}{(1+\nu)^{3/2}} \\ &+ \sigma_0 \int_0^{\sigma_0} f(\sigma_f) \frac{\left(\nu\sigma_0 + \sigma_f\right)^{3/2}\left(3\sigma_f - 2\nu\sigma_0\right)}{(1+\nu)^{3/2}\sigma_0^{5/2}} d\sigma_f\Bigg]_{\max} \end{aligned} \tag{40}$$

Effect of residual thermal stress

Metal-matrix and some polymer-matrix composites are generally synthesized at high or intermediate temperatures, which result in residual thermal stress in fibers during the cooling from the composite synthesis temperature. Assuming that the residual thermal stress in short fibers is σ_t, this stress changes the apparent fiber strength, and makes positive contribution to the tensile strength of a composite if $\sigma_t < 0$ (compressive stress) but negative contribution if $\sigma_t > 0$ (tensile stress). The effect of residual thermal stress is equivalent to changing the average fiber stress by $-\sigma_t$ without changing the scattering of the fiber strength. Among the two Weibull parameters of fiber strength, α is related to the average fiber strength and β is related to the strength scattering. Therefore, we can take into account the effect of residual thermal stress in fibers on composite strength by incorporating it into α.

The average fiber strength for the fiber strength distribution shown in Eq. 1 can be expressed as:

$$\bar{\sigma} = \alpha^{-1/\beta} l^{-1/\beta} \Gamma\left(1 + \frac{1}{\beta}\right). \tag{41}$$

where $\Gamma(1 + 1/\beta)$ is a gamma function. Taking into account the effect of residual thermal stress in fibers, Eq. 41 can be written as:

$$\bar{\sigma} - \sigma_t = \alpha_c^{-1/\beta} l^{-1/\beta} \Gamma\left(1 + \frac{1}{\beta}\right), \tag{42}$$

where α_c is the modified α. α_c can be obtained by substituting Eq. 41 into Eq. 42 and rearranging:

$$\alpha_c = \left[\alpha^{-1/\beta} - l^{1/\beta}\sigma_t / \Gamma(1+1/\beta)\right]^{-\beta}. \tag{43}$$

Taking into account the effect of residual thermal stress in fibers, the total direct fiber strengthening, σ_{ct}^f, can be calculated by substituting α in Eq. 29 with α_c defined in Eq. 43, which yields

$$\sigma_{ct}^f = \frac{\left[P_f(\sigma_0)\right]_{\max}}{A} = V_f\eta\left[\sigma_0\int_0^{\sigma_0}\alpha_c l\beta\sigma_f^{\beta-1}\exp\left(-\alpha_c l\sigma_f^\beta\right)d\sigma_f\int_{\theta_c(\sigma_f)}^{\theta_f}h(\theta)d\theta\right.$$
$$\left.+\sigma_0\exp\left(-\alpha_c l\sigma_f^\beta\right)\int_0^{\theta_f}h(\theta)d\theta + \sigma_0\int_{\pi/2}^{\theta_f}h(\theta)d\theta\right]_{\max} . \tag{44}$$

Correspondingly, σ_{ct}^f for unidirectional short-fiber composites can be expressed as

$$\sigma_{ct}^f = V_f\eta\left(\alpha_c\beta le\right)^{-1/\beta}, \tag{45}$$

σ_{ct}^f for 2-D randomly-oriented short-fiber composites can be expressed as

$$\sigma_{ct}^f = \frac{V_f\eta}{8\pi}\left[B(\nu)\sigma_0 - 16\sigma_0\int_0^{\sigma_0}\alpha_c l\beta\sigma_f^{\beta-1}\exp\left(-\alpha_c l\sigma_f^\beta\right)C\left(\sigma_0,\ \sigma_f\right)d\sigma_f\right]_{\max}, \tag{46}$$

where $B(\nu)$ and $C(\sigma_0, \sigma_f)$ are same as defined in Eqs. 37 and 38, and σ_{ct}^f for 3-D randomly oriented short-fiber composites can be expressed as

$$\sigma_{ct}^f = \frac{V_f\eta}{15}\left[(3-2\nu)\sigma_0\exp\left(-\alpha_c l\sigma_0^\beta\right) + \frac{4\nu^{5/2}\sigma_0}{(1+\nu)^{3/2}}\right.$$
$$\left.+\sigma_0\int_0^{\sigma_0}\alpha_c l\beta\sigma_f^{\beta-1}\exp\left(-\alpha_c l\sigma_f^\beta\right)\frac{\left(\nu\sigma_0+\sigma_f\right)^{3/2}\left(3\sigma_f - 2\nu\sigma_0\right)}{(1+\nu)^{3/2}\sigma_0^{5/2}}d\sigma_f\right]_{\max} . \tag{47}$$

Matrix work hardening

For metal-matrix/short-fiber composites, the matrix may be strengthened by high density thermal stress-induced dislocations, which can be calculated as [22]:

$$\Delta\sigma_{m1} = 2\alpha G_m b \rho_t^{1/2}, \tag{48}$$

where $\alpha = 0.3$ - 0.5 is a constant, G_m is the shear modulus of the matrix, b is the Burgers vector and ρ_t is the density of thermal stress-induced dislocations. Another matrix strengthening mechanism is short-fiber dispersion hardening, which can be calculated as [20, 22]

$$\Delta\sigma_{m2} = 4G_m b\left\{\bar{d}\left(\frac{\pi}{V_f}\right)^{1/6}\left[\left(\frac{\pi}{V_f}\right)^{1/3} - 6^{1/3}\right]\right\}^{-1}. \tag{49}$$

Composite strength calculation

All the strengthening mechanisms discussed above can be incorporated into the calculation of composite strength:

$$\sigma_c = \left(1 - V_f\right)\left(\sigma_m + \Delta\sigma_{m1} + \Delta\sigma_{m2}\right) + \sigma_{ct}^f, \tag{50}$$

where σ_m is the calculated matrix stress at composite failure without the consideration of matrix strengthening by thermal stress-induced dislocations and by dispersion hardening.

Summary and Conclusions

Although comparison of the present composite strength model with experimental results has not been made due to the lack of data, we believe that it is superior to previous models because it takes into account more physical parameters, such as the statistical nature of fiber strength, in the calculation of short-fiber composite strength. Some of the data needed for calculating composite strength, such as the fiber orientation-distribution function $g(\theta)$, average fiber length $\bar{l}$, average fiber diameter $\bar{d}$ and fiber volume fraction V_f, can either be obtained by image analysis [12], or is known before the fabrication of a composite; some other data, such as metal matrix dispersion hardening $\Delta\sigma_{m2}$, matrix shear modulus G_m, residual thermal stress σ_t, dislocation density ρ , and the Weibull distribution parameters for short-fiber strength may be estimated or experimentally determined. However, σ_t and ρ might often need to be estimated because of the experimental difficulty in their determination.

The statistical model developed in this paper for calculating the tensile strength of 3-D oriented short-fiber composites improves upon previous strength models. A maximum load criterion is used for composite failure, which is straight forward and easy to use. The present model can take into account the statistical nature of fiber strength in the calculation of composite strength. The residual thermal stress in fibers is also taken into account in the calculation of composite strength by incorporating it into the Weibull distribution parameter α. We believe that the present model can give a more accurate and realistic estimate of short-fiber composite strength.

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REFERENCES

1. P. E. Chen, Polymer Engineering and Science, 11 (1971), 51-6.
2. P. K. Liaw, J. G. Greggi, and W. A. Logsdon, J. Materials Science, 22 (1987), 1613-7.
3. S. K. Gaggar and L. J. Broutman, Polymer Engineering and Science, 16 (1976), 537-43.
4. L. A. Goetler, Concise Encyclopedia of Composite Materials, revised edition, A. Kelly, Ed. (Elsevier Science, Ltd., Oxford, England, 1994), p. 90.
5. J. V. Milewski, Concise Encyclopedia of Composite Materials, Revised Edition, A. Kelly, Ed. (Elsevier Science Ltd., Oxford, England, 1994), p. 313.
6. Y. Takao, Recent Advances in composites in the United States and Japan, ASTM STP 864, J. R. Vinson and M. Taya, Eds. (American Society for Testing and Materials, Philadelphia, 1985), pp. 685-99.
7. T. F. Klimowicz, Journal of Metals, November 1994, pp. 49-53.
8. M. M. Schwartz, Composite Materials Handbook, 2nd ed. (McGraw-Hill, Inc., New York, 1992), Chapter 7, pp. 54-61.
9. H. Hino, M. Komatsu, Y. Hirasawa, and M. Sasaki, Design and Manufacturing of Advanced Composites, Proceedings of the Fifth Annual ASM/ESD Advanced

Composite Conference, Dearborn, Michigan, 25-28 Sept., 1989, ASM, 1989, pp. 201-8.

10. M. Sasaki, H. Hino, and M. Komatsu, How To Apply Advanced Composites Technology, Proceedings of the Fourth Annual ASM/ESD Advanced Composite Conference, Dearborn, Michigan, 13-15 Sept., 1988, ASM international, 1988, pp. 484-79.

11. D. J. Bay, Composite Applications, T. J. Drozda, Ed. (Society of Manufacture Engineers, Dearborn, Michigan, 1989), pp. 3-23.

12. Y. T. Zhu, W. R. Blumenthal and T. C. Lowe, "Determination of non-symmetric 3-D fiber orientation and average fiber length in short-fiber composites," submitted to Acta Metall. Mater.

13. K. K. Phani, J. Materials Science, 23 (1988) 2424-2428.

14. Y. T. Zhu, B. L. Zhou, G. H. He and Z. G. Zheng, J. Composite Materials, 23 (1989) 280-287.

15. Y. T. Zhu and G. Zong, J. Composite Materials, 27 (1993) 944 - 959.

16. J. B. Jones, J. B. Barr and R. E. Smith, J. Composite Materials, 15 (1980) 2455-2465.

17. J. C. Halpin and J. L. Kardos, Polym. Eng. Sci., 18 (1978), 496.

18. C. M. Friend, J. Materials Science, 22 (1987), 3005.

19. C. M. Friend, Scripta Metall., 23 (1989), 33.

20. Y. T. Zhu, G Zong, A. Manthiram and Z. Eliezer, J. Materials Science, 29 (1994), 6281.

21. Y. T. Zhu and W. R. Blumenthal, "Direct Fiber Strengthening in Three Dimensional Random-Oriented Short-Fiber Composites," Proceedings of Micromechanics of Advanced Materials Symposium, TMS/ASM Materials Week 1995, Cleveland, OH, Oct. 29-Nov. 2, 1995 (in press).

22. Y. T. Zhu, W. R. Blumenthal, and T. C. Lowe, "On the Tensile Strength of Short-Fiber Reinforced Composites," submitted to Acta Metall. Mater.

23. R. W. Hertzberg, Deformation and fracture mechanics of Engineering Materials, p. 136, John Wiley & Sons, New York (1989).

24. R. J. Arsenault and R. M. Fisher, Scripta Metall., 17 (1983), 67.

25. N. Shi, B. Wilner, and R. J. Arsenault, Acta Metall. Mater., 40 (1992), 2841.

26. R. J. Arsenault and N. Shi, Materials Science and Engineering, 81 (1986), 175.

27. L. F. Smith, A. D. Krawitz, P. Clarke, S. Saimoto, N. Shi, and R. J. Arsenault, Materials Science and Engineering, A159 (1992), L13.

28. G. Fisher and P. Eyere, Polymer Composites, 9 (1988), 297.

Figure captions

Fig. 1. Definition of off-axis angle θ.

Fig. 2. A composite sample and its cross-section.

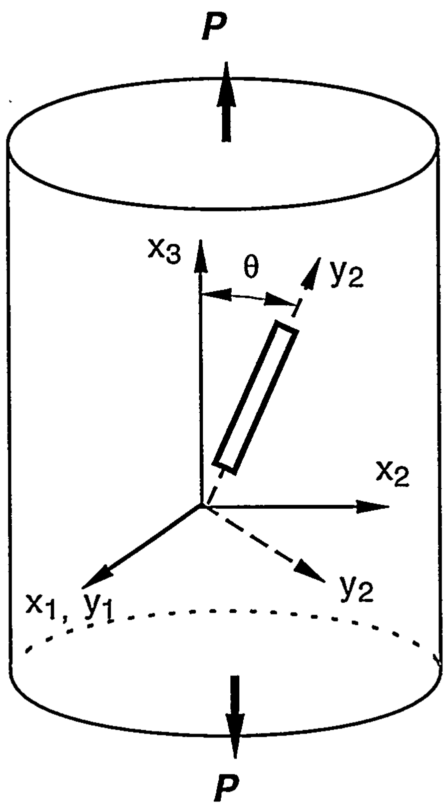

Fig. 1

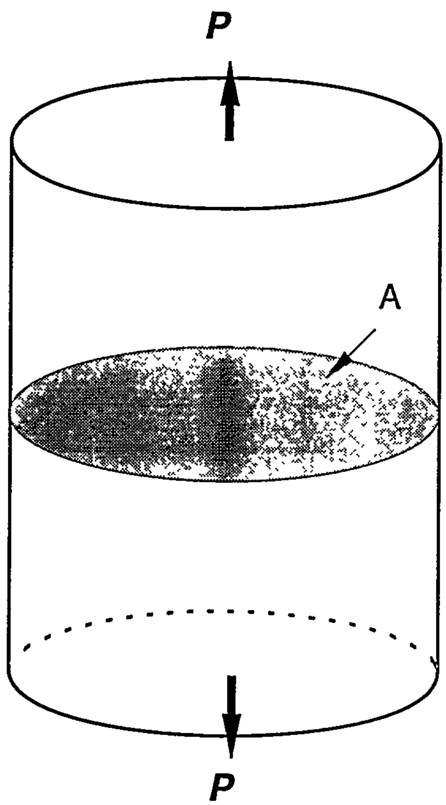

Fig. 2