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SOLUTION OF POTENTIAL FLOW PROBLEMS ON A NETWORK OF WORKSTATIONS

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Solution of Potential Flow Problems on a Network of Workstations

Bart D. Semeraro*

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Abstract

This paper deals with the solution of potential flow problems via a panel or boundary element method on a network of scientific workstations. The idea is to treat the workstation network as a relatively course grained parallel computer and to distribute the work of constructing the influence coefficient matrix over the network. We make use of the Parallel Virtual Machine (PVM) software developed at the Oak Ridge National Laboratory to do the interprocess communications. We specifically address the problem of how to distribute the data across the network as well as the implementation of the parallel linear system solver.

1 Introduction

This draft contains preliminary results of investigations into parallel algorithms for the solution of potential flow problems via panel or boundary element methods. The panel method used here is based on the work of Hess and Smith [1]. These methods consist of two primary subproblems. The first is the construction of a large dense linear system of equations. The second is the solution of the related linear system. This work describes methods for parallelization of both problems.

A major objective of this work is to demonstrate the potential advantages of using distributed parallel computing for engineering design and analysis. This approach is attractive primarily because it allows the engineer to take advantage of the increased speed and memory capacity of parallel computers without the large cost of purchasing new hardware. There are several software packages available that allow one to treat a network of workstations as a parallel computer. We chose the Parallel Virtual Machine software developed at the

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Oak Ridge National Laboratory because it is simple to use, readily available and free public domain software. The reader is referred to papers by Geist and Sunderam [4] as well as Beguelin et al. [5] for more information on PVM.

We briefly describe the panel method of Hess in section 2. This section contains information pertinent to the development of the parallel algorithm. For a more detailed description of panel methods in aerodynamics see Hess and Smith [1] or Hess [2,3]. Section 3 describes the domain decomposition and the communication aspects of the parallel algorithm. The final section presents algorithm performance and results of a representative calculation.

2 Panel Methods

The panel method is a particular technique for calculating potential flow about two or three dimensional bodies. Potential flow theory is well developed. We give here an outline of the theory and the panel method of solution. The text by Karamcheti [6] as well as the text by Kuethe and Chow [7] contain good reference material on potential flow.

Consider the flow of an incompressible inviscid fluid in a region Ω exterior to a surface Γ. Since the flow is irrotational the velocity may be expressed as the gradient of a potential function, ϕ. Furthermore, the potential satisfies Laplace's equation in Ω because the continuity equation dictates a divergence free velocity. The boundary condition of zero normal flow gives the condition that the normal gradient of the potential is equal to the negative of the normal component of the free stream velocity at the surface. Finally the regularity condition of free stream velocity at infinity is imposed. Thus the problem of determining the velocity field is reduced to finding a related scalar field that satisfies Laplace's equation in Ω with appropriate boundary conditions. This problem can be formulated as an integral equation over the boundary. It is this formulation upon which the panel method is based.

It can be shown that the potential at any point in Ω can be expressed as the integral of a surface distribution of sources,

$$\phi = \int_s \frac{\sigma}{4\pi r} ds, \tag{1}$$

where, σ is the source strength density per unit area and r is the distance from the surface element to the point at which the potential is to be determined. In our implementation the surface Γ is represented by a collection of flat quadrilateral or triangular panels. The source density, σ, is assumed to be constant on each panel. The potential induced at a point by the source distribution of a single panel can be found by analytically evaluating (1) over a single panel. The integral (1), is evaluated over the entire surface by summing the influence of each panel. The process we have just described is mathematically equivalent to an inner product of influence coefficients and source densities. By placing a

control point at the centroid of each panel and imposing the boundary condition of no normal flow at that point the source densities σ can be determined by solving the resulting linear system of equations.

$$A\sigma = b \tag{2}$$

Each element of the coefficient matrix A, $a_{i,j}$, represents the normal component of the velocity induced at the control point of panel i by a unit source distribution on panel j. This number is arrived at by taking the inner product of the unit normal at the control point with the gradient of the analytical expression for the induced potential. The analytical expression for this induced velocity is quite complex and involves the evaluation of natural logarithms and inverse tangents. For a body with n panels there are n^2 such evaluations. Assembling the coefficient matrix is an expensive part of the calculation.

Once the source densities have been determined the velocity at each panel control point can be determined by a matrix vector multiply. The pressure coefficient is then directly determined from the velocity on the surface. This calculation is analogous to the exercise in determining the normal component of the velocity. We retain the coefficient matrix of velocity influence coefficients in order to use it to determine the tangential velocity once the source density is determined.

The direct solution of a dense linear system of equations of order n requires $O(n^3)$ operations. The cost of solving the linear system can quickly overwhelm the cost of constructing it. For this reason iterative methods are frequently used [8]. We implemented a version of the GMRES [9] procedure for solving nonsymmetric linear systems of equations.

3 Parallel Implementation

In this work our parallel virtual computer consists of a network of several IBM RS/6000 workstations. The PVM software makes no restriction on the type of workstations or network communications used to make up a virtual computer. There is some heterogeneity in our system in that it is made up of different models of the RS/6000. Many of the machines also have different memory sizes. Future work will involve a more heterogeneous system consisting of Sun workstations in addition to the IBMs.

We begin our discussion of the parallel implementation with the assembly of the coefficient matrix. We deal with four matrices of order n. The first three are the x, y, and z velocity influence matrices. The fourth is the normal velocity matrix and is constructed from the other three. Saving the three component matrices eliminates the need to recalculate the elements when they are needed to find the surface velocity. The evaluation of each integral over the surface of the object requires knowledge of the relative geometry of the particular panel

and control point. Details of the evaluation of a single integral are given in Hess and Smith [1].

The matrices represent the largest data item in the program. The amount of data related to the geometry of the problem, panel corner and control points for example, is relatively small. We chose to decompose the problem by breaking up the matrices by columns. Each process constructs a subset of the columns of the influence matrices. The columns of each matrix are distributed evenly among the processes. In addition each process has all of the geometry data. The program begins by a single process broadcasting the geometry data to all the other processes. The construction of the matrix elements then proceeds in parallel with no communication.

The next phase of the computation involves solving the linear system for the unknown source strengths σ. One can use either direct or iterative solvers for the solution of the linear system. We present results for a parallel version of the GMRES algorithm in this draft. The final paper will contain a comparison of parallel iterative and direct solution methods. The direct solver used is part of the ScaLAPACK [10] library. Details of the GMRES algorithm can be found in Saad and Schultz [9]. We chose to parallelize the algorithm in the simplest way possible. We parallelize only the matrix vector multiply. This is a valid first step since the majority of the work in the GMRES algorithm in this phase. In addition, the calculation of the surface velocities is also essentially a matrix vector product and can be done in parallel also.

Each process contains a local copy of the basis vectors, the upper Hessenberg system and the current solution approximation. Some work is duplicated at each iteration, for example the inner products are not distributed but are done locally. A matrix vector product can be considered as a partitioned multiply of the form:

$$(A_1, A_2, \ldots, A_p) \begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \vdots \\ \sigma_p \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_p \end{pmatrix}.$$

Each A_i contains n/p columns of the matrix A. Each process has a copy of all the σs and all the bs. The first step in the matrix vector multiply is the multiplication $A_i\sigma_i$ in parallel in each process. The result is communicated to a single process and assembled into the vector b which is then broadcast to all the processes. Each matrix vector multiply involves 2 communications between each process and the single process that assembles the b vector.

4 Performance

We describe the performance of the parallel algorithm in this section. Our test case is a swept wing geometry shown in figure (2). The test case consists of 800

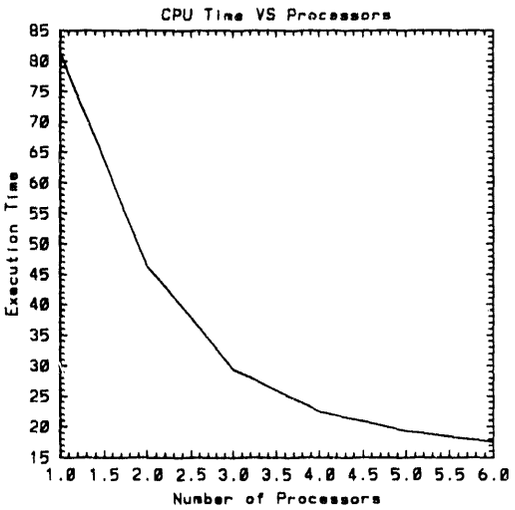

Figure 1: Parallel Performance

quadrilateral panels. This problem required 81.4 seconds to solve on a single IBM RS/6000 530 workstation. A plot of cpu time as a function of the number of processes is shown in figure (1). Notice that the time required to solve the problem is significantly reduced. The curve flattenes out because as we add processes we decrease the work done in each process and increase the amount of communication for a solution. Eventually the communication dominates the calculation. We expect the curve to flatten farther to the right as the problem size increases.

5 Conclusion

We have outlined a parallel algorithm for solving potential flow problems via a panel method. The algorithm was implemented on a network of IBM workstations. The parallel algorithm shows substantial speed increase over the sequential algorithm. Network computing can provide engineers with cheap parallel performance for this and many other types of applications.

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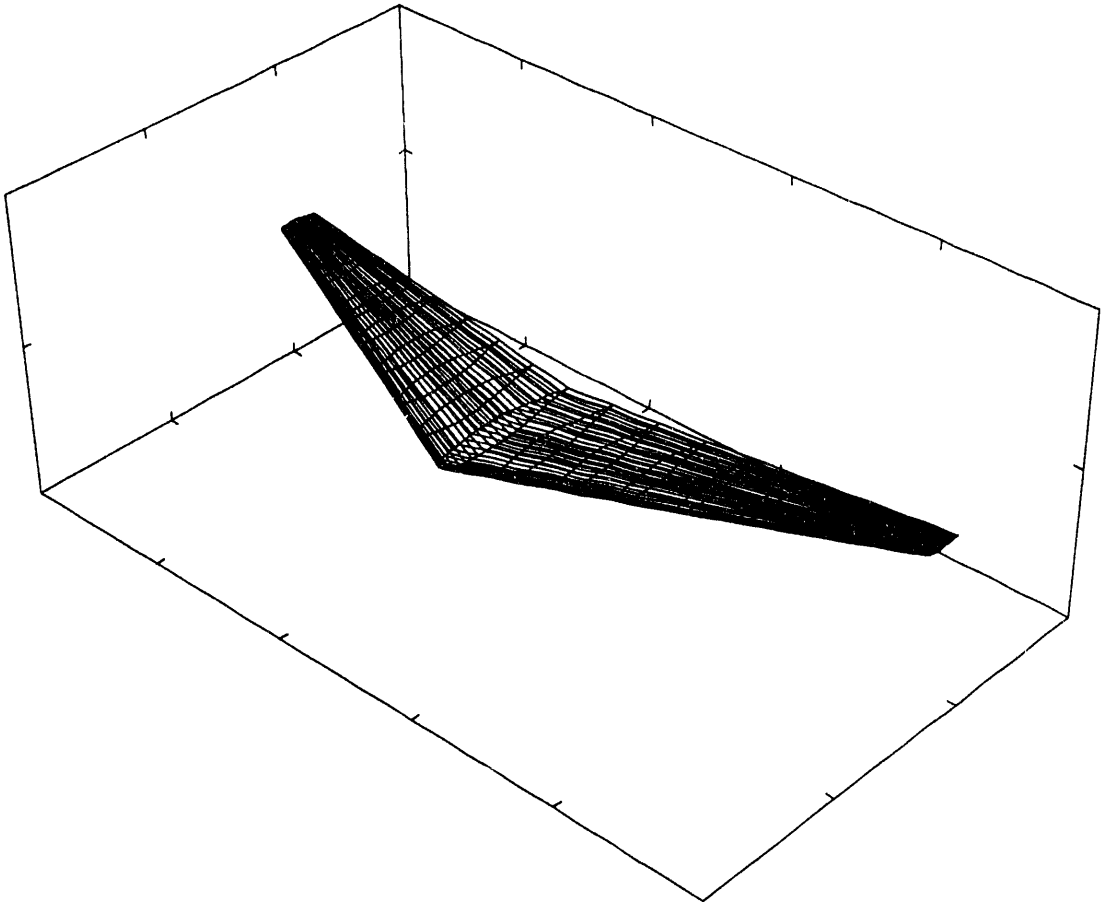

Figure 2: Swept Wing

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